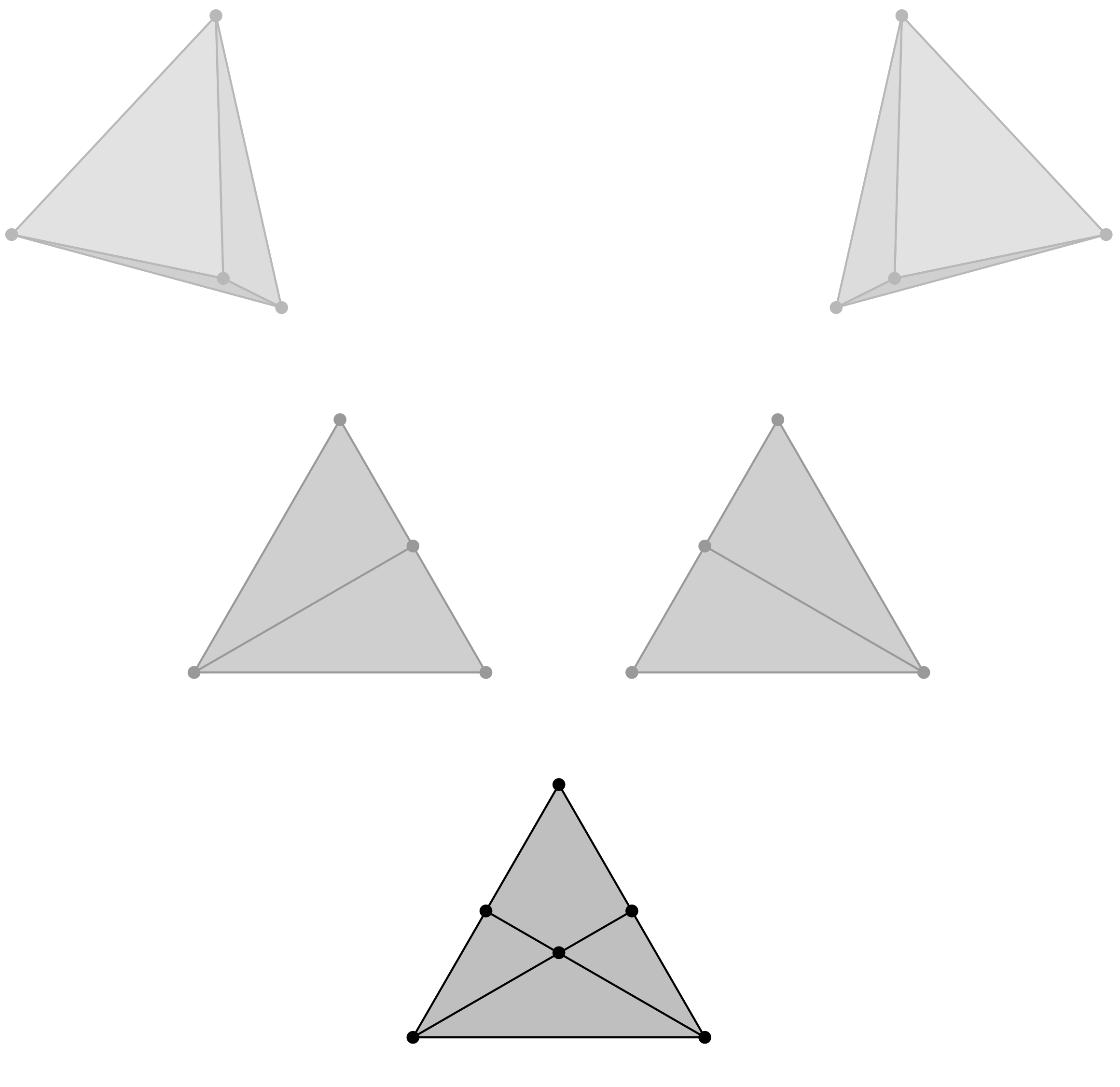
Thorsten Bonato

An algorithmic approach to Tilting Complexes



An algorithmic approach to Tilting complexes

Diplomarbeit von

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Für meine Eltern

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Preface

In 1993 Denis Naddef and Giovanni Rinaldi released their paper *"The graphical relaxation: A new framework for the Symmetric Traveling Salesman Polytope"* (see [NR93]) emphasizing the close connection between the *Graphical Traveling Salesman Polyhedron* GTSP(n) and the *Symmetric Traveling Salesman Polytope* STSP(n), which is a face of GTSP(n). Furthermore, they proved that each non-trivial facet of STSP(n) induces a facet of GTSP(n), but left the question of the converse direction unanswered. The assumption that the converse direction was also true, which will be referred to as *Graphical-Relaxation-conjecture* or *GR-conjecture*, could neither be proven nor disproven for more than ten years.

Until 2005, when Marcus Oswald, Gerhard Reinelt and Dirk Oliver Theis released their paper *"Not Every GTSP Facet Induces an STSP Facet"* (see [ORT05]). As the title indicates, the GR-conjecture turned out to be false. The authors proved that for $n \geq 9$ the TT-type facets of GTSP(n) decompose into two non-empty classes: the *Naddef-Rinaldi-* or *NR-facets* that fulfill the GR-conjecture and the *non-NR facets* that do not. As usual, this paper mainly featured the basic ideas and core statements.

Shortly after, Dirk Oliver Theis released his PhD thesis *"Polyhedra and algorithms for the General Routing Problem"* (see [The05]) containing a detailed description of the whole construction and covering the underlying theory. It was in this thesis that the concept of the *tilting complex* was introduced. The only necessary input for the construction of the tilting complex $\mathcal{T}(F)$ for a given good face F of STSP(n) are all the facets of STSP(n) containing F. The resulting tilting complex provides information about the TT-type faces of GTSP(n) containing this good face. In other words, by providing local data of STSP(n) we get information about parts of the facial structure of GTSP(n), which explains the relevance of tilting complexes.

The aim of this diploma thesis is the implementation of an algorithm for the computation and visualization of tilting complexes. Furthermore, we will provide an algorithm for the computation of the STSP(n) facets containing a face F that will work for arbitrary n. Currently, there is only an implementation that browses a complete description of STSP(n) in search of the facets in question. The major drawbacks of this method are the running time and the fact that up to now there are only proven complete descriptions for $n \leq 9$. The one for $n = 10$ is only conjectured to be complete, hence the algorithm may miss certain facets, and in the case of $n > 10$ there are no descriptions at all. The new algorithm also improves the running time. To give some numbers: for $n = 10$ the computation now takes approximately half a minute instead of more than sixteen hours on an Intel® Xeon™ 2.80 GHz processor with 2 GB of RAM.

Contents

Chapter 1

Preliminaries

Unless otherwise noted, the following general notation is valid throughout the whole diploma thesis:

- lowercase characters for scalars
$$x, y, z$$
- bold lowercase characters for vectors (which also represent (affine) points)
$$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$$
- bold uppercase characters for matrices
$$\mathbf{A}, \mathbf{B}, \mathbf{C}$$

1.1 Polyhedra, polarity and blocking polyhedra

The definitions and results in this section are based on [Brø83, Sch99, Zie98].

1.1.1 Polyhedra and polytopes

The concept of the tilting complex belongs to both the field of polyhedral combinatorics, which deals with facets of polyhedra occurring in combinatorial optimization, and to polyhedral theory. Therefore we start with the definition of a polyhedron.

Definition 1.1.1. *A* polyhedron Q *is the intersection of a finite number of* closed half spaces $H(\boldsymbol{a}_i, \alpha_i)$ *defined by linear inequalities* $\boldsymbol{a}_i^\top \boldsymbol{x} \geq \alpha_i, \boldsymbol{a}_i \neq \boldsymbol{0}$*, or* $(\boldsymbol{a}_i, \alpha_i)$ *for short:* polyhedron $H(\boldsymbol{a}, \alpha)$ $(\boldsymbol{a}, \alpha)$

$$Q = \bigcap_{i=1}^{n} H(\boldsymbol{a}_i, \alpha_i).$$

Every polyhedron is closed and *convex,* i.e. with any two points $\boldsymbol{x}, \boldsymbol{y}$ it also contains the straight line segment $\{\, \lambda \boldsymbol{x} + (1-\lambda)\boldsymbol{y} \mid \lambda \in [0,1] \,\}$ between them. convex

Another basic module in polyhedral theory is the polytope. But prior to its definition we have to introduce some variants of vector combinations besides the well-known linear combination that will be encountered repeatedly in the course of this diploma thesis.

Definition 1.1.2. *Let* $\boldsymbol{x}_0, \ldots, \boldsymbol{x}_n \in \mathbb{R}^d$, $\mathbf{X} := (\boldsymbol{x}_0, \ldots, \boldsymbol{x}_n)$ *and* $\boldsymbol{\lambda} \in \mathbb{R}^{n+1}$. *A vector*

$$\mathbf{X}\boldsymbol{\lambda} = \sum_{i=0}^{n} \lambda_i \boldsymbol{x}_i, \text{ with } \mathbf{1}^\top \boldsymbol{\lambda} = \sum_{i=0}^{n} \lambda_i = 1$$

affine / conical combination

is called an affine combination *of* $\boldsymbol{x}_0, \ldots, \boldsymbol{x}_n$. *Similarly, we define the* conical combination

$$\mathbf{X}\boldsymbol{\lambda}, \text{ with } \boldsymbol{\lambda} \geq \mathbf{0}$$

convex combination

and the convex combination

$$\mathbf{X}\boldsymbol{\lambda}, \text{ with } \boldsymbol{\lambda} \geq \mathbf{0} \wedge \mathbf{1}^\top \boldsymbol{\lambda} = 1.$$

affine hull

The affine hull *of* $\boldsymbol{x}_0, \ldots, \boldsymbol{x}_n$ *is the set of all affine combinations of* $\boldsymbol{x}_0, \ldots, \boldsymbol{x}_n$ *and will be denoted by*

$$\operatorname{aff}(\boldsymbol{x}_0, \ldots, \boldsymbol{x}_n).$$

conical hull (cone)

Analogously, we define the conical hull *(also referred to as* cone*)*

$$\operatorname{cone}(\boldsymbol{x}_0, \ldots, \boldsymbol{x}_n)$$

convex hull

and the convex hull

$$\operatorname{conv}(\boldsymbol{x}_0, \ldots, \boldsymbol{x}_n).$$

polytope

Definition 1.1.3. *A* polytope P *is defined as the convex hull of a finite number of vectors*

$$P = \operatorname{conv}(\boldsymbol{x}_0, \ldots, \boldsymbol{x}_n).$$

Obviously, the concepts of polyhedron and polytope are closely related. This is made more precise in the following *Decomposition theorem* for polyhedra and the subsequent corollary.

Theorem 1.1.4 (Decomposition theorem)**.** *A set of vectors* $Q \subseteq \mathbb{R}^d$ *is a polyhedron, if and only if* $Q = P + C$ *for some polytope* P *and some cone* C, *i.e.*

$$Q = \operatorname{conv}(\boldsymbol{x}_0, \ldots, \boldsymbol{x}_n) + \operatorname{cone}(\boldsymbol{y}_0, \ldots, \boldsymbol{y}_m).$$

In this case we say that Q *is generated by the points* $\boldsymbol{x}_0, \ldots, \boldsymbol{x}_n$ *and the directions* $\boldsymbol{y}_0, \ldots, \boldsymbol{y}_m$.

Corollary 1.1.5. *A non-empty subset* P *of* $\mathbb{R}^d$ *is a polytope, if and only if it is a bounded polyhedron.*

inner / outer description

As a consequence, there are two ways of describing a polytope: as convex hull of a finite number of vectors *(inner description)* or as intersection of a finite number of closed half spaces *(outer description)*.

We continue with the definition of the face of a polyhedron.

Definition 1.1.6. *Let $P \subseteq \mathbb{R}^d$ be a polyhedron. A linear inequality $(\boldsymbol{a}, \alpha)$ is called* valid *for P, if it is satisfied for all points $\boldsymbol{x} \in P$. A* face *of P is any set of the form* valid face

$$F = P \cap \{\, \boldsymbol{x} \in \mathbb{R}^d \mid \boldsymbol{a}^\top \boldsymbol{x} = \alpha \,\},$$

where $(\boldsymbol{a}, \alpha)$ is a valid inequality for P. The dimension *of a face is the dimension of its affine hull:* $\dim(F) := \dim\big(\operatorname{aff}(F)\big)$*. Since every affine space is a translation of a linear subspace uniquely defined by the affine space, its dimension is equal to that of the corresponding subspace.* dimension of a face

The valid inequalities $(\mathbf{0}, 0)$ and $(\mathbf{0}, 1)$ evince that P and $\emptyset$ are faces of P. All faces $F \subsetneq P$ are called *proper faces*. Those with dimension $0, 1, \dim(P) - 2$ and $\dim(P) - 1$ are referred to as *vertices, edges, ridges* and *facets* respectively. proper face vertex, edge, ridge, facet

Sometimes it is more convenient to give the dimension of a face F of a polyhedron P in relation to the dimension of P itself. In those cases we will use the *co-dimension* of F in P, which is defined as co-dimension

$$\operatorname{codim}(F) := \dim(P) - \dim(F),$$

i.e. 1 for facets, 2 for ridges and so on.

At a later point it will also be necessary to refer to those points lying in the interior of a polyhedron, i.e. the points of the polyhedron that are not contained in its facets. Unfortunately, we cannot directly apply the concept of the interior known from the field of topology, since it fails in some cases as will be shown subsequent to the next definition. Instead we will use the relative interior, which is based on the topological interior.

Definition 1.1.7. *The* relative interior $\operatorname{relint}(P)$ *of a polyhedron P is defined as the interior of P within the affine hull of P.* $\operatorname{relint}(P)$

For example, let

$$P := \operatorname{conv}\left\{ \tbinom{0}{0}, \tbinom{1}{0} \right\} \subseteq \mathbb{R}^2.$$

The topological interior of P in $\mathbb{R}^2$ is empty whereas the relative interior is

$$\left\{ \tbinom{x}{0} \mid x \in (0,1) \right\} = (0,1) \times \{0\},$$

since $\operatorname{aff}\left\{ \tbinom{0}{0}, \tbinom{1}{0} \right\}$ is the straight line through the two points.

Next we consider the set of all faces of a given polyhedron. It is easy to see that the inclusion relation induces a partial order on this set. A structure taking advantage of this fact is the so-called face lattice. It contains information about the faces of a polyhedron and their interrelation. But before we can formally define this structure, we have to cover some poset definitions.

Definition 1.1.8. *A* poset $(S, \preccurlyeq)$ *is a finite partially ordered set, i.e. a finite set S equipped with a relation "$\preccurlyeq$", which is reflexive ($x \preccurlyeq x, \forall\, x \in S$), transitive ($x \preccurlyeq y$ and $y \preccurlyeq z$ implies $x \preccurlyeq z$) and antisymmetric ($x \preccurlyeq y$ and $y \preccurlyeq x$ implies $x = y$). We say that z* covers x*, if $x \preccurlyeq z$ and there exists no third element y in the poset, for which $x \preccurlyeq y \preccurlyeq z$.* poset z covers x

A poset is bounded*, if it has a unique minimal element, denoted by $\hat{0}$, and a unique maximal element, denoted by $\hat{1}$. A poset is a* lattice*, if it is bounded and every two* bounded poset lattice

join
meet

elements $x, y \in S$ have a unique minimal upper bound, called the join $x \vee y$, *and a unique maximal lower bound, called the* meet $x \wedge y$, *in S.*

(anti-) isomorphism

A mapping ϕ from one lattice $(S_1, \preccurlyeq)$ onto another lattice $(S_2, \preccurlyeq)$ is called an isomorphism, *if it is bijective and we have $x \preccurlyeq y \Leftrightarrow \phi(x) \preccurlyeq \phi(y)$, for all $x, y \in S_1$. Accordingly, a bijective mapping ψ with $x \preccurlyeq y \Leftrightarrow \psi(x) \succcurlyeq \psi(y)$, for all $x, y \in S_1$, is called an* anti-isomorphism.

(anti-) isomorphic

We say that $(S_1, \preccurlyeq)$ and $(S_2, \preccurlyeq)$ are (anti-) isomorphic, *if there exists an (anti-) isomorphism from $(S_1, \preccurlyeq)$ onto $(S_2, \preccurlyeq)$.*

face lattice $\mathscr{L}(P)$

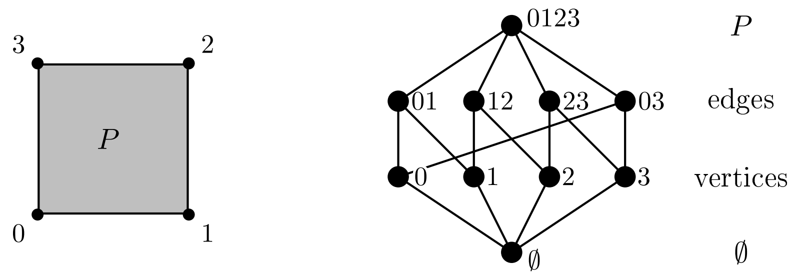
Definition 1.1.9. *The* face lattice $\mathscr{L}(P)$ *of a polyhedron $P \subseteq \mathbb{R}^d$ is the poset of all faces of P partially ordered by inclusion, where $\hat{0} = \emptyset$ and $\hat{1} = P$.*

In order to facilitate the work with face lattices, we introduce the following graphical representation called Hasse diagram.

Hasse diagram

Definition 1.1.10. *The* Hasse diagram *of a poset is a directed graph, whose vertices are the elements of the set and whose directed edges $y \to x$ are precisely those ordered pairs such that y covers x.*

In all Hasse diagrams throughout this diploma thesis we will always assume a downward direction of the edges and therefore use undirected edges for reasons of lucidity. Figure 1.1.1 shows an example of such a diagram for the face lattice of a square in $\mathbb{R}^2$.



(a) A polytope $P \subseteq \mathbb{R}^2$ (b) Hasse diagram of the face lattice $\mathscr{L}(P)$

Figure 1.1.1: Hasse diagram of the face lattice of a square in $\mathbb{R}^2$

Apart from providing information about the set of faces of a polyhedron, the face lattice is also of importance when dealing with structural similarities of polytopes.

combinatorially equivalent

Definition 1.1.11. *Two polytopes are said to be* combinatorially equivalent, *if their face lattices are isomorphic, i.e. there exists an isomorphism that maps one face lattice onto the other.*

We conclude this subsection with the introduction of a second kind of similarity between polytopes that is relevant for tilting complexes.

Definition 1.1.12. *Two polytopes $P \subseteq \mathbb{R}^d$ and $Q \subseteq \mathbb{R}^e$ are* affinely isomorphic, *if there exists an affine mapping* affinely isomorphic

$$f\colon \mathbb{R}^d \longrightarrow \mathbb{R}^e$$
$$\boldsymbol{x} \longmapsto \mathbf{M}\boldsymbol{x} + \boldsymbol{x}_0, \text{ with } \mathbf{M} \in \mathbb{M}(e \times d),\ \boldsymbol{x}_0 \in \mathbb{R}^e,$$

that is a bijection between the points of the two polytopes.

1.1.2 Polarity

Polarity plays an important role in both polyhedral theory and the concept of the tilting complex.

Definition 1.1.13. *For any subset $P \subseteq \mathbb{R}^d$ the* polar set, *or* polar *for short, is defined by* polar

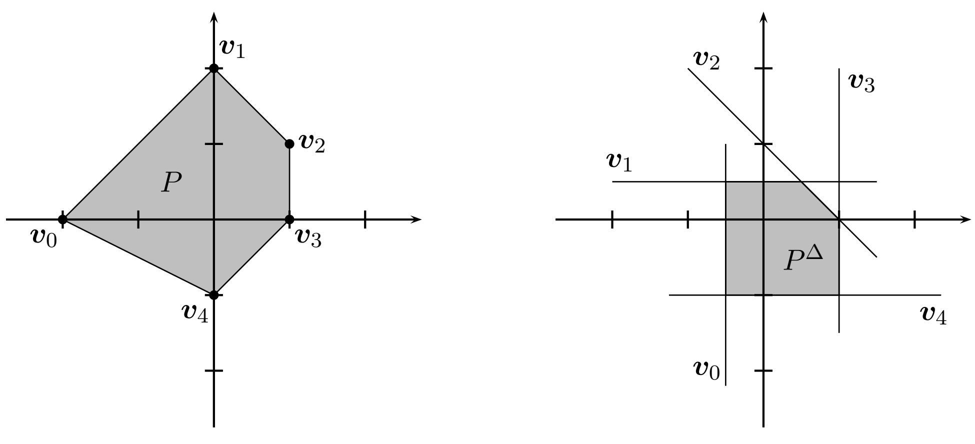
$$P^\Delta := \{\, \boldsymbol{a} \in \mathbb{R}^d \mid \boldsymbol{a}^\top \boldsymbol{x} \le 1,\ \forall\, \boldsymbol{x} \in P \,\}.$$

The polar is sometimes also referred to as *dual set*. Its construction can be iterated to get the polar of the polar (or bipolar) $P^{\Delta\Delta}$. For the purpose of clarification and in order to derive some basic properties of the polar, we give a brief example.

Example 1.1.14. *Figure 1.1.2*[1] *(a) shows a polytope $P \subseteq \mathbb{R}^2$ determined by its five vertices, that is $P = \operatorname{conv}\{\, \boldsymbol{v}_0, \dots, \boldsymbol{v}_4 \,\}$. All $\boldsymbol{a} \in P^\Delta$ must satisfy $\boldsymbol{a}^\top \boldsymbol{x} \le 1$ for all points in P, thus in particular for the vertices of P. Hence each vertex defines a valid inequality for P^Δ, namely*

$$\boldsymbol{v}_i^\top \boldsymbol{a} \le 1,\ \forall\, \boldsymbol{a} \in P^\Delta,\ \forall\, i = 0, \dots, 4.$$

Figure 1.1.2 (b) shows the polar polytope P^Δ determined by the above inequalities.



(a) A polytope $P \subseteq \mathbb{R}^2$

(b) The polar polytope of P

Figure 1.1.2: Example of mutually polar sets in $\mathbb{R}^2$

The most important characteristics of the polar are combined in the following

[1] based on a figure on page 61 of [Zie98]

Theorem 1.1.15. *Let $P, Q \subseteq \mathbb{R}^d$. Then we have*

(i) $P \subseteq Q$ *implies* $P^\Delta \supseteq Q^\Delta$ *and* $P^{\Delta\Delta} \subseteq Q^{\Delta\Delta}$.

(ii) $P \subseteq P^{\Delta\Delta}$.

(iii) P^Δ *and* $P^{\Delta\Delta}$ *are convex.*

(iv) $\mathbf{0} \in P^\Delta$ *and* $\mathbf{0} \in P^{\Delta\Delta}$.

(v) if P *is closed and convex and* $\mathbf{0} \in P$*, then* $P = P^{\Delta\Delta}$.

(vi) if a polytope P *with* $\mathbf{0} \in \operatorname{relint}(P)$ *is given by* $P = \operatorname{conv}(V)$*, where* $V = \{\boldsymbol{v}_0, \dots, \boldsymbol{v}_k\}$*, then*

$$P^\Delta = \{\, \boldsymbol{a} \mid \boldsymbol{a}^\top \boldsymbol{v}_i \le 1, \forall \boldsymbol{v}_i \in V \,\}.$$

(vii) if a polytope P *with* $\mathbf{0} \in \operatorname{relint}(P)$ *is given by* $P = \{\, \boldsymbol{x} \in \mathbb{R}^d \mid \mathbf{A}^\top \boldsymbol{x} \le \mathbf{1} \,\}$*, where* $\mathbf{A} = (\boldsymbol{a}_0, \dots, \boldsymbol{a}_{m-1}) \in \mathbb{M}(d \times m)$*, then*

$$P^\Delta = \operatorname{conv}(\mathbf{A}).$$

An obvious question is the following one: what structure of the polar P^Δ corresponds to a face F of P? Before answering this question we present a mathematical description of this structure.

d-polytope

conjugate face $F^\diamond$

Definition 1.1.16. *Let* $P \subseteq \mathbb{R}^d$ *be a d-dimensional polytope (henceforth referred to as* d-polytope*) with* $\mathbf{0} \in \operatorname{relint}(P)$*. For all faces* F *of* P *we define the* conjugate face $F^\diamond$ *as*

$$F^\diamond := \{\, \boldsymbol{a} \in \mathbb{R}^d \mid \boldsymbol{a}^\top \boldsymbol{x} \le 1, \forall \boldsymbol{x} \in P \text{ and } \boldsymbol{a}^\top \boldsymbol{x} = 1, \forall \boldsymbol{x} \in F \,\}.$$

The name "conjugate face" already indicates the answer to our question. And in fact each face F of a polytope P with $\mathbf{0}$ as a relative interior point corresponds to a face $F^\diamond$ of P^Δ. In order to confirm this assertion, we revisit Example 1.1.14 and take a look at the conjugate face $F^\diamond$ for some faces F of P.

Example 1.1.17. *As faces we choose the vertices* $\boldsymbol{v}_0, \dots, \boldsymbol{v}_4$ *and the facets*

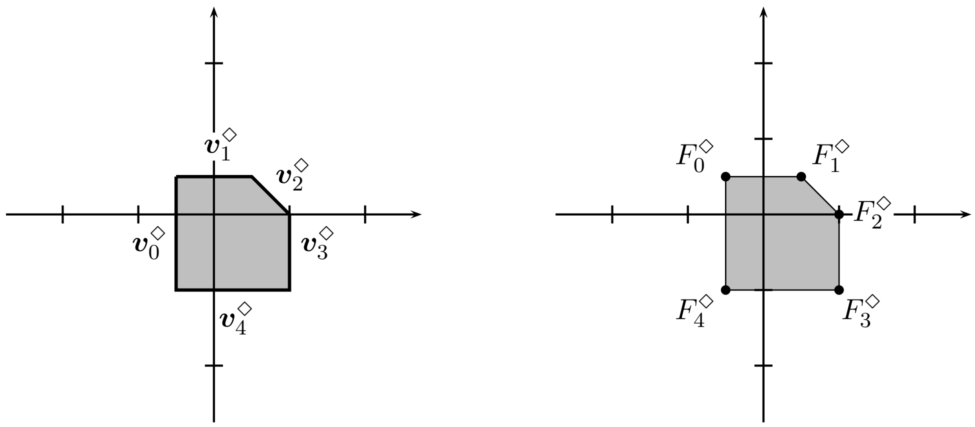
$$F_i := \{\, \boldsymbol{v}_i, \boldsymbol{v}_{(i+1) \bmod 5} \,\}$$

of P *respectively. Figure 1.1.3 shows that the* $\boldsymbol{v}_i^\diamond$ *define the facets of* P^Δ *whereas the* $F_i^\diamond$ *define the vertices.*

The main results concerning conjugate faces are summed up in the following theorem and the subsequent corollary. The theorem provides an interesting variant of the description of the conjugate face that will be used in Chapter 3.

Theorem 1.1.18. *Assume that* $P = \operatorname{conv}(V) = \{\, \boldsymbol{x} \in \mathbb{R}^d \mid \mathbf{A}^\top \boldsymbol{x} \le \mathbf{1} \,\} \subseteq \mathbb{R}^d$ *with* $V, \mathbf{A}$ *as in Theorem 1.1.15 (vi) and (vii) respectively. Let*

$$F = \operatorname{conv}(V_F) = \{\, \boldsymbol{x} \in \mathbb{R}^d \mid \mathbf{A}_{F\le}^\top \boldsymbol{x} \le \mathbf{1} \wedge \mathbf{A}_{F=}^\top \boldsymbol{x} = \mathbf{1} \,\}$$



(a) $\boldsymbol{v}_i^\diamond$ for the vertices $\boldsymbol{v}_i$ of P (b) $F_i^\diamond$ for the facets F_i of P

Figure 1.1.3: Examples of $F^\diamond$ for some faces F of P

be a face of P. ($V = V_F \uplus V_F^\complement$, where V_F is the set of vertices of P contained in F and $V_F^\complement$ is the complement of V_F within V. Similarly, $\mathbf{A} = \mathbf{A}_{F\leq} \uplus \mathbf{A}_{F=}$, where $\mathbf{A}_{F=}$ represents the set of normalized inequalities, whose corresponding faces contain F. For the inequalities in $\mathbf{A}_{F\leq}$ the face F is only part of the valid half space.) Then

$$\begin{aligned} F^\diamond &= \operatorname{conv}(\mathbf{A}_{F=}) \\ &= \{ \boldsymbol{a} \mid \boldsymbol{a}^\top \boldsymbol{v} \leq 1, \forall \boldsymbol{v} \in V_F^\complement \text{ and} \\ &\qquad \boldsymbol{a}^\top \boldsymbol{v} = 1, \forall \boldsymbol{v} \in V_F \}. \end{aligned}$$

Corollary 1.1.19. *Let P be a polytope with $\mathbf{0} \in \operatorname{relint}(P)$ and let F, G be faces of P.*

(i) $F^\diamond$ is a face of P^Δ.

(ii) $F^{\diamond\diamond} = F$.

(iii) $F \subseteq G$ if and only if $F^\diamond \supseteq G^\diamond$.

Statement (iii) directly implies the next result regarding the face lattices of a polytope and its corresponding polar.

Corollary 1.1.20. *The face lattices of P and P^Δ are anti-isomorphic, i.e. the face lattice of P^Δ is the opposite of the face lattice of P.*

$$\begin{aligned} \emptyset &\longleftrightarrow P \\ \text{vertices} &\longleftrightarrow \text{facets} \\ \text{edges} &\longleftrightarrow \text{ridges} \\ \ldots &\longleftrightarrow \ldots \end{aligned}$$

Figure 1.1.4 shows a schematic diagram that visualizes the essence of the previous results.

In order to derive further properties of $F^\diamond$, we first need to define the face figures of a d-polytope P. To simplify matters, we start with a special case of face figures: the

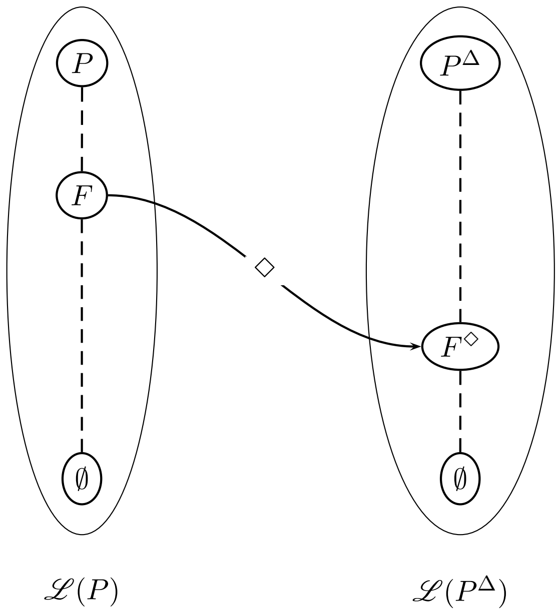


Figure 1.1.4: Schematic diagram of the face lattices of a polytope and its corresponding polar

so-called vertex figures. These are certain $(d-1)$-polytopes, each containing information about the "local" facial structure of P "near" one of its vertices. A vertex figure is obtained by cutting a polytope by a hyperplane that cuts off one single vertex.

vert(P) **Example 1.1.21.** *Consider a d-polytope P. Let $V := \mathrm{vert}(P)$ be the set of its vertices and $\boldsymbol{v} \in V$. Furthermore, let $(\boldsymbol{c}, c_0)$ be a valid inequality with*

$$\{\boldsymbol{v}\} = P \cap \{\, \boldsymbol{x} \mid \boldsymbol{c}^\top \boldsymbol{x} = c_0 \,\}.$$

Now we choose some $c_1 > c_0$ with $\boldsymbol{c}^\top \boldsymbol{v}' > c_1$, $\forall\, \boldsymbol{v}' \in \mathrm{vert}(P) \backslash \{\boldsymbol{v}\}$. Then the corresponding
vertex figure vertex figure *of P at $\boldsymbol{v}$ is the polytope*

$$(P/\boldsymbol{v})_{(\boldsymbol{c},c_1)} := P \cap \{\, \boldsymbol{x} \mid \boldsymbol{c}^\top \boldsymbol{x} = c_1 \,\}.$$

In the geometrical sense the vertex figure depends on the choice of $(\boldsymbol{c}, c_1)$. Figure 1.1.5 displays two different vertex figures in dark-gray, each being a one-dimensional polytope with two vertices.

It is quite evident that these vertex figures only differ in the coordinates of their vertices, hence the polytopes themselves are combinatorially equivalent, i.e. their face lattices are isomorphic. So, without loss of generality we can choose a representative face lattice *for all the vertex figures of P at $\boldsymbol{v}$ and denote it by $\mathscr{L}(P/\boldsymbol{v})$. Obviously, $\mathscr{L}(P/\boldsymbol{v})$ is a sublattice of $\mathscr{L}(P)$, namely the lattice of all faces of P containing $\boldsymbol{v}$, partially ordered by inclusion. As depicted in Figure 1.1.6, it can be obtained from $\mathscr{L}(P)$ by simply deleting all faces not containing $\boldsymbol{v}$. The result is a sublattice of $\mathscr{L}(P)$ with $\hat{0} = \boldsymbol{v}$ (since $\boldsymbol{v}$ has the smallest dimension of all faces of P that contain $\boldsymbol{v}$) and $\hat{1} = P$. It yields a representative face lattice for both the vertex figures in Figure 1.1.5.*

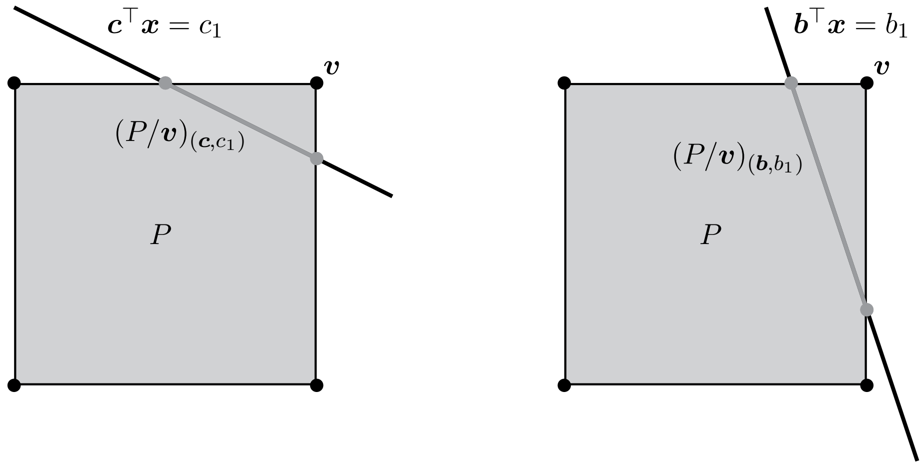
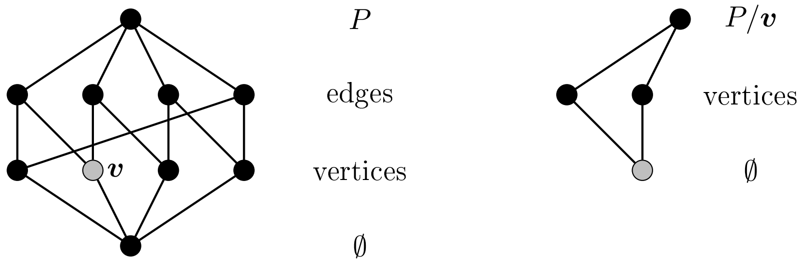


Figure 1.1.5: Two examples of vertex figures



(a) Hasse diagram of the face lattice of a square $P \subseteq \mathbb{R}^2$

(b) Hasse diagram of the face lattice of a vertex figure $P/\boldsymbol{v}$

Figure 1.1.6: Example of a face lattice $\mathscr{L}(P/\boldsymbol{v})$

The above example works analogously for arbitrary faces F of P, which leads to the previously mentioned face figures.

face figure P/F

Definition 1.1.22. *Let P be a d-polytope and F be a face of P. The* face figure *P/F of P at F is defined by the face lattice*

$$\mathscr{L}(P/F) := \big(\{ G \in \mathscr{L}(P) \mid F \subseteq G \}, \subseteq\big).$$

Sometimes the face figure is also defined as

$$P/F := \big(F^{\diamond}\big)^{\Delta}. \tag{1.1.1}$$

When using this notation we always assume a combinatorial point of view, i.e. instead of the polytopes we consider their face lattices. This is due to the fact that in the strict sense this alternative definition is only correct, if $F^{\diamond}$ has $\mathbf{0}$ as a relative interior point and is full-dimensional.

Finally, we are able to gain additional information on $F^{\diamond}$. Due to Corollary 1.1.20 the bipolar of $F^{\diamond}$ is combinatorially equivalent to $F^{\diamond}$. Using (1.1.1) we get

$$F^{\diamond} = \big(F^{\diamond}\big)^{\Delta\Delta} = \big(P/F\big)^{\Delta}. \tag{1.1.2}$$

For clarification we apply this description to the vertex figure $P/\boldsymbol{v}$ in Figure 1.1.6 (b). Corollary 1.1.20 states that the face lattice of $\big(P/F\big)^{\Delta}$ is simply the face lattice of P/F turned upside down. Figure 1.1.7 shows the result of this operation.

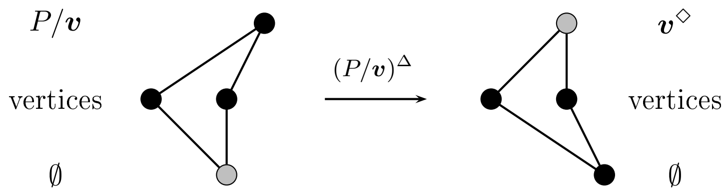


Figure 1.1.7: Obtaining the face lattice $\mathscr{L}(\boldsymbol{v}^{\diamond})$ from $\mathscr{L}(P/\boldsymbol{v})$

Now the following lemma is rather obvious.

Lemma 1.1.23. *The vertices of $F^{\diamond}$ correspond exactly to those facets of P containing F and vice versa.*

Before we deal with the dimension of $F^{\diamond}$, we need to define the dimension of a face lattice.

dimension of a face lattice

Definition 1.1.24. *Let $\mathscr{L}(P)$ be the face lattice of a polytope P. Then the* dimension *$\dim\big(\mathscr{L}(P)\big)$ of the face lattice is defined as the length k of a maximal chain $(x_1, \ldots, x_k)$ with*

- *$x_i \in \mathscr{L}(P)$, $\forall\, i = 1, \ldots, k$.*
- *x_{i+1} covers x_i, $\forall\, i = 1, \ldots, k-1$.*

Obviously, $x_1 = \emptyset$ and $x_k = P$. In other words, the dimension of a face lattice is simply the number of different "levels" in the corresponding Hasse diagram. Level 1 only contains the empty set, which is the (-1)-dimensional face of P. Levels 2 and 3 contain the vertices and edges with dimension 0 and 1 respectively. The elements of level $k-1$ are the facets with dimension $\dim(P)-1$, whereas the single element in level k is the polytope itself. So, the index of the level is equal to the dimension of the faces in this level plus two. Hence, $k = \dim(P)+2$, which yields a formula for the dimension of a polytope based on the dimension of its face lattice:

$$\dim(P) = \dim\big(\mathscr{L}(P)\big) - 2. \tag{1.1.3}$$

With this formula we can easily prove the following result for the dimension of the conjugate face $F^{\diamond}$.

Lemma 1.1.25. *Let P be a polytope and let F be a face of P. Then*

$$\dim(F^{\diamond}) = \operatorname{codim}(F) - 1.$$

Proof.

$$\begin{aligned}
\dim\big(\mathscr{L}(F^{\diamond})\big) &= \dim\big(\mathscr{L}\big((P/F)^{\Delta}\big)\big) && \big(\text{see (1.1.2)}\big)\\
&= \dim\big(\mathscr{L}(P/F)\big) && (\text{Cor. 1.1.20})\\
&= \dim\big(\mathscr{L}(P)\big) - \dim\big(\mathscr{L}(F)\big) + 1 && (\text{easy to verify})\\
&= \dim(P) + 2 - \dim(F) - 2 + 1 && \big(\text{see (1.1.3)}\big)\\
&= \operatorname{codim}(F) + 1
\end{aligned}$$

Hence

$$\dim(F^{\diamond}) = \dim\big(\mathscr{L}(F^{\diamond})\big) - 2 = \operatorname{codim}(F) - 1.$$

□

1.1.3 Blocking polyhedra

Besides the classical polarity discussed in the previous section, there is the blocking relation between polyhedra of a certain class.

Definition 1.1.26. *A polyhedron $P \subseteq \mathbb{R}^d$ is said to be of* blocking type, *if $P \subseteq \mathbb{R}^d_+$ and if $y \geq x \in P$ implies $y \in P$.* polyhedron of blocking type

It follows directly from the Decomposition theorem for polyhedra that a polyhedron P in $\mathbb{R}^d$ is of blocking type if and only if there exist vectors $\boldsymbol{x}_0, \dots, \boldsymbol{x}_n$ in $\mathbb{R}^d_+$ such that

$$P = \operatorname{conv}(\boldsymbol{x}_0, \dots, \boldsymbol{x}_n) + \mathbb{R}^d_+.$$

Similarly, P is of blocking type if and only if there are vectors $\boldsymbol{d}_0, \dots, \boldsymbol{d}_m$ in $\mathbb{R}^d_+$ such that

$$P = \{\, \boldsymbol{x} \in \mathbb{R}^d_+ \mid \boldsymbol{d}_j^{\top} \boldsymbol{x} \geq 1,\ \forall\, j = 0, \dots, m \,\}.$$

In this context we define a structure that is essential for the understanding of the significance of tilting complexes: the so-called blocking polyhedron.

blocking polyhedron $B(P)$

Definition 1.1.27. *For any polyhedron P in $\mathbb{R}^d$ we define its* blocking polyhedron $B(P)$ *by*

$$B(P) := \{\, \boldsymbol{z} \in \mathbb{R}^d_+ \mid \boldsymbol{z}^\top \boldsymbol{x} \geq 1,\ \forall\, \boldsymbol{x} \in P \,\}.$$

We now specify the most important properties of the blocking polyhedron.

Theorem 1.1.28. *Let $P \subseteq \mathbb{R}^d$ be a polyhedron of blocking type. Then*

(i) $B(P)$ *is again a polyhedron of blocking type.*

(ii) $B\big(B(P)\big) = P$.

(iii) if $P = \operatorname{conv}(\boldsymbol{x}_0, \ldots, \boldsymbol{x}_n) + \mathbb{R}^d_+$, *then*

$$B(P) = \{\, \boldsymbol{z} \in \mathbb{R}^d_+ \mid \boldsymbol{z}^\top \boldsymbol{x}_i \geq 1,\ \forall\, i = 0, \ldots, n \,\}$$

and conversely.

Since the blocking relation is similar to the classical polarity, the characteristics of the face lattice of the polar can mostly be transferred to the face lattice of the blocking polyhedron. In this context we also refer to the following

Proposition 1.1.29 (cf. Proposition 0.1.1 (c) of [The05])**.** *Let $\mathscr{L}$ denote the face lattice $\mathscr{L}(P)$ of P and let $\mathscr{N} \subseteq \mathscr{L}$ be the set of all trivial faces of P, i.e. the proper and non-empty faces that are only contained in non-negativity facets (cf. page 17). Furthermore, let $\mathscr{L}^B$ denote the face lattice $\mathscr{L}\big(B(P)\big)$ of $B(P)$ and let $\mathscr{N}^B \subseteq \mathscr{L}^B$ be the set of all trivial faces of $B(P)$. Then the posets $\mathscr{L} \setminus \mathscr{N}$ and $\mathscr{L}^B \setminus \mathscr{N}^B$ are anti-isomorphic. We denote this anti-isomorphism by*

$F^\sharp$

$$\begin{aligned} \cdot^\sharp \colon \mathscr{L} \setminus \mathscr{N} &\longrightarrow \mathscr{L}^B \setminus \mathscr{N}^B \\ F &\longmapsto F^\sharp. \end{aligned}$$

A depiction of this proposition is shown in Figure 1.1.8.

1.2 Polytopal complexes and subdivisions

As the name “tilting complex” indicates, it is a special case of a complex (or polytopal complex to be precise). This term shall now be explained. The following definitions and results are mainly based on the section “Polyhedral Complexes” of [Zie98].

polyhedral complex

Definition 1.2.1. *A* polyhedral complex $\mathcal{C}$ *is a finite collection of polyhedra in $\mathbb{R}^d$ such that*

(i) if $P \in \mathcal{C}$, then all faces of P are also in $\mathcal{C}$. In particular the empty polyhedron is therefore in $\mathcal{C}$.

(ii) the intersection $P \cap Q$ of two polyhedra $P, Q \in \mathcal{C}$ is a face of both P and Q.

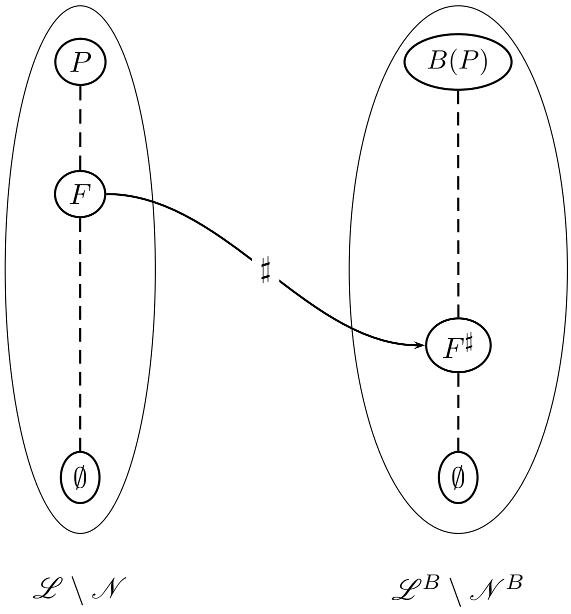


Figure 1.1.8: Schematic diagram of the face lattices $\mathscr{L} \setminus \mathscr{N}$ and $\mathscr{L}^B \setminus \mathscr{N}^B$

$\mathcal{C}$ *is a* polytopal complex, *if all the polyhedra in* $\mathcal{C}$ *are bounded (i.e. polytopes). The* underlying set *of* $\mathcal{C}$ *is the point set* $|\mathcal{C}| := \bigcup_{P \in \mathcal{C}} P$. *The* vertex set $\operatorname{vert}(\mathcal{C})$ *of a polytopal complex* $\mathcal{C}$ *is defined as* [polytopal complex] [underlying set]

$$\operatorname{vert}(\mathcal{C}) := \bigcup_{P \in \mathcal{C}} \operatorname{vert}(P).$$

[vert($\mathcal{C}$)]

A straightforward example of a polytopal complex is the complex of all faces of a polytope P, which is denoted by $\mathcal{C}(P)$. Another obvious example is the so-called *boundary complex* $\mathcal{C}(\partial P)$ formed by all proper faces of a polytope P. [boundary complex $\mathcal{C}(\partial P)$]

Similar to the face lattice of a polyhedron we can define the *face poset* of a polyhedral complex. In general, we cannot assume this poset to be a lattice, because of the following counter-example: [face poset]

Example 1.2.2. *Consider the polytopal complex consisting of the empty set and two distinct vertices*

$$\mathcal{C} := \{\, \emptyset, \{\boldsymbol{v}_1\}, \{\boldsymbol{v}_2\} \,\}.$$

It is easy to verify that this is indeed a polytopal complex. In a lattice every two elements must have a join. But in this example the join $\boldsymbol{v}_1 \vee \boldsymbol{v}_2$ *does not exist. Hence, the face poset of* $\mathcal{C}$ *cannot be a lattice.*

Using the face poset of a polyhedral complex we can generalize the previously introduced concept of combinatorial equivalence.

Definition 1.2.3. *Two polyhedral complexes are called* combinatorially equivalent, *if their face posets are isomorphic.*

A special form of a polytopal complex is the subdivision, which will dominate the Sections 2.3 and 2.4. A subdivision is defined as follows:

(regular) subdivision

Definition 1.2.4. *A* subdivision *of a polytope P is a polytopal complex $\mathcal{C}$ with the underlying set $|\mathcal{C}| = P$. A subdivision $\mathcal{C}$ of a polytope $P \subseteq \mathbb{R}^d$ is called* regular, *if it arises from a polytope $Q \subseteq \mathbb{R}^{d+1} = \operatorname{span}(\boldsymbol{e}_0, \ldots, \boldsymbol{e}_d)$ in the following way:*

(i) $P = \pi(Q)$ via the canonical projection

$$\pi\colon \mathbb{R}^{d+1} \longrightarrow \mathbb{R}^d$$
$$\begin{pmatrix} \boldsymbol{x} \\ x_d \end{pmatrix} \longmapsto \boldsymbol{x}$$

that "deletes" the last coordinate.

(ii) $\mathcal{C}$ is the set of all lower faces of Q projected down to P. The lower faces F are the ones satisfying

$$\boldsymbol{x} - \lambda\, \boldsymbol{e}_d \notin Q,\ \forall\, \boldsymbol{x} \in F,\ \forall\, \lambda > 0.$$

Figure 1.2.1 shows a simple example of a regular subdivision for $d = 1$. The lower faces of Q are drawn thicker.

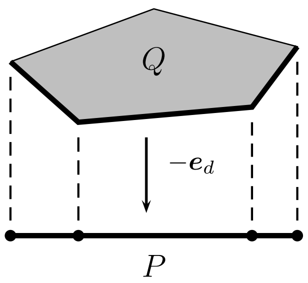


Figure 1.2.1: Example of a regular subdivision for $d = 1$

As Figure 1.2.1 motivates, regular subdivisions arise from piecewise linear convex functions. Given the projection $\pi\colon Q \longrightarrow P$, the function

$$f\colon P \longrightarrow \mathbb{R}$$
$$\boldsymbol{x} \longmapsto \min_{y \in \mathbb{R}} \{\, (\boldsymbol{x}^\top, y) \in Q \,\}$$

is piecewise linear and convex. It describes exactly the lower faces of Q.

Thus, every regular subdivision defines a piecewise linear convex function. But even more important is the converse result.

Lemma 1.2.5. *Every piecewise linear convex function f over a polytope P determines a regular subdivision of P by a canonical projection of the lower faces of the polytope*

$$Q := \operatorname{conv}\{\, \big(\boldsymbol{x}^\top, f(\boldsymbol{x})\big) \mid \boldsymbol{x} \in P \,\}.$$

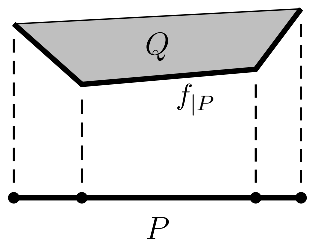


Figure 1.2.2: $f_{|P}$ defines a regular subdivision of P

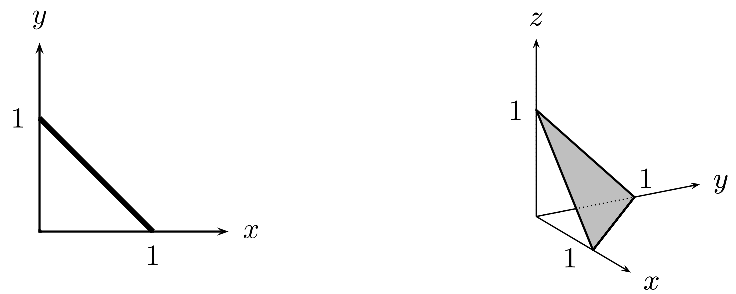
Figure 1.2.2 illustrates the idea of the lemma.

We conclude this section with the definition of the standard simplex. Although it doesn't seem to fit in this context, we will introduce it at this point due to its close relation to the term of a subdivision in the concept of tilting complexes. For a detailed explanation we refer to Section 2.3.

Definition 1.2.6. *The* k-dimensional standard simplex $\mathbb{A}^k \subsetneq \mathbb{R}^{k+1}$ *is the convex hull of the unit vectors* $\mathbb{A}^k$

$$\mathbb{A}^k := \operatorname{conv}\{\, \boldsymbol{e}_0, \ldots, \boldsymbol{e}_k \,\}.$$

Figure 1.2.3 shows some examples of standard simplices. Note that the boundary of the two-dimensional standard simplex consists of three one-dimensional standard simplices. In general, the boundary of the k-dimensional standard simplex in $\mathbb{R}^{k+1}$ consists of $k+1$ standard simplices with dimension $k-1$.



(a) one-dimensional standard simplex $\mathbb{A}^1$

(b) two-dimensional standard simplex $\mathbb{A}^2$

Figure 1.2.3: Standard simplices $\mathbb{A}^k$ for $k = 1, 2$

1.3 Definitions and notation for STSP and GTSP

Probably the best-known combinatorial optimization problem is the *Traveling Salesman Problem* or TSP. Its description is fairly easy:

> A salesman has to visit $n-1$ cities. The task consists in finding a preferably "efficient" tour through all cities that starts and ends in his hometown and visits each of the cities exactly once.

But the simplicity of the description is deceptive, since the *Hamiltonian Cycle Problem*, which is known to be $\mathcal{NP}$-complete (see [Kar72]), reduces to TSP. Hence, the Traveling Salesman Problem is $\mathcal{NP}$-hard. Provided that $\mathcal{P} \neq \mathcal{NP}$, this implies that there exists no deterministic method for solving the problem in polynomial time.

There are many derivatives of the TSP, but in the context of this diploma thesis we will only consider the following ones:

1. the *Symmetric Traveling Salesman Problem* and
2. the *Graphical Traveling Salesman Problem*

The former is simply a special case of the TSP with symmetric travel costs, i.e. the costs for traveling from A to B equal the costs for the reverse direction. The latter is a relaxation of the Symmetric Traveling Salesman Problem allowing the salesman to visit cities more than once.

After this rather descriptive definition of the problems we will now provide a mathematical formulation. The content of the remainder of this section is mainly based on [NR93, The05].

incidence vector $\boldsymbol{\chi}^E$

Definition 1.3.1. *For a given set E the* incidence vector $\boldsymbol{\chi}^E$ *is a vector in* $\{0,1\}^{|E|}$ *with*

$$\chi_e^E := \begin{cases} 1, & \text{if } e \in E; \\ 0, & \text{otherwise.} \end{cases}$$

complete graph K_n

$\delta(U)$

Hamiltonian cycle

STSP(n)

Let $K_n := (V_n, E_n)$ be the complete graph*, where $V_n := \{\, 0, \dots, n-1 \,\}$ and E_n is the set of all two-element subsets of V_n. For a set $U \subseteq V_n$, we denote by $\delta(U)$ the set of all edges of K_n with precisely one end node in U, and we define $\delta(u) := \delta(\{u\})$. A* Hamiltonian cycle *is the edge set of a connected spanning subgraph of K_n, for which $\delta(u) = 2$, $\forall\, u \in V_n$.*

The Symmetric Traveling Salesman Polytope STSP(n) *is the convex hull of all incidence vectors of edge sets of Hamiltonian cycles of K_n.*

GTSP(n)

The Graphical Traveling Salesman Polyhedron GTSP(n) *is equal to the convex hull of all non-negative integral vectors $(x_e)_{e \in E_n}$ satisfying the properties that the graph with node set V_n and edge set $\{\, e \mid x_e \neq 0 \,\}$ is connected and that the number $\left(\boldsymbol{\chi}^{\delta(u)}\right)^\top \boldsymbol{x}$ is even for all $u \in V_n$.*

As stated in [GP79], STSP(n) is an $(\,|E_n| - n)$-dimensional polytope in $\mathbb{R}^{|E_n|}$, i.e.

$$\dim\bigl(\mathrm{STSP}(n)\bigr) = \binom{n}{2} - n.$$

GTSP(n) is full-dimensional and of blocking type. The observation that STSP(n) is the face of GTSP(n), which is the intersection of the so-called *degree facets* defined by the *degree inequalities* $(\boldsymbol{\chi}^{\delta(u)}, 2), u \in V_n$, is fairly obvious. degree facets / inequalities

We continue with some technical definitions that are essential for both the definition and the analysis of the so-called tilting functions in the next chapter.

Definition 1.3.2. *For a node $u \in V_n$ and an edge $e := vw \in E_n$, which forms a triangle with u (i.e. $u \notin e$), we define the* shortcut $\boldsymbol{s}_{u,e}$ *by* shortcut $\boldsymbol{s}_{u,e}$

$$\boldsymbol{s}_{u,e} := \boldsymbol{\chi}^{e} - \boldsymbol{\chi}^{uv} - \boldsymbol{\chi}^{uw} \in \{0, \pm 1\}^{|E_n|}.$$

Let $\boldsymbol{a} \in \mathbb{R}^{|E_n|}$. We denote the triangle slack *for $u \notin e$ by* triangle slack $\bar{t}_{u,e}(\boldsymbol{a})$

$$\bar{t}_{u,e}(\boldsymbol{a}) := -\boldsymbol{s}_{u,e}^{\top}\boldsymbol{a},$$

which is obviously linear in $\boldsymbol{a}$. For a given $u \in V_n$ we define the vector of all triangle slacks for the node u

$$\bar{\boldsymbol{t}}_u(\boldsymbol{a}) := \left(\bar{t}_{u,e}(\boldsymbol{a})\right)_{e \in E_n \setminus \delta(u)}.$$ $\bar{\boldsymbol{t}}_u(\boldsymbol{a})$

We will also use the vector

$$\bar{\boldsymbol{t}}(\boldsymbol{a}) := \left(\bar{t}_{u,e}(\boldsymbol{a})\right)_{\substack{u \in V_n \\ e \in E_n \setminus \delta(u)}}$$ $\bar{\boldsymbol{t}}(\boldsymbol{a})$

consisting of all possible triangle slacks.

Definition 1.3.3. *A vector $\boldsymbol{a} \in \mathbb{R}^{|E_n|}$ is called* metric, *if it satisfies the triangle inequality, i.e. $\bar{\boldsymbol{t}}(\boldsymbol{a}) \geq \boldsymbol{0}$. For all $u \in V_n$ we define the* Tight-Triangular-set *or* TT-set $\Delta_u(\boldsymbol{a})$ *by* metric TT-set $\Delta_u(\boldsymbol{a})$

$$\Delta_u(\boldsymbol{a}) := \{\, e \in E_n \mid u \notin e \wedge \bar{t}_{u,e}(\boldsymbol{a}) = 0 \,\}.$$

A vector $\boldsymbol{a} \in \mathbb{R}^{|E_n|}$ is said to be in TT-form, *if it is metric and $\Delta_u(\boldsymbol{a}) \neq \emptyset$, $\forall u \in V_n$. An inequality $(\boldsymbol{a}, \alpha)$ is in TT-form, if this is true for $\boldsymbol{a}$. A face F of* GTSP(n) *is of* TT-type, *if it is not contained in a* non-negativity facet *defined by $(\boldsymbol{\chi}^e, 0)$ or a degree facet. Accordingly, an inequality defining a TT-type face is also said to be of TT-type.* TT-form TT-type non-negativity facet

Finally, a set of TT-type facets defined by the inequalities $\{(\boldsymbol{a}_i, \alpha_i)\}_{i \in I}$ is said to be TT-disjoint *at a node u, if* TT-disjoint

$$\bigcap_{i \in I} \Delta_u(\boldsymbol{a}_i) \;=\; \emptyset.$$

In order to understand the connection between inequalities in TT-form and inequalities of TT-type, we need a result from [NR93].

Proposition 1.3.4 (Proposition 2.2 of [NR93])**.** *An inequality $(\boldsymbol{a}, \alpha)$ facet-defining for* GTSP(n) *falls in one of the following three categories:*

(i) trivial inequalities: $(\boldsymbol{\chi}^e, 0)$, $\forall\, e \in E_n$

(ii) degree inequalities: $(\boldsymbol{\chi}^{\delta(u)}, 2)$, $\forall\, u \in V_n$

(iii) inequalities in TT-form

As a consequence, a *facet*-defining inequality is of TT-type, if and only if it is in TT-form. This is due to the inclusion maximality of facets. For faces F with $\operatorname{codim}(F) > 1$ on the other hand, this conclusion is generally false, since a face can be *properly* contained in a facet.

Next, we want to provide a technical definition for certain kinds of TT-type facets that were shortly mentioned in the preface: the so-called NR- and non-NR facets.

(non-) NR-facet

Definition 1.3.5. *A TT-type facet G of* GTSP(n) *with the property that $G \cap$* STSP(n) *is a facet of* STSP(n), *is called a* Naddef-Rinaldi facet *or* NR-facet. *A TT-type facet, which is not an NR-facet, is called a non-NR facet.*

good face

A good face *of* STSP(n) *is a proper face, which is not contained in a non-negativity facet.*

Finally, we introduce the standard scaling, which is a specific form a valid inequality for STSP(n) can have.

standard scaling

Definition 1.3.6. *Let $\boldsymbol{x}^*$ be an arbitrary relative interior point of* STSP(n) *and let $(\boldsymbol{a}, \alpha)$ be a valid inequality for* STSP(n). *If $\boldsymbol{a}^\top \boldsymbol{x}^* - \alpha = 1$, we say that $(\boldsymbol{a}, \alpha)$ is in* standard scaling *with respect to $\boldsymbol{x}^*$ (we will omit to mention the $\boldsymbol{x}^*$).*

In the next chapter we will need certain NR-facets to be in standard scaling. This is a consequence of the definition of the conjugate face (cf. Definition 1.1.16), which requires the polytope P to be full-dimensional and to contain the origin as a relative interior point. Hence, it is not possible to directly apply the polar theory to STSP(n), since this polytope fulfills neither of these two properties.

This problem can be solved by translating STSP(n) in such a way that a fixed relative interior point $\boldsymbol{x}^*$ is mapped to the origin and then projecting the translated STSP(n) appropriately in order to assure the image to be full-dimensional. During this process the valid inequalities for STSP(n) are also transformed, namely into standard scaling with respect to $\boldsymbol{x}^*$.

With regard to the fact that the standard scaling is rather a technical detail and not vital for a general understanding, this explanation shall suffice.

Chapter 2

Tilting complexes

2.1 General explanation of the concept

After the introduction of the necessary terminology, we first give an overview of the concept of tilting complexes in order to provide a general understanding. After that we will continue with a brief outline of the forthcoming construction.

A *tilting complex* $\mathcal{T}(F)$ of a good face F of $\mathrm{STSP}(n)$ is a subdivision of the conjugate face $F^{\diamond}$ of the polar. We will henceforth omit to mention F, if it is clear which face is meant. The construction of $\mathcal{T}$ requires information about all facets of $\mathrm{STSP}(n)$ containing F. The reason why this concept may be valuable will become clear after the following theorem, which is one of the core results of [The05]. $\mathcal{T}$

Theorem 2.1.1. *Let F be a good face of* $\mathrm{STSP}(n)$*. Then the tilting complex $\mathcal{T}(F)$ is combinatorially equivalent to the subcomplex of the boundary complex of the blocking polyhedron $B\big(\mathrm{GTSP}(n)\big)$ corresponding to the TT-type faces of* $\mathrm{GTSP}(n)$ *that contain F.*

Apart from corroborating the close relationship between $\mathrm{STSP}(n)$ and $\mathrm{GTSP}(n)$, the theorem indicates that by solely providing local data of the well-studied $\mathrm{STSP}(n)$ we can get information about parts of the facial structure of $\mathrm{GTSP}(n)$.

Let's have a look at Figure 2.1.1. It shows those parts of the Hasse diagram of $\mathscr{L}\big(\mathrm{GTSP}(n)\big)$ that are relevant in this context. Solid lines indicate a dimension difference of 1 between the faces of $\mathrm{GTSP}(n)$ corresponding to the vertices, whereas dashed lines mean a difference strictly greater 1. According to Proposition 1.3.4 each facet-defining inequality for $\mathrm{GTSP}(n)$ falls in one of the following three categories:

(i) trivial inequalities: $(\boldsymbol{\chi}^e, 0)$, $\forall\, e \in E_n$,

(ii) degree inequalities: $(\boldsymbol{\chi}^{\delta(u)}, 2)$, $\forall\, u \in V_n$,

(iii) inequalities in TT-form.

The facets defined by the TT-form inequalities decompose into NR- and non-NR facets.

In the figure we omitted the trivial facets, since they are of no interest. The degree facets are denoted by $F_{\boldsymbol{\partial}_i}$, the NR-facets by H_i and the non-NR facets by G_i. The

TT-type facets were restricted to those containing the face F. STSP(n) as intersection of all degree facets is obviously a face of every $F_{\boldsymbol{\partial}_i}$. The intersections of the NR-facets with STSP(n) are by definition facets of the latter, unlike the $G_i \cap \mathrm{STSP}(n)$, which are therefore located below the $H_i \cap \mathrm{STSP}(n)$.

The subcomplex of the boundary complex of $B\big(\mathrm{GTSP}(n)\big)$ mentioned in Theorem 2.1.1 is shown in Figure 2.1.2. Its dimension, i.e. the maximal dimension of a face of $B\big(\mathrm{GTSP}(n)\big)$ contained in the subcomplex, depends on the co-dimension of the face F. Before stating this more precisely, we consider an example of a tilting complex $\mathcal{T}$ of a face F of STSP(10) with co-dimension 3 as shown in Figure 2.1.3[1] (a).

Since each subdivision is in particular a polytopal complex, we can apply Definition 1.2.1 to obtain the vertex set vert$(\mathcal{T})$ of the tilting complex. In this case we have

$$\mathrm{vert}(\mathcal{T}) = \{\, \boldsymbol{a}_0, \boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_{02}, \boldsymbol{a}_{12}, \boldsymbol{a}_{012} \,\}.$$

According to Lemma 1.1.23 the vertices $\boldsymbol{a}_0, \boldsymbol{a}_1$ and $\boldsymbol{a}_2$ of $F^\diamond$ correspond to the facets of STSP(10) containing F, which are exactly the NR-facets H_0, H_1 and H_2 intersected with STSP(10). More precisely, each vertex $\boldsymbol{a}_i$ of $F^\diamond$ is the conjugate face of $H_i \cap \mathrm{STSP}(10)$. The remaining vertices of $\mathcal{T}$

$$\mathrm{vert}(\mathcal{T}) \setminus \mathrm{vert}(F^\diamond) = \{\, \boldsymbol{a}_{02}, \boldsymbol{a}_{12}, \boldsymbol{a}_{012} \,\}$$

correspond to $G_{02} \cap \mathrm{STSP}(10), G_{12} \cap \mathrm{STSP}(10)$ and $G_{012} \cap \mathrm{STSP}(10)$, where the G_i are the non-NR facets of GTSP(10) containing F. We see that the maximal dimension of faces of the tilting complex – and thus the dimension of the combinatorially equivalent subcomplex of the boundary complex – is equal to the dimension of $F^\diamond$. Due to Lemma 1.1.25 this is exactly

$$\mathrm{codim}(F) - 1.$$

Furthermore, the adjacency of the vertices in the tilting complex can be directly transferred to the corresponding faces. In order to understand this, we must first define the term of adjacency for facets of a polyhedron.

adjacent facets

Definition 2.1.2. *Two facets of a polyhedron are said to be* adjacent, *if their intersection is a ridge of the polyhedron, that is a face with co-dimension* 2.

With this in mind, we take a closer look at Figure 2.1.3 (a). For example, we can read from the tilting complex that the non-NR facet G_{012} is adjacent to H_0, H_1, G_{02} and G_{12}, but not to H_2. Similarly, we see that the two NR-facets H_0, H_1 are adjacent, but none of them is adjacent to the last NR-facet H_2.

Another interesting conclusion is the fact that $F = G_{012} \cap \mathrm{STSP}(10)$. For an explanation we refer to Lemma 2.3.12 at the end of Section 2.3.1, where we will dispose of the necessary means to prove this assertion.

Finally, we can derive results regarding certain intersections. For example, $H_1 \cap H_2$ does not correspond to a face of the tilting complex, since there exists no face of $\mathcal{T}$ that contains both $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$. Due to the combinatorial equivalence stated in Theorem 2.1.1 the intersection of these two NR-facets is therefore not an element of the subcomplex of

[1] based on Figure 5.6 (b) on page 62 of [The05]

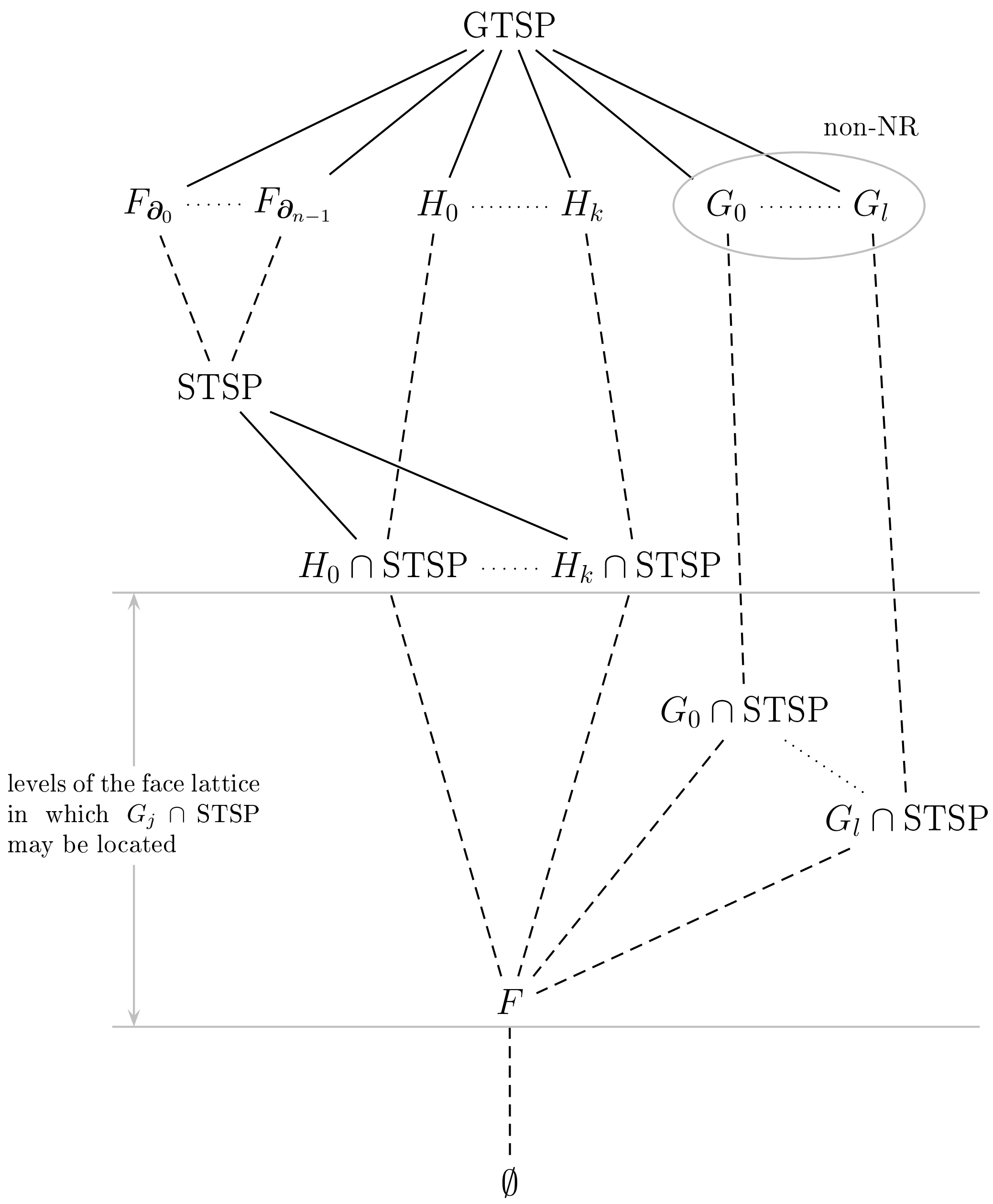


Figure 2.1.1: Structure of the face lattice of GTSP(n)

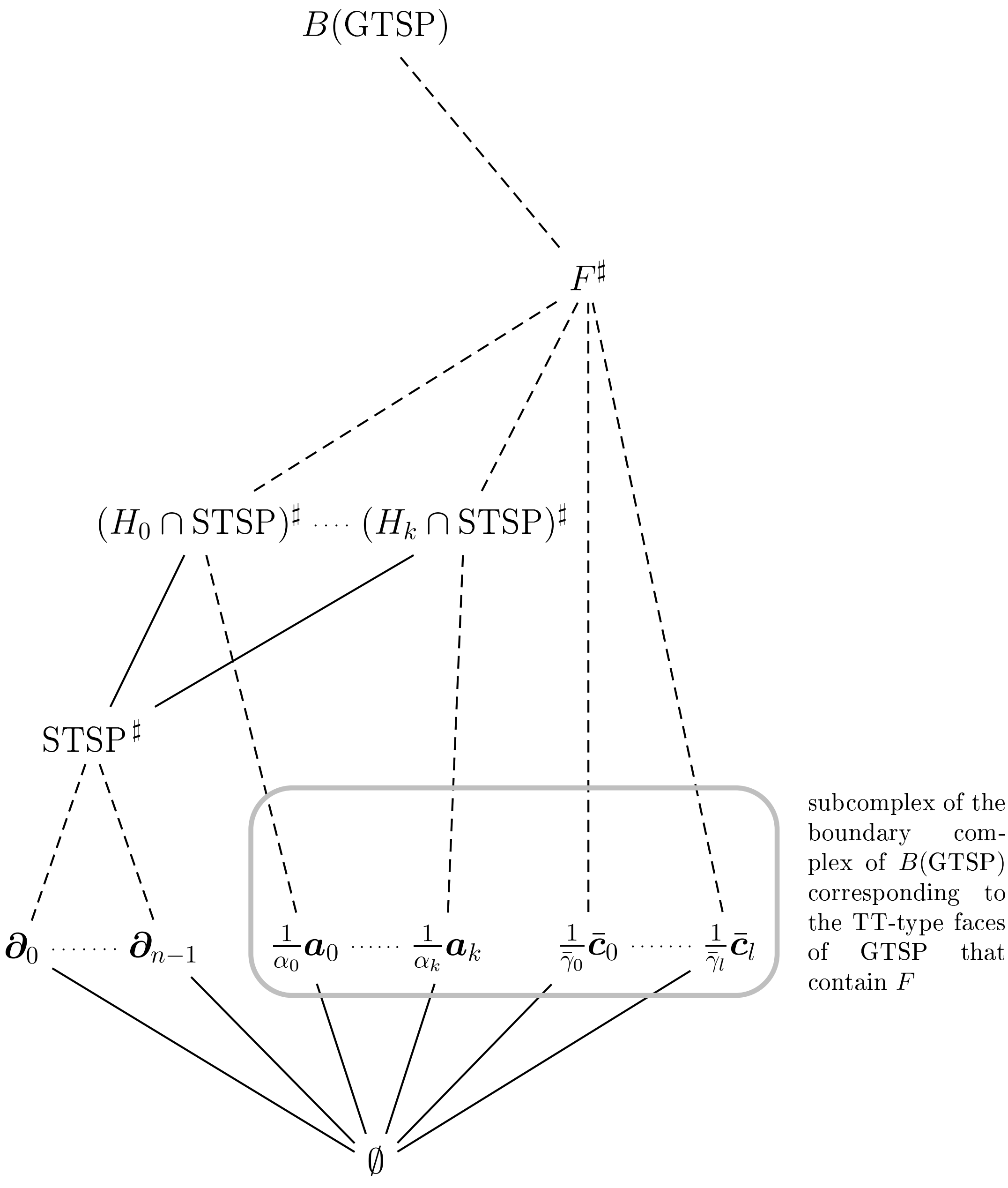
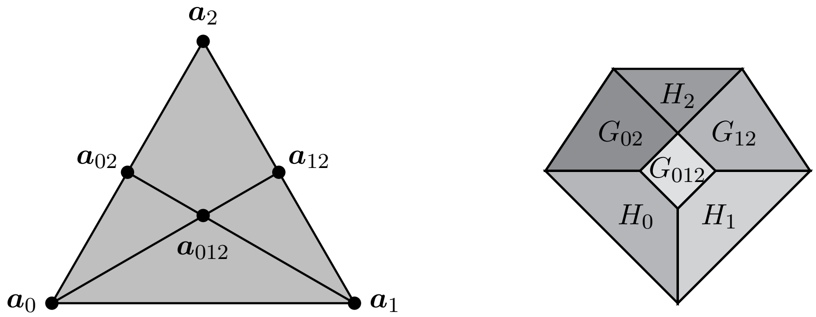


Figure 2.1.2: Structure of the face lattice of $B\big(\mathrm{GTSP}(n)\big)$



(a) $F^\diamond$ with $\mathcal{T}(F)$ (b) local facial structure of GTSP(10)

Figure 2.1.3: Example of $F^\diamond$ with corresponding tilting complex as well as the implied facial structure of GTSP(10)

the boundary complex of $B\big(\text{GTSP}(n)\big)$ corresponding to the TT-type faces of GTSP(n) that contain F. But since the H_i are exactly the NR-facets containing F, we have

$$F \subseteq H_1 \cap H_2.$$

Hence, $H_1 \cap H_2$ cannot be a TT-type face.

By extracting all these pieces of information we get a description of the local facial structure of GTSP(10). Figure 2.1.3 (b) attempts to visualize the results. However, the picture is merely illustrative, since GTSP(10) has dimension 45.

2.1.1 Outline of the construction of tilting complexes

Now that we are familiar with the concept of the tilting complex and some of its interesting properties, the question arises how we can determine $\mathcal{T}(F)$ for a given face F.

The following list is intended to be a guideline for the forthcoming sections dealing with the construction and computation of tilting complexes. It only sums up the basic steps without further explanations.

- Definition of the tilting functions $\lambda_u, u \in V_n$, and the proof that they define regular subdivisions of the k-dimensional standard simplex $\mathbb{A}^k$.
- Computation of the regular subdivisions $\mathcal{C}_u, u \in V_n$, of the standard simplex $\mathbb{A}^k$ defined by the tilting functions λ_u.
- Computation of the subdivisions $\mathcal{T}_u, u \in V_n$, of the conjugate face $F^\diamond$ as images of the regular subdivisions $\mathcal{C}_u$ under a certain projection:
  - Determination of the orthogonal projection $\mathcal{C}'_u$ of $\mathcal{C}_u$ onto $\Big(\text{pr}_{\boldsymbol{\vartheta}}\big(\ker(\widetilde{\mathbf{A}})\big)\Big)^\perp$.
  - Projection of $\mathcal{C}'_u$ onto the linear subspace defined by its affine hull.
- Intersection of all $\mathcal{T}_u$ in order to get the tilting complex $\mathcal{T} = \bigcap_{u\in V_n} \mathcal{T}_u$.

2.2 Definition of tilting functions

The following construction is taken from [The05].

Let F be a fixed good face of STSP(n) and let $\{H_j\}_{j=0,\dots,k}$ be the set of NR-facets of GTSP(n) containing F. This means that $\{H_j \cap \text{STSP}(n)\}_{j=0,\dots,k}$ is the set of facets of STSP(n) containing F. For all $j = 0,\dots,k$ let $(\boldsymbol{a}_j, \alpha_j)$ be an inequality defining H_j. We now define the following matrix

matrix $\mathbf{A}$

$$\mathbf{A} := \begin{pmatrix} \boldsymbol{a}_0 & \cdots & \boldsymbol{a}_k \\ \alpha_0 & \cdots & \alpha_k \end{pmatrix} \in \mathbb{M}\big((|E_n|+1) \times (k+1)\big),$$

whose columns are the aforementioned inequalities $(\boldsymbol{a}_j, \alpha_j)$.

For $u \in V_n$ and $e := vw \in E_n$ with $u \notin e$ we define the function

$t_{u,e}(\boldsymbol{\mu})$

$$\begin{aligned} t_{u,e}\colon \mathbb{R}^{k+1} &\longrightarrow \mathbb{R} \\ \boldsymbol{\mu} &\longmapsto \sum_{j=0}^{k} \mu_j \, \bar{t}_{u,e}(\boldsymbol{a}_j) \; = \; \sum_{j=0}^{k} \mu_j \left(a_j^{uv} + a_j^{uw} - a_j^{vw}\right) \end{aligned}$$

with triangle slacks $\bar{t}_{u,e}(\boldsymbol{a}_j)$. Obviously, $t_{u,e}(\cdot)$ is linear in $\boldsymbol{\mu}$. Finally, for $u \in V_n$ we define the vector $\boldsymbol{t}_u(\boldsymbol{\mu}) := \big(t_{u,e}(\boldsymbol{\mu})\big)_{e \in E_n \setminus \delta(u)}$ and the *tilting functions*

$\boldsymbol{t}_u(\boldsymbol{\mu})$

tilting function λ_u

$$\begin{aligned} \lambda_u\colon \mathbb{R}^{k+1} &\longrightarrow \mathbb{R} \\ \boldsymbol{\mu} &\longmapsto \min_{e \not\ni u} t_{u,e}(\boldsymbol{\mu}) \end{aligned}$$

2.3 Subdivisions of $\mathbb{\Delta}^k$

The next step is to show that each tilting function λ_u defines a regular subdivision of $\mathbb{\Delta}^k$, where $k+1$ is the number of NR-facets containing F.

Lemma 2.3.1. *Let $u \in V_n$. Then $-\lambda_{u|\mathbb{\Delta}^k}$ defines a regular subdivision of $\mathbb{\Delta}^k$.*

Proof. Due to Lemma 1.2.5 it is sufficient to show that λ_u is piecewise linear and concave. According to Lemma 5.4.3 of [The05], λ_u has indeed these claimed properties. Since no proof for the lemma was included, we provide one now.

- The piecewise linearity follows immediately from the definition of λ_u and the fact that the $t_{u,e}(\cdot)$ are linear in $\boldsymbol{\mu}$.
- Let $m \in (0,1)$ and $\boldsymbol{\mu}^{(1)}, \boldsymbol{\mu}^{(2)} \in \mathbb{R}^{k+1}$.

$$\begin{aligned} \lambda_u\big(m\,\boldsymbol{\mu}^{(1)} + (1-m)\,\boldsymbol{\mu}^{(2)}\big) &= \min_{e \not\ni u} t_{u,e}\big(m\,\boldsymbol{\mu}^{(1)} + (1-m)\,\boldsymbol{\mu}^{(2)}\big) \\ &= \min_{e \not\ni u} \left(m\, t_{u,e}(\boldsymbol{\mu}^{(1)}) + (1-m)\, t_{u,e}(\boldsymbol{\mu}^{(2)})\right) \\ &\geq m\big(\min_{e \not\ni u} t_{u,e}(\boldsymbol{\mu}^{(1)})\big) + (1-m)\big(\min_{e \not\ni u} t_{u,e}(\boldsymbol{\mu}^{(2)})\big) \\ &= m\,\lambda_u(\boldsymbol{\mu}^{(1)}) + (1-m)\,\lambda_u(\boldsymbol{\mu}^{(2)}). \end{aligned}$$

Hence, λ_u is piecewise linear and concave as asserted. ☐

Since the reflection of λ_u in aff($\mathbb{A}^k$) and the restriction of its domain to $\mathbb{A}^k$ yields a regular subdivision, we can reformulate this lemma as follows:

Theorem 2.3.2. *Let $u \in V_n$. Then λ_u defines a regular subdivision of $\mathbb{A}^k$.*

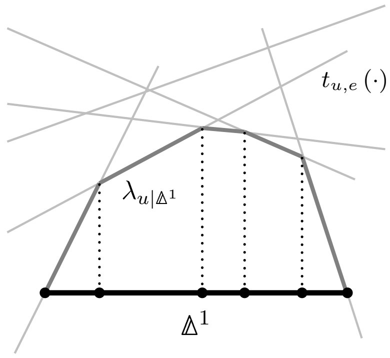


Figure 2.3.1: Example of a regular subdivision of $\mathbb{A}^1$ defined by λ_u

Figure 2.3.1 shows a possible regular subdivision of $\mathbb{A}^1$ defined by the tilting function λ_u. The fact that certain functions $t_{u,e}(\cdot)$ intersect the vertices of $\mathbb{A}^k$, which are exactly the unit vectors of $\mathbb{R}^{k+1}$, is not a coincidence but a general result.

Lemma 2.3.3. *Let $u \in V_n$. Then $\lambda_u(\boldsymbol{e}_i) = 0$ for all unit vectors $\boldsymbol{e}_0, \dots, \boldsymbol{e}_k$ of $\mathbb{R}^{k+1}$.*

Proof. First note that $\lambda_u(\boldsymbol{\mu}) \geq 0$, $\forall\, \boldsymbol{\mu} \in \mathbb{A}^k$, since

1) $\boldsymbol{\mu} \in \mathbb{A}^k \Rightarrow \boldsymbol{\mu} \geq \mathbf{0}$.

2) Each $(\boldsymbol{a}_j, \alpha_j)$ defines an NR-facet. Hence $\boldsymbol{a}_j$ is metric, i.e. $\bar{t}_{u,e}(\boldsymbol{a}_j) \geq 0$, $\forall\, e \not\ni u$.

The non-negativity of $\lambda_{u|\mathbb{A}^k}$ now follows directly from the definition of λ_u.

Furthermore, we have

$$\lambda_u(\boldsymbol{e}_i) = \min_{e \not\ni u} \sum_{j=0}^{k} (\boldsymbol{e}_i)_j \, \bar{t}_{u,e}(\boldsymbol{a}_j) = \min_{e \not\ni u} \bar{t}_{u,e}(\boldsymbol{a}_i) = 0$$

due to the fact that $(\boldsymbol{a}_i, \alpha_i)$ defines a NR-facet, which implies $\Delta_u(\boldsymbol{a}_i) \neq \emptyset$. ☐

Let's have a closer look at those $t_{u,e}(\cdot)$ intersecting the unit vectors, i.e. with

$$t_{u,e}(\boldsymbol{e}_i) = 0, \text{ for appropriate } i \in \{0, \dots, k\}.$$

As already seen in the proof of Lemma 2.3.3, we have

$$t_{u,e}(\boldsymbol{e}_i) = \sum_{j=0}^{k} (\boldsymbol{e}_i)_j \, \bar{t}_{u,e}(\boldsymbol{a}_j) = \bar{t}_{u,e}(\boldsymbol{a}_i) = 0,$$

which directly implies that $e \in \Delta_u(\boldsymbol{a}_i)$. This observation leads to the following result.

Corollary 2.3.4. *Let $u \in V_n$. Then the functions $t_{u,e}(\cdot)$ intersecting the unit vector $\boldsymbol{e}_i$ correspond to the edges in the TT-set $\Delta_u(\boldsymbol{a}_i)$ and vice versa.*

Now the question arises whether the existence of a $\bar{\boldsymbol{\mu}} \in \mathbb{A}^k$ with $\lambda_u(\bar{\boldsymbol{\mu}}) > 0$ can be proven. Unfortunately, there is no straightforward way to do so, because of the close relationship to the question of the existence of non-NR facets. However, there is a criterion for $\lambda_{u|\mathbb{A}^k} \equiv 0$.

Lemma 2.3.5. *Let $u \in V_n$. Then*

$$\bigcap_{j=0}^{k} \Delta_u(\boldsymbol{a}_j) \neq \emptyset \quad \Rightarrow \quad \lambda_u(\boldsymbol{\mu}) = 0, \forall\, \boldsymbol{\mu} \in \mathbb{A}^k.$$

In other words: if the NR-facets containing the good face F are **not** *TT-disjoint at node u, then $\lambda_{u|\mathbb{A}^k} \equiv 0$.*

Proof. The assumption $\bigcap_{j=0}^{k} \Delta_u(\boldsymbol{a}_j) \neq \emptyset$ is equivalent to the following formulation

$$\exists\, e^\star \not\ni u : e^\star \in \Delta_u(\boldsymbol{a}_j)\,, \forall\, j = 0, \ldots, k.$$

Applying the definition of the TT-sets leads to

$$\bar{t}_{u,e^\star}(\boldsymbol{a}_j) = 0\,, \forall\, j = 0, \ldots, k,$$

which directly implies

$$\sum_{j=0}^{k} \mu_j\, \bar{t}_{u,e^\star}(\boldsymbol{a}_j) = 0\,, \forall \boldsymbol{\mu} \in \mathbb{A}^k.$$

Hence, $\lambda_{u|\mathbb{A}^k} \leq 0$ due to the definition of the tilting functions. On the other hand, we know from the proof of Lemma 2.3.3 that $\lambda_{u|\mathbb{A}^k} \geq 0$. This concludes the proof. ☐

Before we deal with the actual computation of the subdivisions of $\mathbb{A}^k$, we make a short excursus in order to examine a certain mapping that appears in this context. It is negligible for the computation itself, but it yields some interesting theoretical results and also provides additional information for the understanding of Theorem 2.1.1.

2.3.1 Excursus: The mapping $(\boldsymbol{c}_{\boldsymbol{\cdot}}, \gamma_{\boldsymbol{\cdot}})$

The definition of $(\boldsymbol{c}_{\boldsymbol{\cdot}}, \gamma_{\boldsymbol{\cdot}})$ is taken from [The05].

Let $u \in V_n$. We abbreviate the normalized left hand side of the degree inequality for u by

$\boldsymbol{\partial}_u$

$$\boldsymbol{\partial}_u := \tfrac{1}{2}\,\boldsymbol{\chi}^{\delta(u)}$$

and define the matrix

matrix $\mathbf{D}$

$$\mathbf{D} := \big(\boldsymbol{\partial}_0 \;\ldots\; \boldsymbol{\partial}_{n-1}\big) \in \mathbb{M}\big(|E_n| \times |V_n|\big),$$

which is equal to the transpose of the *node-edge incidence matrix*.

Now consider the mapping

$$\begin{aligned}(\boldsymbol{c}_{\boldsymbol{\cdot}}, \gamma_{\boldsymbol{\cdot}})\colon \mathbb{R}^{k+1} &\longrightarrow \mathbb{R}^{|E_n|} \times \mathbb{R} \\ \boldsymbol{\mu} &\longmapsto (\boldsymbol{c}_{\boldsymbol{\mu}}, \gamma_{\boldsymbol{\mu}}) := \mathbf{A}\boldsymbol{\mu} - \left(\begin{smallmatrix}\mathbf{D}\\ \mathbf{1}^\top\end{smallmatrix}\right)\boldsymbol{\lambda}(\boldsymbol{\mu}),\end{aligned}$$

$\boldsymbol{\mu} \mapsto (\boldsymbol{c}_{\boldsymbol{\mu}}, \gamma_{\boldsymbol{\mu}})$

where $\mathbf{A}$ is the matrix defined in Section 2.2 and $\boldsymbol{\lambda}(\boldsymbol{\mu}) := \big(\lambda_u(\boldsymbol{\mu})\big)_{u\in V_n}$ is the vector of all tilting functions. If $\boldsymbol{\mu}$ is chosen correctly, then $(\boldsymbol{c}_{\boldsymbol{\mu}}, \gamma_{\boldsymbol{\mu}})$ defines a valid inequality for GTSP(n).

Lemma 2.3.6 (cf. Lemma 5.4.4 in [The05])**.** *For all $\boldsymbol{\mu} \geq \mathbf{0}$ the inequality $(\boldsymbol{c}_{\boldsymbol{\mu}}, \gamma_{\boldsymbol{\mu}})$ is valid for* GTSP(n) *and of TT-type.*

Proof. First, we deal with the validity of $(\boldsymbol{c}_{\boldsymbol{\mu}}, \gamma_{\boldsymbol{\mu}})$ for GTSP(n). For this purpose we start with the proof that $\boldsymbol{c}_{\boldsymbol{\mu}} = (\,\boldsymbol{a}_0\ \ldots\ \boldsymbol{a}_k\,)\,\boldsymbol{\mu} - \mathbf{D}\,\boldsymbol{\lambda}(\boldsymbol{\mu})$ is metric.

Let $u \in V_n$. $\bar{t}_{u,e}(\cdot)$ is linear, so

$$\begin{aligned}\bar{\boldsymbol{t}}_u(\boldsymbol{c}_{\boldsymbol{\mu}}) &= \underbrace{\sum_{j=0}^{k} \mu_j\, \bar{\boldsymbol{t}}_u(\boldsymbol{a}_j)}_{=\,\boldsymbol{t}_u(\boldsymbol{\mu})} - \sum_{v\in V_n} \lambda_v(\boldsymbol{\mu})\, \bar{\boldsymbol{t}}_u\big(\tfrac{1}{2}\,\boldsymbol{\chi}^{\delta(v)}\big) \\ &= \boldsymbol{t}_u(\boldsymbol{\mu}) - \lambda_u(\boldsymbol{\mu})\, \bar{\boldsymbol{t}}_u\big(\tfrac{1}{2}\,\boldsymbol{\chi}^{\delta(u)}\big) - \sum_{v\neq u} \lambda_v(\boldsymbol{\mu})\, \bar{\boldsymbol{t}}_u\big(\tfrac{1}{2}\,\boldsymbol{\chi}^{\delta(v)}\big).\end{aligned} \tag{2.3.1}$$

Note that for an arbitrary $e := vw \in E_n$ with $u \notin e$ we have

$$\bar{t}_{u,e}\big(\tfrac{1}{2}\,\boldsymbol{\chi}^{\delta(u)}\big) = \tfrac{1}{2}\Big(\underbrace{(\boldsymbol{\chi}^{\delta(u)})^{vu}}_{=1} + \underbrace{(\boldsymbol{\chi}^{\delta(u)})^{uw}}_{=1} - \underbrace{(\boldsymbol{\chi}^{\delta(u)})^{vw}}_{=0}\Big) = 1.$$

Using a similar argument, it can be shown that $\bar{\boldsymbol{t}}_u\big(\tfrac{1}{2}\,\boldsymbol{\chi}^{\delta(v)}\big) = \mathbf{0}$. Thus, (2.3.1) is reduced to

$$\bar{\boldsymbol{t}}_u(\boldsymbol{c}_{\boldsymbol{\mu}}) = \boldsymbol{t}_u(\boldsymbol{\mu}) - \lambda_u(\boldsymbol{\mu}) \cdot \mathbf{1} \geq \mathbf{0}. \tag{2.3.2}$$

The non-negativity follows directly from the definition of λ_u, implying that the vector $\boldsymbol{c}_{\boldsymbol{\mu}}$ is metric. Furthermore, $\arg\min_{e\not\ni u} t_{u,e}(\boldsymbol{\mu})$ satisfies the corresponding inequality of (2.3.2) with equality. Hence, $\Delta_u(\boldsymbol{c}_{\boldsymbol{\mu}}) \neq \emptyset$ for all $u \in V_n$, meaning that $\boldsymbol{c}_{\boldsymbol{\mu}}$ is even in TT-form.

Since $\mathbf{A}\boldsymbol{\mu}$ is a conical combination of valid inequalities and each $\boldsymbol{x} \in$ STSP(n) satisfies the degree inequalities with equality, $(\boldsymbol{c}_{\boldsymbol{\mu}}, \gamma_{\boldsymbol{\mu}})$ is valid for STSP(n). Similar to the proof of Lemma 2.5 in [NR93], one can show that each tour $\boldsymbol{x} \in$ GTSP(n) can be transformed to a Hamiltonian cycle $\boldsymbol{x}' \in$ STSP(n) by successively adding shortcuts $\boldsymbol{s}_{\tilde{u},\tilde{e}}$ for nodes $\tilde{u} \in V_n$ with $\delta(\tilde{u}) > 2$. Because of $\boldsymbol{c}_{\boldsymbol{\mu}}$ being metric, we have

$$\boldsymbol{c}_{\boldsymbol{\mu}}^\top \boldsymbol{x} \;\geq\; \boldsymbol{c}_{\boldsymbol{\mu}}^\top \boldsymbol{x}'.$$

So, the validity for GTSP(n) follows directly from the validity for STSP(n).

Let $R_{\boldsymbol{\mu}}$ be the face defined by $(\boldsymbol{c}_{\boldsymbol{\mu}}, \gamma_{\boldsymbol{\mu}})$. We finally want to show that $R_{\boldsymbol{\mu}}$ is of TT-type, i.e. it is not contained in a non-negativity facet or a degree facet.

(i) Let F be the good face of STSP(n) that is contained in the NR-facets defined by the columns of the matrix $\mathbf{A}$. Then each $\boldsymbol{x} \in F$ fulfills the inequalities $(\boldsymbol{a}_j, \alpha_j)$ for $j = 0, \ldots, k$ and $(\boldsymbol{\partial}_u, 1)$ for $u \in V_n$ with equality, resulting in $F \subseteq R_{\boldsymbol{\mu}}$. Since F is a good face, $R_{\boldsymbol{\mu}}$ can not be contained in a non-negativity facet.

(ii) Let u be an arbitrary node in V_n and $e \in \Delta_u(\boldsymbol{c}_{\boldsymbol{\mu}})$. Such an e exists due to $\boldsymbol{c}_{\boldsymbol{\mu}}$ being in TT-form. Since F is a good face of $\mathrm{STSP}(n)$, it exists a Hamiltonian cycle $\boldsymbol{x} \in F$ with $x^e > 0$. As stated in (i), $\boldsymbol{x}$ fulfills $(\boldsymbol{c}_{\boldsymbol{\mu}}, \gamma_{\boldsymbol{\mu}})$ with equality. Now we subtract the shortcut $\boldsymbol{s}_{u,e}$ from $\boldsymbol{x}$. Because of $e \in \Delta_u(\boldsymbol{c}_{\boldsymbol{\mu}})$, we have

$$\boldsymbol{c}_{\boldsymbol{\mu}}^\top(\boldsymbol{x} - \boldsymbol{s}_{u,e}) = \boldsymbol{c}_{\boldsymbol{\mu}}^\top \boldsymbol{x} - \boldsymbol{c}_{\boldsymbol{\mu}}^\top \boldsymbol{s}_{u,e} = \boldsymbol{c}_{\boldsymbol{\mu}}^\top \boldsymbol{x} + \underbrace{\bar{t}_{u,e}(\boldsymbol{c}_{\boldsymbol{\mu}})}_{=0} = \gamma_{\boldsymbol{\mu}}.$$

Hence, $\boldsymbol{x} - \boldsymbol{s}_{u,e}$ is also an element of $R_{\boldsymbol{\mu}}$. But the subtraction of the shortcut also caused

$$\left(\boldsymbol{\chi}^{\delta(u)}\right)^\top (\boldsymbol{x} - \boldsymbol{s}_{u,e}) \geq 4.$$

Thus, $R_{\boldsymbol{\mu}}$ can not be contained in the degree facet defined by $(\boldsymbol{\partial}_u, 1)$.

□

An interesting connection between $\lambda_u(\boldsymbol{\mu})$ and the TT-set $\Delta_u(\boldsymbol{c}_{\boldsymbol{\mu}})$ is stated in the following

Lemma 2.3.7. *Let $u \in V_n$ and $e \in E_n$ with $u \notin e$. Then*

$$\lambda_u(\boldsymbol{\mu}) = t_{u,e}(\boldsymbol{\mu}) \;\Leftrightarrow\; e \in \Delta_u(\boldsymbol{c}_{\boldsymbol{\mu}}).$$

Proof. From (2.3.2) we already know that

$$\bar{\boldsymbol{t}}_u(\boldsymbol{c}_{\boldsymbol{\mu}}) = \boldsymbol{t}_u(\boldsymbol{\mu}) - \lambda_u(\boldsymbol{\mu}) \cdot \mathbf{1}.$$

For a given edge $e \not\ni u$ this implies

$$\bar{t}_{u,e}(\boldsymbol{c}_{\boldsymbol{\mu}}) = t_{u,e}(\boldsymbol{\mu}) - \lambda_u(\boldsymbol{\mu}).$$

Hence, the assertion follows directly from the definition of the TT-sets, since

$$e \in \Delta_u(\boldsymbol{c}_{\boldsymbol{\mu}}) \;\Leftrightarrow\; \bar{t}_{u,e}(\boldsymbol{c}_{\boldsymbol{\mu}}) = 0.$$

□

As a consequence, given a $\boldsymbol{\mu} \in \mathbb{R}^{k+1}$ and a $u \in V_n$ we can determine the edges contained in the TT-set $\Delta_u(\boldsymbol{c}_{\boldsymbol{\mu}})$. In addition, we can derive a result regarding the TT-sets $\Delta_u(\boldsymbol{c}_{\boldsymbol{\mu}})$ and $\Delta_u(\boldsymbol{c}_{\boldsymbol{\mu}'})$ for a certain choice of $\boldsymbol{\mu}$ and $\boldsymbol{\mu}'$.

$\mathcal{C}_u$ **Lemma 2.3.8.** *Let $u \in V_n$ and $\mathcal{C}_u$ be the regular subdivision of $\mathbb{A}^k$ defined by λ_u. Furthermore, assume that $P \in \mathcal{C}_u$ is a polytope and P' is a facet of P. For arbitrary $\boldsymbol{\mu} \in \mathrm{relint}(P)$ and $\boldsymbol{\mu}' \in \mathrm{relint}(P')$ with $\boldsymbol{\mu}, \boldsymbol{\mu}' \in \mathrm{relint}(\mathbb{A}^k)$, we have*

$$\Delta_u(\boldsymbol{c}_{\boldsymbol{\mu}}) \;\subsetneq\; \Delta_u(\boldsymbol{c}_{\boldsymbol{\mu}'}).$$

Proof. Due to the construction of $\mathcal{C}_u$ from λ_u, P is the canonical projection of a proper upper face F of the polytope $Q := \mathrm{conv}\big\{ \big(\boldsymbol{\xi}^\top, \lambda_u(\boldsymbol{\xi})\big) \mid \boldsymbol{\xi} \in \mathbb{A}^k \big\}$, whose valid inequalities are defined by the functions $t_{u,e}(\cdot), e \in E_n \setminus \delta(u)$, and have the form

$$t_{u,e}(\boldsymbol{\nu}) \geq \nu_{k+1},$$

where $\boldsymbol{\nu} := (\nu_0, \dots, \nu_k) \in \mathbb{R}^{k+1}$ (cf. Figure 2.3.1). So, $\boldsymbol{\mu} \in \operatorname{relint}(P)$ is the canonical projection of a point $(\boldsymbol{\mu}^\top, \mu_{k+1}) \in F$ satisfying

$$t_{u,e}(\boldsymbol{\mu}) \geq \mu_{k+1}, \forall\, e \not\ni u.$$

Since F is a face of Q, some of these inequalities are fulfilled with equality. According to Lemma 2.3.7 those tight inequalities correspond exactly to the edges $e \in \Delta_u(\boldsymbol{c_\mu})$.

We now consider a point $\boldsymbol{\mu}' \in \operatorname{relint}(P')$. Since $\boldsymbol{\mu}'$ is in particular an element of P, it satisfies all the aforementioned (in-) equalities, implying that

$$\Delta_u(\boldsymbol{c_\mu}) \subseteq \Delta_u(\boldsymbol{c_{\mu'}}).$$

But as a consequence of P' being a facet of P, $\boldsymbol{\mu}'$ is also the canonical projection of a relative interior point $\big((\boldsymbol{\mu}')^\top, \mu'_{k+1}\big)$ of a facet F' of F. Let this facet be defined by the inequality corresponding to the edge e'. Unlike the relative interior points of F, $\big((\boldsymbol{\mu}')^\top, \mu'_{k+1}\big)$ satisfies this inequality with equality. Thus,

$$e' \in \Delta_u(\boldsymbol{c_{\mu'}}), \text{ but } e' \notin \Delta_u(\boldsymbol{c_\mu}).$$

This concludes the proof. □

So, when $\boldsymbol{\mu}$ moves from a relative interior point of a polytope $P \in \mathcal{C}_u$ to a relative interior point of one of its facets, then the corresponding TT-set $\Delta_u(\boldsymbol{c_\mu})$ expands. But even more important is the fact that the dimension of the face $R_{\boldsymbol{\mu}}$ defined by the inequality $(\boldsymbol{c_\mu}, \gamma_{\boldsymbol{\mu}})$ increases as stated in the following

Lemma 2.3.9. *Let $R_{\boldsymbol{\mu}}$ and $R_{\boldsymbol{\mu}'}$ be the faces of* GTSP(n) *defined by $(\boldsymbol{c_\mu}, \gamma_{\boldsymbol{\mu}})$ and $(\boldsymbol{c_{\mu'}}, \gamma_{\boldsymbol{\mu}'})$ respectively, with $\boldsymbol{\mu}, \boldsymbol{\mu}'$ as in Lemma 2.3.8. Then*

$$\dim(R_{\boldsymbol{\mu}'}) > \dim(R_{\boldsymbol{\mu}}).$$

For the proof of this lemma we will use the following consideration. In [NR93] it is shown how STSP(n) solutions can be obtained from GTSP(n) solutions via so-called *shortcut-reduction*, i.e. by successively adding appropriate shortcuts $\boldsymbol{s}_{u,e}$ for nodes $u \in V_n$ with $\delta(u) > 2$ to the current solution. This process is reversible, which means that subtraction of shortcuts transforms a solution of STSP(n) into one of GTSP(n). This will be referred to as *reverse-shortcut-reduction.* Since we are interested in the dimension of the faces $R_{\boldsymbol{\mu}}$ and $R_{\boldsymbol{\mu}'}$, we need to determine the GTSP(n) solutions contained in this faces.

The first step is to specify the sets of STSP(n) solutions contained in $R_{\boldsymbol{\mu}}$ and $R_{\boldsymbol{\mu}'}$ respectively. It will turn out that these sets are identical. In order to construct the GTSP(n) solutions contained in $R_{\boldsymbol{\mu}}$ and $R_{\boldsymbol{\mu}'}$ via reverse-shortcut-reduction it is imperative that in each step of this process the current solution remains contained in the given face, i.e. it fulfills the corresponding inequality $(\boldsymbol{c_\mu}, \gamma_{\boldsymbol{\mu}})$ or $(\boldsymbol{c_{\mu'}}, \gamma_{\boldsymbol{\mu}'})$ with equality. This can be assured by only applying shortcuts for triangles specified by the TT-sets $\Delta_u(\boldsymbol{c_\mu})$ or $\Delta_u(\boldsymbol{c_{\mu'}})$ and thus not altering the value of the current solution regarding this inequality. Since the STSP(n) solutions in the two faces are identical, the dimension of a particular face depends solely on the number of affinely independent shortcuts available. Thus, the application of Lemma 2.3.8 leads to the conclusion of the proof.

Proof of Lemma 2.3.9. Consider an arbitrary $\boldsymbol{\xi} \in \operatorname{relint}(\mathbb{A}^k)$. We will show that for all $\boldsymbol{x} \in \mathrm{STSP}(n)$ we have

$$\boldsymbol{x} \in R_{\boldsymbol{\xi}} \;\Leftrightarrow\; \boldsymbol{x} \in \bigcap_{j=0}^{k} H_j,$$

where the H_j are the NR-facets containing the good face F, which are defined by the inequalities $(\boldsymbol{a}_j, \alpha_j)$. For convenience, we write out the definition of $(\boldsymbol{c}_{\boldsymbol{\xi}}, \gamma_{\boldsymbol{\xi}})$ in full:

$$(\boldsymbol{c}_{\boldsymbol{\xi}}, \gamma_{\boldsymbol{\xi}}) := \mathbf{A}\boldsymbol{\xi} - \binom{\mathbf{D}}{\mathbf{1}^\top}\boldsymbol{\lambda}(\boldsymbol{\xi}) = \sum_{j=0}^{k} \xi_j \, (\boldsymbol{a}_j, \alpha_j) - \sum_{u \in V_n} \lambda_u(\boldsymbol{\xi}) \, (\boldsymbol{\partial}_u, 1).$$

Since each $\boldsymbol{x} \in \mathrm{STSP}(n)$ satisfies all degree inequalities with equality, $\boldsymbol{x}$ only has to fulfill $\sum_{j=0}^{k} \xi_j \, (\boldsymbol{a}_j, \alpha_j)$ with equality in order to be an element of $R_{\boldsymbol{\xi}} = \{\, \boldsymbol{y} \mid \boldsymbol{c}_{\boldsymbol{\xi}}^\top \boldsymbol{y} = \gamma_{\boldsymbol{\xi}} \,\}$. For all $\boldsymbol{x} \in \bigcap_{j=0}^{k} H_j$ this is obviously true, because

$$\boldsymbol{x} \in \bigcap_{j=0}^{k} H_j \;\Leftrightarrow\; \boldsymbol{a}_j^\top \boldsymbol{x} = \alpha_j \,,\; \forall\, j = 0, \ldots, k.$$

For the converse direction we first note that $\boldsymbol{\xi} \in \operatorname{relint}(\mathbb{A}^k)$ implies that the vector $\boldsymbol{\xi}$ has positive entries only. Thus, $\sum_{j=0}^{k} \xi_j \, (\boldsymbol{a}_j, \alpha_j)$ is a conical combination of the inequalities $(\boldsymbol{a}_j, \alpha_j)$. Now consider an $\boldsymbol{x} \notin \bigcap_{j=0}^{k} H_j$, i.e. $\exists\, j_0 : \boldsymbol{x} \notin H_{j_0}$. $\boldsymbol{x}$ is an element of $\mathrm{STSP}(n)$ and thus particularly contained in $\mathrm{GTSP}(n)$, which implies

$$\boldsymbol{a}_{j_0}^\top \boldsymbol{x} > \alpha_{j_0}$$

as well as

$$\boldsymbol{a}_j^\top \boldsymbol{x} \geq \alpha_j, \; \forall j \neq j_0.$$

Hence,

$$\left(\sum_{j=0}^{k} \xi_j \boldsymbol{a}_j \right)^\top \boldsymbol{x} \; > \; \sum_{j=0}^{k} \xi_j \alpha_j,$$

which results in $\boldsymbol{x}$ not being an element of $R_{\boldsymbol{\xi}}$.

This proves that the faces $R_{\boldsymbol{\mu}}$ and $R_{\boldsymbol{\mu}'}$ contain exactly the same solutions of $\mathrm{STSP}(n)$. Their dimension therefore solely depends on the number of affinely independent shortcuts that can be applied to these solutions.

Now let $e' \in \Delta_{u'}(\boldsymbol{c}_{\boldsymbol{\mu}'}) \setminus \Delta_{u'}(\boldsymbol{c}_{\boldsymbol{\mu}})$ for a $u' \in V_n$. Such an e' exists due to Lemma 2.3.8. The claim is that the corresponding shortcut $\boldsymbol{s}_{u',e'}$ is affinely independent from the shortcuts for triangles specified by the TT-sets $\Delta_u(\boldsymbol{c}_{\boldsymbol{\mu}}), u \in V_n$. For the proof assume the converse, i.e.

$$\boldsymbol{s}_{u',e'} \;=\; \sum_{\substack{u \in V_n \\ e \in \Delta_u(\boldsymbol{c}_{\boldsymbol{\mu}})}} \lambda_{u,e} \, \boldsymbol{s}_{u,e}, \;\text{ for an appropriate } \boldsymbol{\lambda} \text{ with } \mathbf{1}^\top \boldsymbol{\lambda} = 1.$$

Hence,

$$\begin{aligned}
\bar{t}_{u',e'}(\boldsymbol{c_\mu}) &= -\,\boldsymbol{c}_{\boldsymbol{\mu}}^\top \boldsymbol{s}_{u',e'} \\
&= -\,\boldsymbol{c}_{\boldsymbol{\mu}}^\top \sum_{u,e} \lambda_{u,e}\, \boldsymbol{s}_{u,e} \\
&= \sum_{u,e} \lambda_{u,e}\, \underbrace{(-\,\boldsymbol{c}_{\boldsymbol{\mu}}^\top \boldsymbol{s}_{u,e})}_{=\bar{t}_{u,e}(\boldsymbol{c_\mu})} \\
&= 0,
\end{aligned}$$

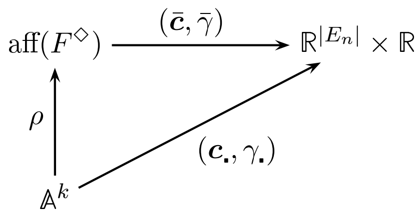
since each e is an element of the TT-set $\Delta_u(\boldsymbol{c_\mu})$ and therefore $\bar{t}_{u,e}(\boldsymbol{c_\mu}) = 0$ according to Definition 1.3.3. But $\bar{t}_{u',e'}(\boldsymbol{c_\mu}) = 0$ directly implies $e' \in \Delta_{u'}(\boldsymbol{c_\mu})$, which contradicts the assumption that e' is not an element of this particular TT-set. □

In Section 2.1 we said that the vertices in $\mathrm{vert}(\mathcal{T}) \setminus \mathrm{vert}(F^\diamond)$ correspond to the non-NR facets containing F. The formal proof of this assertion would go beyond the scope of this diploma thesis. But Lemma 2.3.9 indicates why this is actually the case, since iterating the process of moving from a polytope in $\mathcal{C}_u$ to one of its facets leads to the vertices of the subdivision. And in each step the dimension of the corresponding face increases. The complexity of the complete proof is a consequence of the fact that we are currently operating on the standard simplex, but the aspired result refers to the conjugate face $F^\diamond$. Hence, an appropriate projection has to be taken into account. Apart from that, Lemma 2.3.9 requires relative interior points and is consequently not applicable to the final iteration, since vertices don't have any relative interior points.

Now that we have revealed some of the interesting characteristics of the mapping $(\boldsymbol{c}_\bullet, \gamma_\bullet)$, it is time to introduce another mapping that is important in this context. It is also necessary for a better understanding of Theorem 2.1.1. Proposition 5.5.7 of [The05] states that there exists a mapping

$$\begin{aligned}
(\bar{\boldsymbol{c}}, \bar{\gamma})\colon \mathrm{aff}(F^\diamond) &\longrightarrow \mathbb{R}^{|E_n|} \times \mathbb{R} \\
\boldsymbol{a} &\longmapsto \big(\bar{\boldsymbol{c}}(\boldsymbol{a}), \bar{\gamma}(\boldsymbol{a})\big)
\end{aligned}$$ $(\bar{\boldsymbol{c}}, \bar{\gamma})$

with $(\bar{\boldsymbol{c}}, \bar{\gamma}) \circ \rho = (\boldsymbol{c}_\bullet, \gamma_\bullet)$ (the image under the mapping ρ is mainly an affine combination of the vertices of $F^\diamond$. For details we refer to [The05]). This connection is depicted in the following diagram[2]



$\mathbb{A}^k$ denotes the affine space of dimension k, i.e. the set of all points $\boldsymbol{x} \in \mathbb{R}^{k+1}$ satisfying $\mathbf{1}^\top \boldsymbol{x} = 1$, which is simply the affine hull of the k-dimensional standard simplex. With this additional information we are able to formulate a more precise version of Theorem 2.1.1.

[2]based on a diagram on page 66 of [The05]

Theorem 2.3.10 (cf. Theorem 5.5.20 in [The05]). *The mapping*

$$\begin{aligned} \varphi\colon F^{\diamond} &\longrightarrow B\big(\mathrm{GTSP}(n)\big) \\ \boldsymbol{a} &\longmapsto \tfrac{1}{\bar{\gamma}(\boldsymbol{a})}\,\bar{\boldsymbol{c}}(\boldsymbol{a}) \end{aligned}$$

induces an isomorphism from the face poset of the tilting complex $\mathcal{T}(F)$ onto that of the subcomplex of the boundary complex of $B\big(\mathrm{GTSP}(n)\big)$ corresponding to the TT-type faces of GTSP(n) *that contain F.*

This theorem finally enables us to address the yet unproven assertion from Section 2.1 that $F = G_{012} \cap \mathrm{STSP}(10)$. In order to facilitate the proof, we need the following auxiliary result.

Lemma 2.3.11. *Let $P \subseteq \mathbb{R}^d$ be a polyhedron and $\{(\boldsymbol{b}_i, \beta_i) \mid i \in I\}$ a set of valid inequalities defining the faces F_i of P. Furthermore, let F be the face of P defined by $\sum_{i\in I} \xi_i(\boldsymbol{b}_i, \beta_i)$ with positive scaling factors ξ_i. Then*

$$F = \bigcap_{i\in I} F_i.$$

Proof. It is evident that $\bigcap_{i\in I} F_i \subseteq F$, since every $\boldsymbol{x} \in \bigcap_{i\in I} F_i$ fulfills each inequality $(\boldsymbol{b}_i, \beta_i)$ with equality. For the converse direction we consider an arbitrary $\boldsymbol{x} \in F$. Apart from satisfying $\sum_{i\in I} \xi_i(\boldsymbol{b}_i, \beta_i)$ with equality, it is in particular an element of P, thus

$$\boldsymbol{b}_i^{\top}\boldsymbol{x} \geq \beta_i,\ \forall\, i \in I. \tag{2.3.3}$$

$\boldsymbol{x} \notin \bigcap_{i\in I} F_i$ would imply the existence of at least one index $i_0 \in I$ with $\boldsymbol{x} \notin F_{i_0}$, i.e.

$$\boldsymbol{b}_{i_0}^{\top}\boldsymbol{x} > \beta_{i_0}.$$

But due to (2.3.3) this results in $\sum_{i\in I} \xi_i(\boldsymbol{b}_i, \beta_i)$ not being fulfilled with equality, which leads to a contradiction. ☐

We now dispose of all necessary means to prove a generalization of the assertion that the face $G_{012} \cap \mathrm{STSP}(10)$ is equal to F.

Lemma 2.3.12. *Let $\mathcal{T} := \mathcal{T}(F)$ be the tilting complex of a good face F of* STSP(n). *Furthermore, let $\boldsymbol{a} \in \mathrm{vert}(\mathcal{T}) \cap \mathrm{relint}(F^{\diamond})$ be a relative interior vertex of $\mathcal{T}$ corresponding to the non-NR facet G. Then*

$$F = G \cap \mathrm{STSP}(n).$$

Proof. G is a non-NR facet. As a consequence of Theorem 2.3.10, G is defined by the inequality $\big(\varphi(\boldsymbol{a}), 1\big)$, which is equivalent to $\big(\bar{\boldsymbol{c}}(\boldsymbol{a}), \bar{\gamma}(\boldsymbol{a})\big)$. According to the diagram on page 31, this inequality can also be represented by $(\boldsymbol{c}_{\boldsymbol{\mu}}, \gamma_{\boldsymbol{\mu}})$, where $\boldsymbol{\mu}$ is the preimage of $\boldsymbol{a}$ under ρ.

We now consider the inequality $(\boldsymbol{b}, \beta) := (\boldsymbol{c}_{\boldsymbol{\mu}}, \gamma_{\boldsymbol{\mu}}) + \sum_{u\in V_n} (\boldsymbol{\partial}_u, 1)$. According to Lemma 2.3.11 it defines the face $G \cap \mathrm{STSP}(n)$. Applying the definition of $(\boldsymbol{c}_{\boldsymbol{\mu}}, \gamma_{\boldsymbol{\mu}})$ leads to

$$(\boldsymbol{b}, \beta) = \sum_{j=0}^{k} \mu_j(\boldsymbol{a}_j, \alpha_j) - \sum_{u \in V_n} \lambda_u(\boldsymbol{\mu})\,(\boldsymbol{\partial}_u, 1) + \sum_{u \in V_n} (\boldsymbol{\partial}_u, 1)$$
$$= \sum_{j=0}^{k} \mu_j(\boldsymbol{a}_j, \alpha_j) + \sum_{u \in V_n} \big(1 - \lambda_u(\boldsymbol{\mu})\big)(\boldsymbol{\partial}_u, 1).$$

When restricted to STSP(n), all degree inequalities are satisfied with equality. Hence, $(\boldsymbol{b}, \beta)$ is equivalent to $\sum_{j=0}^{k} \mu_j(\boldsymbol{a}_j, \alpha_j)$, where $\boldsymbol{\mu} > \mathbf{0}$, since $\boldsymbol{a}$ is a relative interior point of $F^{\diamond}$. Thus, in STSP(n) the inequality $(\boldsymbol{b}, \beta)$ also defines the face $\bigcap_{j=0}^{k} H_j \cap \mathrm{STSP}(n)$, which is equal to F, since the $H_j \cap \mathrm{STSP}(n)$ are exactly the facets of STSP(n) containing F. □

This concludes our excursus. We continue with the description of how the subdivisions $\mathcal{C}_u$ of the standard simplex are computed in practice.

2.3.2 Computation of the subdivisions $\mathcal{C}_u$

Since the definition of the tilting function λ_u is rather unsuitable for the purpose of computation, we choose another approach. Instead of evaluating the minimum over all functions $t_{u,e}(\cdot)$ for all edges e not containing u, we determine the corresponding inequalities for all $t_{u,e}(\cdot)$ and add the constraints for the standard simplex Δ^k. This yields a polytope $\mathcal{P}_u$ whose canonical projection defines a regular subdivision of Δ^k.

The hyperplane defining the function $t_{u,e}(\cdot)$ is given by all points $(\boldsymbol{\mu}, \mu_{k+1}) \in \mathbb{R}^{k+2}$ with $\boldsymbol{\mu} := (\mu_0, \dots, \mu_k)$ and

$$\mu_{k+1} = t_{u,e}(\boldsymbol{\mu}) = \sum_{j=0}^{k} \mu_j\, \bar{t}_{u,e}(\boldsymbol{a}_j).$$

The corresponding valid half space is defined by the inequality

$$t_{u,e}(\boldsymbol{\mu}) \geq \mu_{k+1} \;\Leftrightarrow\; t_{u,e}(\boldsymbol{\mu}) - \mu_{k+1} \geq 0.$$

Figure 2.3.2 gives an idea of how such an inequality looks like in the case $k = 1$.

Algorithm 2.3.1 uses the aforementioned idea to compute the subdivisions $\mathcal{C}_u$ of Δ^k. Note that all algorithms written for this diploma thesis were implemented with exact arithmetic using the *GNU Multiple Precision Number Package (GMP)*[3]. Besides, we are utilizing *polymake,* which is a tool for the algorithmic treatment of convex polyhedra and finite simplicial complexes. For more information on *polymake* we refer to [GJ00, GJ01].

[3]See `http://www.swox.com/gmp/` for more information

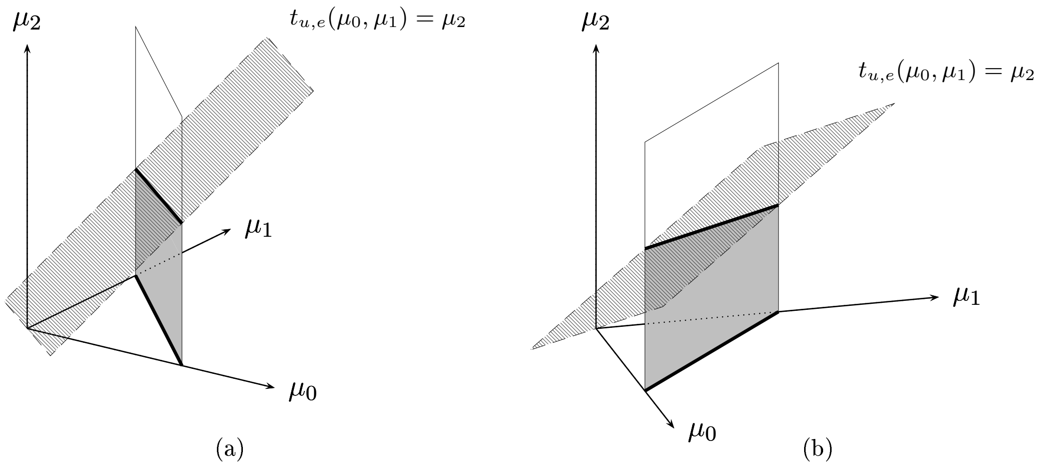


Figure 2.3.2: Example of a hyperplane defining $t_{u,e}(\cdot)$ shown from different angles

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Algorithm 2.3.1 Compute the subdivisions of $\mathbb{A}^k$

Input: Number of nodes n
NR-facets $(\boldsymbol{a}_j, \alpha_j), j = 0, \ldots, k$

Output: Subdivision $\mathcal{C}_u$ of $\mathbb{A}^k$ defined by λ_u, $\forall\, u \in V_n$

1: Convert all $(\boldsymbol{a}_j, \alpha_j)$ to standard scaling with respect to the relative interior point $\boldsymbol{x}^* := \frac{2}{n-1} \cdot \mathbf{1}$;
2: **for** ($u \in V_n$) **{**
3:    **for** ($e \in E_n \setminus \delta(u)$) **{**
4:       **for** ($j = 0, \ldots, k$) **{**
5:          Compute $\bar{t}_{u,e}(\boldsymbol{a}_j)$;
6:       **}** // for
7:       Formulate the constraint $t_{u,e}(\boldsymbol{\mu}) - \mu_{k+1} \geq 0$;
8:    **}** // for
9:    Add constraints $\boldsymbol{\mu} \geq \mathbf{0}$ and $\mathbf{1}^\top \boldsymbol{\mu} = 1$ defining $\mathbb{A}^k$. Together with the inequalities from line 7 they yield the polytope $\mathcal{P}_u$;
   // if NR-facets are TT-disjoint at node u
10:    **if** ($\mathcal{P}_u \neq \mathbb{A}^k$) **{**
11:       Compute vertex and facet information of $\mathcal{P}_u$ using *polymake*;
12:       Determine canonical projection of $\mathcal{P}_u$;
13:    **}** // if
14: **}** // for

2.3.3 A practical example for GTSP(10) [Part 1]

We consider the non-NR facet of GTSP(10) defined by the following inequality

$$\begin{aligned}
&3x_1 + 5x_2 + 6x_3 + 6x_4 + 7x_5 + 8x_6 + 9x_7 + 9x_8 + 9x_9 + 8x_{10}\\
&+ 9x_{11} + 9x_{12} + 6x_{13} + 5x_{14} + 6x_{15} + 8x_{16} + 8x_{17} + 9x_{18} + 9x_{19} + 8x_{20}\\
&+ 3x_{21} + 8x_{22} + 6x_{23} + 6x_{24} + 6x_{25} + 6x_{26} + 6x_{27} + 9x_{28} + 3x_{29} + 9x_{30}\\
&+ 9x_{31} + 6x_{32} + 3x_{33} + 9x_{34} + 6x_{35} + 9x_{36} + 6x_{37} + 9x_{38} + 3x_{39} + 9x_{40}\\
&+ 9x_{41} + 7x_{42} + 9x_{43} + 9x_{44} + 6x_{45} \geq 47.
\end{aligned}$$

Its intersection with STSP(10) has co-dimension 3 and is contained in three NR-facets given by the following inequalities

$$\begin{aligned}
&1x_1 + 2x_2 + 3x_3 + 4x_4 + 3x_5 + 3x_6 + 4x_7 + 4x_8 + 4x_9 + 3x_{10}\\
&+ 4x_{11} + 5x_{12} + 2x_{13} + 2x_{14} + 3x_{15} + 3x_{16} + 3x_{17} + 3x_{18} + 4x_{19} + 3x_{20}\\
&+ 1x_{21} + 4x_{22} + 2x_{23} + 2x_{24} + 3x_{25} + 2x_{26} + 2x_{27} + 5x_{28} + 1x_{29} + 3x_{30}\\
&+ 5x_{31} + 3x_{32} + 2x_{33} + 4x_{34} + 4x_{35} + 4x_{36} + 3x_{37} + 3x_{38} + 1x_{39} + 5x_{40}\\
&+ 3x_{41} + 3x_{42} + 4x_{43} + 4x_{44} + 2x_{45} \geq 20,
\end{aligned}$$

$$\begin{aligned}
&1x_1 + 2x_2 + 3x_3 + 2x_4 + 3x_5 + 3x_6 + 3x_7 + 5x_8 + 4x_9 + 3x_{10}\\
&+ 4x_{11} + 3x_{12} + 2x_{13} + 2x_{14} + 2x_{15} + 4x_{16} + 3x_{17} + 5x_{18} + 4x_{19} + 3x_{20}\\
&+ 1x_{21} + 3x_{22} + 3x_{23} + 2x_{24} + 3x_{25} + 4x_{26} + 4x_{27} + 4x_{28} + 2x_{29} + 5x_{30}\\
&+ 3x_{31} + 3x_{32} + 1x_{33} + 5x_{34} + 2x_{35} + 4x_{36} + 2x_{37} + 4x_{38} + 1x_{39} + 4x_{40}\\
&+ 4x_{41} + 3x_{42} + 4x_{43} + 3x_{44} + 3x_{45} \geq 20,
\end{aligned}$$

$$\begin{aligned}
&4x_1 + 6x_2 + 7x_3 + 7x_4 + 8x_5 + 10x_6 + 11x_7 + 9x_8 + 10x_9 + 10x_{10}\\
&+ 11x_{11} + 11x_{12} + 8x_{13} + 6x_{14} + 7x_{15} + 9x_{16} + 10x_{17} + 11x_{18} + 11x_{19} + 10x_{20}\\
&+ 4x_{21} + 9x_{22} + 7x_{23} + 8x_{24} + 8x_{25} + 7x_{26} + 7x_{27} + 10x_{28} + 4x_{29} + 11x_{30}\\
&+ 11x_{31} + 7x_{32} + 4x_{33} + 10x_{34} + 7x_{35} + 10x_{36} + 7x_{37} + 11x_{38} + 4x_{39} + 9x_{40}\\
&+ 11x_{41} + 8x_{42} + 10x_{43} + 11x_{44} + 7x_{45} \geq 58.
\end{aligned}$$

Consequently, k is equal to 2. The computation of $\mathcal{P}_u$ for all $u = 0, \ldots, 9$ shows that the NR-facets are TT-disjoint at the nodes 3 and 4. Hence, the polytope $\mathcal{P}_u$ differs from the two-dimensional standard simplex $\mathbb{\Delta}^2$ for these two nodes. In this example we only consider $\mathcal{P}_3$, since $\mathcal{P}_4$ looks very similar.

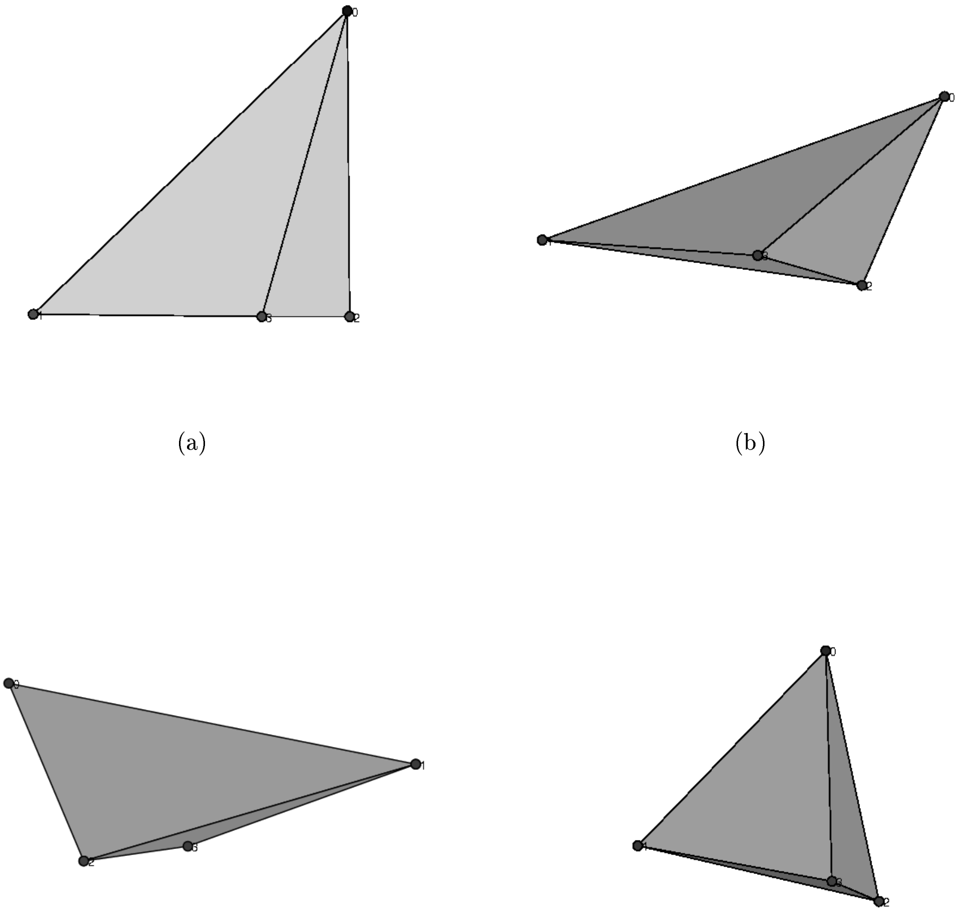
Figure 2.3.3 shows a visualization of $\mathcal{P}_3$ created with *polymake*. The upper face in Figure 2.3.3 (c) is equal to $\mathbb{\Delta}^2$ and the top view in Figure 2.3.3 (a) gives an idea of how the canonical projection of $\mathcal{P}_3$ looks like.

2.4 Subdivisions of $F^\diamond$

Given the subdivisions $\mathcal{C}_u$ of the standard simplex from the previous section, we are able to obtain the subdivisions of $F^\diamond$ by applying a result from [The05]. But for the formulation of this result we have to introduce some additional terminology first.

With the matrix $\mathbf{A}$ from Section 2.2 and the matrix $\mathbf{D}$ from the excursus we compose the matrix

$$\widetilde{\mathbf{A}} := \left(\mathbf{A}, -\left(\begin{smallmatrix}\mathbf{D}\\ \mathbf{1}^\top\end{smallmatrix}\right)\right) \in \mathbb{M}\big((|E_n|+1) \times (|V_n|+k+1)\big),$$ matrix $\widetilde{\mathbf{A}}$



(a) (b)

(c) (d)

Figure 2.3.3: Visualization of the polytope $\mathcal{P}_3$ from different angles

whose first $k+1$ columns are the inequalities $(\boldsymbol{a}_j, \alpha_j)$ defining the NR-facets of GTSP(n) that contain F. The remaining $|V_n|$ columns are the negative coefficients of the normalized degree inequalities. For a vector $\boldsymbol{x} \in \mathbb{R}^{|V_n|+k+1}$ we abbreviate the first $k+1$ coefficients by $\boldsymbol{\vartheta}$ and the last $|V_n|$ coefficients by $\boldsymbol{\xi}$

$$\boldsymbol{x} = \binom{\boldsymbol{\vartheta}}{\boldsymbol{\xi}}.$$

For instance, using this notation the inequality $(\boldsymbol{c}_{\boldsymbol{\mu}}, \gamma_{\boldsymbol{\mu}}) = \mathbf{A}\boldsymbol{\mu} - \binom{\mathbf{D}}{\mathbf{1}^\top}\boldsymbol{\lambda}(\boldsymbol{\mu})$ from the excursus could be written as

$$\widetilde{\mathbf{A}} \binom{\boldsymbol{\mu}}{\boldsymbol{\lambda}(\boldsymbol{\mu})}.$$

Finally, we introduce the projection

$\mathrm{pr}_{\boldsymbol{\vartheta}}$

$$\mathrm{pr}_{\boldsymbol{\vartheta}} \colon \mathbb{R}^{|V_n|+k+1} \longrightarrow \mathbb{R}^{k+1}$$
$$\boldsymbol{x} = \binom{\boldsymbol{\vartheta}}{\boldsymbol{\xi}} \longmapsto \boldsymbol{\vartheta}$$

and define

$$\Theta := \mathrm{pr}_{\boldsymbol{\vartheta}}\big(\ker(\widetilde{\mathbf{A}})\big).$$ Θ

We are now ready to formulate the result from [The05].

Theorem 2.4.1. *The orthogonal projection of $\mathbb{A}^k$ onto $\Theta^{\perp}$, which is subsequently projected onto the linear subspace defined by its affine hull, is affinely isomorphic to $F^{\diamond}$. Regular subdivisions of $\mathbb{A}^k$ are transformed into subdivisions of $F^{\diamond}$, although regularity is generally lost.*

The remainder of this section will deal with the different steps of computing a subdivision of $F^{\diamond}$ out of a subdivision $\mathcal{C}_u$ of $\mathbb{A}^k$.

2.4.1 Orthogonal projection of $\mathcal{C}_u$ onto $\Theta^{\perp}$

For the calculation of an orthogonal projection onto $\Theta^{\perp}$ we need either an orthogonal or an orthonormal basis of $\Theta^{\perp}$. The latter would require the use of the Euclidean norm and hence the extraction of roots. Since this operation is not supported by the *GMP* package, we utilize an orthogonal basis.

We start with the determination of a basis of $\ker(\widetilde{\mathbf{A}})$. This can be obtained by applying Gaussian elimination to $\widetilde{\mathbf{A}}$, which is equivalent to solving the homogeneous linear equation system $\widetilde{\mathbf{A}}\binom{\boldsymbol{\vartheta}}{\boldsymbol{\xi}} = \mathbf{0}$. Since $\widetilde{\mathbf{A}}$ is a non-square matrix, it will be transformed into reduced row-echelon form. And it is a well-known fact from the field of linear algebra that a basis of the kernel can easily be extracted from this form. Let

$$\mathbf{B} := (\boldsymbol{b}_0, \dots, \boldsymbol{b}_l)$$

be this basis. Then

$$\big(\mathrm{pr}_{\boldsymbol{\vartheta}}(\boldsymbol{b}_0), \dots, \mathrm{pr}_{\boldsymbol{\vartheta}}(\boldsymbol{b}_l)\big)$$

forms a generating system of Θ. In general, we cannot assume this generating system to be a basis, since the projection $\mathrm{pr}_{\boldsymbol{\vartheta}}$ may destroy the linear independence of the basis vectors of $\ker(\widetilde{\mathbf{A}})$. Let $\mathbf{P}$ be the matrix, whose columns are the vectors $\mathrm{pr}_{\boldsymbol{\vartheta}}(\boldsymbol{b}_0), \dots, \mathrm{pr}_{\boldsymbol{\vartheta}}(\boldsymbol{b}_l)$. matrix $\mathbf{P}$
Then

$$\Theta = \mathrm{im}(\mathbf{P}).$$

At this point we need another result from the field of linear algebra.

Lemma 2.4.2. *Let $\mathbf{M} \in \mathbb{M}(m \times n)$ define a linear mapping. Then*

$$\big(\mathrm{im}(\mathbf{M})\big)^{\perp} = \ker(\mathbf{M}^{\top}).$$

Proof. Let $\boldsymbol{y} \in \big(\mathrm{im}(\mathbf{M})\big)^{\perp}$. This is equivalent to the following formulation

$$\boldsymbol{y}^{\top}\mathbf{M}\boldsymbol{x} \;=\; 0, \forall\, \boldsymbol{x} \in \mathbb{R}^n,$$

which is the same as

$$\big(\mathbf{M}^{\top}\boldsymbol{y}\big)^{\top}\boldsymbol{x} \;=\; 0, \forall\, \boldsymbol{x} \in \mathbb{R}^n.$$

Since the above equation must be fulfilled by *every* vector $\boldsymbol{x} \in \mathbb{R}^n$, this is true if and only if

$$\mathbf{M}^{\top}\boldsymbol{y} = 0,$$

which means that $\boldsymbol{y} \in \ker(\mathbf{M}^{\top})$. □

As a consequence of this lemma, we get a representation of $\Theta^\perp$ subject to the matrix $\mathbf{P}$, namely

$$\Theta^\perp \;=\; \big(\mathrm{im}(\mathbf{P})\big)^\perp \;=\; \ker(\mathbf{P}^\top).$$

With this representation we can compute a basis of $\Theta^\perp$ using Gaussian elimination as explained before, which is then transformed into an orthogonal basis $\overline{\mathbf{B}} := (\bar{\boldsymbol{b}}_0, \dots, \bar{\boldsymbol{b}}_t)$ via the Gram-Schmidt procedure.

Given this basis, we can determine the orthogonal projection of a subdivision $\mathcal{C}_u$ of $\mathbb{A}^k$ onto $\Theta^\perp$ by projecting all the vertices $\boldsymbol{v} \in \mathrm{vert}(\mathcal{C}_u)$. The image of each $\boldsymbol{v}$ is given by

$$\boldsymbol{v}' \;:=\; \mathrm{pr}_\perp(\boldsymbol{v}) \;=\; \sum_{i=0}^{t} \frac{\boldsymbol{v}^\top \bar{\boldsymbol{b}}_i}{\|\bar{\boldsymbol{b}}_i\|_2^2}\, \bar{\boldsymbol{b}}_i \;=\; \sum_{i=0}^{t} \frac{\boldsymbol{v}^\top \bar{\boldsymbol{b}}_i}{\bar{\boldsymbol{b}}_i^\top \bar{\boldsymbol{b}}_i}\, \bar{\boldsymbol{b}}_i. \tag{2.4.1}$$

We denote this orthogonal projection of $\mathcal{C}_u$ by $\mathcal{C}'_u$. In general, $\mathcal{C}'_u$ may not be full-dimensional. Thus, the next step consists in the projection of $\mathcal{C}'_u$ onto the linear subspace defined by its affine hull.

2.4.2 Projection of $\mathcal{C}'_u$ onto the linear subspace defined by its affine hull

For this purpose we first need to determine an orthogonal basis of $\mathrm{aff}(\mathcal{C}'_u)$. Then we can calculate the coordinates of the projected vertices $\boldsymbol{v}'$ with respect to this basis. This will reduce the dimension of the vertices to that of $\mathrm{aff}(\mathcal{C}'_u)$.

The basic idea for getting a basis of $\mathrm{aff}(\mathcal{C}'_u)$ is to take a fixed vertex $\boldsymbol{v}'_*$ and compute the vectors from this vertex to all the other vertices of $\mathcal{C}'_u$. These vectors yield a generating system of $\mathrm{aff}(\mathcal{C}'_u)$, which can be transformed into an orthogonal basis using the Gram-Schmidt procedure.

First we note that we don't need all the vertices of $\mathcal{C}'_u$ for the computation of the generating system. Since $\mathbb{A}^k$ is the convex hull of the unit vectors of $\mathbb{R}^{k+1}$, it is sufficient to only consider the projected unit vectors $\boldsymbol{e}'_0, \dots, \boldsymbol{e}'_k$. To simplify matters, we additionally assume that the vertices of $\mathcal{C}'_u$ are sorted such that the first $k+1$ vertices correspond to the unit vectors in reverse lexicographical order, i.e. if $\boldsymbol{v}'_0, \dots, \boldsymbol{v}'_s, s > k$, are the vertices of $\mathcal{C}'_u$, then

$$\boldsymbol{v}'_i \;=\; \boldsymbol{e}'_i\,,\ \forall\, i = 0, \dots, k.$$

We choose $\boldsymbol{v}'_0$ as fixed vertex and get $\big((\boldsymbol{v}'_1 - \boldsymbol{v}'_0), \dots, (\boldsymbol{v}'_k - \boldsymbol{v}'_0)\big)$ as generating system of $\mathrm{aff}(\mathcal{C}'_u)$. If the vertices were not sorted this way, the generating system and thus the resulting basis would vary depending on the order of the projected unit vectors and a coordinate transformation would be necessary in order to compare the results for different nodes $u \in V_n$. Hence, it is easier to sort the vertices in the forefront of the computation.

Next, we apply the Gram-Schmidt procedure to the vectors $(\boldsymbol{v}'_1 - \boldsymbol{v}'_0), \dots, (\boldsymbol{v}'_k - \boldsymbol{v}'_0)$. In its standard version it requires the input vectors to be linearly independent, but with a slight modification it is able to handle linearly dependent vectors as well.

In each step the Gram-Schmidt procedure takes one of the remaining input vectors and calculates the difference of this vector and its orthogonal projection onto the linear space that is spanned by all orthogonal basis vectors computed up to this time. In the standard version a problem occurs, if the current vector lies within this space. Then it is identical to its orthogonal projection, thus the difference is $\mathbf{0}$. Obviously, this case can easily be detected. Each such vector is simply removed from the input list and the computation continues with the next input vector.

So, let $\mathbf{B}^{\text{aff}}$ be the resulting orthogonal basis of $\text{aff}(\mathcal{C}'_u)$. For the calculation of the coordinates of the vertices $\boldsymbol{v}'$ with respect to this basis, we revisit (2.4.1) and notice that the coefficients

$$\frac{(\boldsymbol{v}')^\top \boldsymbol{b}_i^{\text{aff}}}{\|\boldsymbol{b}_i^{\text{aff}}\|_2^2} = \frac{(\boldsymbol{v}')^\top \boldsymbol{b}_i^{\text{aff}}}{(\boldsymbol{b}_i^{\text{aff}})^\top \boldsymbol{b}_i^{\text{aff}}}$$

of the orthogonal projection of $\boldsymbol{v}'$ onto $\text{aff}(\mathcal{C}'_u)$ with respect to the basis $\mathbf{B}^{\text{aff}}$ are exactly the coordinates we are looking for. Finally, we have computed a subdivision of the conjugate face $F^\diamond$, also referred to as *tilting complex at node* u, which we will denote by $\mathcal{T}_u$. $\mathcal{T}_u$

For the visualization we would like to give the results with respect to an ortho*normal* basis, so there is one more step to be made, namely the i-th coordinate of each vertex has to be multiplied by the Euclidean norm of the i-th basis vector of $\mathbf{B}^{\text{aff}}$. As stated before, the *GMP* package does not support the extraction of roots, hence a typecast to *double* is necessary, accompanied by the inevitable precision loss. But the visual representation of $\mathcal{T}_u$ is the final output, i.e. the results are no longer needed for future computation. Therefore the precision loss is tolerable.

An abstract of this whole process in pseudocode is given in Algorithm 2.4.1.

As soon as all $\mathcal{T}_u$ are computed, we can obtain the tilting complex $\mathcal{T}$ by intersecting all tilting complexes at the different nodes, which means

$$\mathcal{T} = \bigcap_{u \in V_n} \mathcal{T}_u.$$

Note that the union of the vertex sets $\big(\text{vert}(\mathcal{T}_u)\big)_{u \in V_n}$ may be a proper subset of $\text{vert}(\mathcal{T})$, as depicted in Figure 2.4.1.

At this point it is necessary to explain how the good faces for the computation of tilting complexes were chosen. We started with known TT-type facets G of $\text{GTSP}(n)$. For each of them the dimension of the intersection $F := G \cap \text{STSP}(n)$ was calculated. Those F with $\text{codim}(F) > 1$, i.e. the intersections of non-NR facets with $\text{STSP}(n)$, were taken as good faces for further computation.

The idea behind this approach is the following. The intersection of a TT-type facet with $\text{STSP}(n)$ cannot be contained in a non-negativity facet, thus being a good face of $\text{STSP}(n)$. And only the non-NR facets lead to reasonable results, since for an NR-facet the number $k+1$ of NR-facets containing this particular facet is one. Accordingly, k is equal to zero, which results in the zero-dimensional standard simplex $\mathbb{\Delta}^0$ that consists of one single vertex and thus cannot be subdivided.

So far, we have only encountered non-NR facets, whose corresponding face F of $\text{STSP}(n)$ has $\text{codim}(F) \leq 3$. As a consequence, we have $\dim(F^\diamond) \leq 2$ due to Lemma 1.1.25. The tilting complex can therefore easily be obtained by simply overlaying all $\mathcal{T}_u$. In the case that non-NR facets should be discovered, whose intersection with $\text{STSP}(n)$ has $\text{codim}(F) > 3$, one would need a more sophisticated method for the intersection of the $\mathcal{T}_u$.

Algorithm 2.4.1 Compute the tilting complex at node $u \in V_n$

Input: Number of nodes n
NR-facets $(\boldsymbol{a}_j, \alpha_j), j = 0, \ldots, k$
Subdivision $\mathcal{C}_u$ of $\mathbb{A}^k$

Output: $\mathcal{T}_u$

Generate the matrix $\widetilde{\mathbf{A}} := \Big(\mathbf{A}, -\big(\begin{smallmatrix}\mathbf{D}\\ \mathbf{1}^\top\end{smallmatrix}\big)\Big)$;

// Compute generating system of $\Theta := \mathrm{pr}_{\boldsymbol{\vartheta}}\big(\ker(\widetilde{\mathbf{A}})\big)$

Determine a basis $\mathbf{B} := (\boldsymbol{b}_0, \ldots, \boldsymbol{b}_l)$ of $\ker(\widetilde{\mathbf{A}})$ by solving the homogeneous linear equation system $\widetilde{\mathbf{A}}\big(\begin{smallmatrix}\boldsymbol{\vartheta}\\ \boldsymbol{\xi}\end{smallmatrix}\big) = \mathbf{0}$ via Gaussian elimination;

$\Theta = \mathrm{im}(\mathbf{P})$, with $\mathbf{P} := \big(\mathrm{pr}_{\boldsymbol{\vartheta}}(\boldsymbol{b}_0), \ldots, \mathrm{pr}_{\boldsymbol{\vartheta}}(\boldsymbol{b}_l)\big)$;

// Compute an orthogonal basis of $\Theta^\perp$

$\Theta^\perp = \mathrm{im}(\mathbf{P})^\perp = \ker\big(\mathbf{P}^\top\big)$. Determine a basis of $\Theta^\perp$ by solving $\mathbf{P}^\top \boldsymbol{x} = \mathbf{0}$ via Gaussian elimination and obtain an orthogonal basis $\overline{\mathbf{B}}$ of $\Theta^\perp$ by applying the Gram-Schmidt procedure to it;

// Compute orthogonal projection of $\mathcal{C}_u$ onto $\Theta^\perp$

Sort the vertices $\boldsymbol{v}_i$ of $\mathcal{C}_u$ such that the first $k+1$ vertices are the unit vectors in reverse lexicographical order;

for ($\boldsymbol{v}_i \in \mathrm{vert}(\mathcal{C}_u)$) **{**

  Determine the orthogonal projection $\boldsymbol{v}'_i$ of $\boldsymbol{v}_i$ onto $\Theta^\perp$ with respect to $\overline{\mathbf{B}}$;

} // for

// Project the above projection $\mathcal{C}'_u$ of $\mathcal{C}_u$ onto the linear subspace defined by its affine // hull

Compute the vectors $\big((\boldsymbol{v}'_1 - \boldsymbol{v}'_0), \ldots, (\boldsymbol{v}'_k - \boldsymbol{v}'_0)\big)$ in order to obtain a generating system of $\mathrm{aff}(\mathcal{C}'_u)$. Apply the Gram-Schmidt procedure to get an orthogonal basis $\mathbf{B}^{\mathrm{aff}}$;

Determine the coordinates of the projected vertices $\boldsymbol{v}'_i$ with respect to $\mathbf{B}^{\mathrm{aff}}$. These are exactly the coefficients of the orthogonal projection of $\boldsymbol{v}'_i$ onto $\mathrm{aff}(\mathcal{C}'_u)$;

For the purpose of visualization, multiply the j-th coordinate of $\boldsymbol{v}'_i$ by $\big\|\boldsymbol{b}^{\mathrm{aff}}_j\big\|_2$;

2.4.3 A practical example for GTSP(10) [Part 2]

We now conclude the example from Section 2.3.3, where we found out that the polytopes $\mathcal{P}_3$ and $\mathcal{P}_4$ were the only ones differing from the standard simplex. Applying Algorithm 2.4.1 to their canonical projections $\mathcal{C}_3$ and $\mathcal{C}_4$ results in the subdivisions $\mathcal{T}_3$ and $\mathcal{T}_4$ of $F^\diamond$ as shown in Figure 2.4.1. The intersection of the tilting complexes at nodes 3 and 4 leads to the tilting complex $\mathcal{T}$, which we have already seen in Figure 2.1.3 (a).

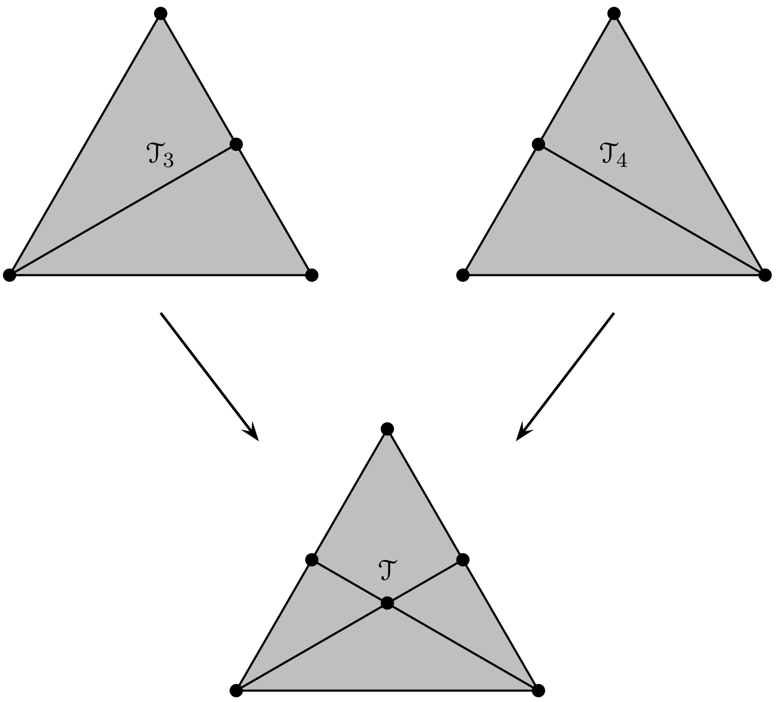


Figure 2.4.1: $F^\diamond$ with $\mathcal{T}_3$, $\mathcal{T}_4$ and $\mathcal{T}$

Chapter 3

Computation of NR-facets

Since their introduction in section 2.2, the NR-facets containing the good face F of $\mathrm{STSP}(n)$ were always assumed to be a given input. This chapter will finally answer the question how to actually compute them.

For this purpose we remember that according to Lemma 1.1.23 the vertices of $F^{\diamond}$ correspond exactly to those facets of $\mathrm{STSP}(n)$ containing F. Thus, we need an outer description of $F^{\diamond}$ in order to compute the vertices of $F^{\diamond}$ using *polymake*. At this point we get back to Theorem 1.1.18, which explains how this description can be obtained. Once we have these vertices of $F^{\diamond}$, we apply Theorem 1.1.15 (vi) to get the corresponding facet-defining inequalities of $\mathrm{STSP}(n)$. Finally, we transform the inequalities into TT-form. The main advantage of this method is the fact that solely information about $\mathrm{STSP}(n)$ is needed for the computation.

The remainder of this chapter will give a detailed explanation of how to implement the above construction.

3.1 An outer description of $F^{\diamond}$

As stated before, we want to use Theorem 1.1.15 (vi) in the course of this implementation, which explicitly requires $\mathbf{0}$ to be a relative interior point of the polytope we are considering. Since this is not the case for $\mathrm{STSP}(n)$, we have to translate it in such a way, that a relative interior point is mapped to the origin. Again we choose $\boldsymbol{x}^* := \frac{2}{n-1} \cdot \mathbf{1}$ to be this point. Hence, the translated version of $\mathrm{STSP}(n)$, which we will denote by $S_\downarrow$, is obtained by mapping every vertex $\boldsymbol{v}$ of $\mathrm{STSP}(n)$ to $S_\downarrow$

$$\boldsymbol{v}_\downarrow := \boldsymbol{v} - \boldsymbol{x}^*.$$

Let $F_\downarrow$ be the face of $S_\downarrow$ corresponding to F. According to Theorem 1.1.18 an outer description of $F_\downarrow^{\diamond}$ is given by $F_\downarrow$

$$F_\downarrow^{\diamond} = \{\, \boldsymbol{a} \mid \boldsymbol{a}^\top \boldsymbol{v}_\downarrow \leq 1,\ \forall\, \boldsymbol{v}_\downarrow \in V_{F_\downarrow}^{\mathsf{c}} \ \text{ and } \\ \boldsymbol{a}^\top \boldsymbol{v}_\downarrow = 1,\ \forall\, \boldsymbol{v}_\downarrow \in V_{F_\downarrow} \,\},$$

where $V_{F_\downarrow} := \mathrm{vert}(F_\downarrow)$ and $V_{F_\downarrow} \uplus V_{F_\downarrow}^{\mathsf{c}} = \mathrm{vert}\big(S_\downarrow\big) =: V$. Since $\mathrm{STSP}(n)$ is defined as the convex hull of the incidence vectors $\boldsymbol{\chi}^H$ of all Hamiltonian cycles, we have

- $V \quad = \{\, \boldsymbol{\chi}^H_{\downarrow} \mid H \text{ is Hamiltonian cycle} \,\}$
- $V_{F_{\downarrow}} = \{\, \boldsymbol{\chi}^H_{\downarrow} \mid H \text{ is Hamiltonian cycle } \wedge \boldsymbol{\chi}^H \in F \,\}$
- $V^{\mathsf{c}}_{F_{\downarrow}} = \{\, \boldsymbol{\chi}^H_{\downarrow} \mid H \text{ is Hamiltonian cycle } \wedge \boldsymbol{\chi}^H \notin F \,\}$

Consequently,

$$\boldsymbol{v}_{\downarrow}^{\top} \boldsymbol{a} \leq 1, \ \forall\, \boldsymbol{v}_{\downarrow} \in V^{\mathsf{c}}_{F_{\downarrow}}$$

yield the inequalities and

$$\boldsymbol{v}_{\downarrow}^{\top} \boldsymbol{a} = 1, \ \forall\, \boldsymbol{v}_{\downarrow} \in V_{F_{\downarrow}}$$

the equalities of the outer description. In the current form the description contains a lot of redundant inequalities. Although *polymake* is able to handle redundancy, we have to reduce the number of dominated inequalities, because otherwise the *polymake* input file may become too large. For example, an input file for GTSP(12) would have a size of over 6 GB!

In order to reduce redundancy, we use the following approach:

Algorithm 3.1.1 Eliminate redundant inequalities

```
Let I be the index set of V^c_{F↓}, J the index set of V_{F↓} and N := ∅ the index set of
  translated vertices defining non-redundant inequalities;
Choose an arbitrary i ∈ I. Set N = {i} and I = I \ {i};
while ( I ≠ ∅ ) {
    Choose an arbitrary i ∈ I and set I = I \ {i};
    Test for the existence of coefficients λ_n ≥ 0, n ∈ N, and μ_j, j ∈ J, such that
      (i)   Σ_{n∈N} λ_n v↓^(n) + Σ_{j∈J} μ_j v↓^(j) = v↓^(i)
      (ii)  Σ_{n∈N} λ_n        + Σ_{j∈J} μ_j        ≤ 1
    by solving a linear program (LP) with CPLEX;
    if ( LP is infeasible ) {
        Add (v↓^(i))^T a ≤ 1 to outer description of F↓^◇ and set N = N ∪ {i};
    } // if
} // while
```

where step (i) and (ii) read:

$$\text{(i)} \quad \textstyle\sum_{n \in N} \lambda_n \boldsymbol{v}_{\downarrow}^{(n)} + \sum_{j \in J} \mu_j \boldsymbol{v}_{\downarrow}^{(j)} = \boldsymbol{v}_{\downarrow}^{(i)}$$

$$\text{(ii)} \quad \textstyle\sum_{n \in N} \lambda_n + \sum_{j \in J} \mu_j \leq 1$$

Note that in each cycle of the while loop the LP hardly alters. Only the right hand side is changed and at most one new column is added. Hence, Algorithm 3.1.1 can be efficiently implemented using the *CPLEX* C-interface. The results are convincing. In the case of GTSP(10) the above method has reduced

- the size of the *polymake* input files from 34 MB to 13 KB.
- the overall running time from 10 minutes to less than 30 seconds on an Intel® Xeon™ 2.80 GHz processor with 2 GB of RAM.

But the outer description is not complete, yet. Although the face F is a subset of the $|E_n|$-dimensional space, it is still embedded in the affine hull of STSP(n), which is determined by the degree equalities

$$\boldsymbol{\partial}_u^\top \boldsymbol{x} = 1\,,\ \forall\, u \in V_n.$$

Thus, these equalities have to be added to the outer description. Of course they have to be translated accordingly, resulting in

$$\boldsymbol{\partial}_u^\top \boldsymbol{a} = 0\,,\ \forall\, u \in V_n. \tag{3.1.1}$$

Now we have all the necessary constraints to describe $F_\downarrow^\diamond$ and can compute its vertices using *polymake*. Let $\boldsymbol{a}_0, \dots, \boldsymbol{a}_k$ be those vertices. As stated before, they correspond to the facets of $S_\downarrow$ containing $F_\downarrow$. The next step is the calculation of the inequalities defining those facets as well as their inverse translation in order to obtain facet-defining inequalities for $\mathrm{STSP}(n)$.

3.2 Facet-defining inequalities for STSP

By applying Theorem 1.1.15 (vi) to the vertices $\boldsymbol{a}_0, \dots, \boldsymbol{a}_k$ of $F_\downarrow^\diamond$ we obtain facet-defining inequalities for the polar of $F_\downarrow^\diamond$. Due to (1.1.1) we have

$$\left(F_\downarrow^\diamond\right)^\Delta = S_\downarrow / F_\downarrow,$$

where $S_\downarrow/F_\downarrow$ is the face figure of $S_\downarrow$ at the face $F_\downarrow$. Its face lattice $\mathscr{L}(S_\downarrow/F_\downarrow)$ is the sublattice of $\mathscr{L}(S_\downarrow)$ consisting of all faces that contain $F_\downarrow$. Hence, the facets of $S_\downarrow/F_\downarrow$ correspond exactly to those facets of $S_\downarrow$ containing $F_\downarrow$, which are defined by the inequalities

$$\boldsymbol{a}_j^\top \boldsymbol{x}_\downarrow \le 1\,,\ \forall\, j = 0, \dots, k.$$

In order to obtain inequalities for $\mathrm{STSP}(n)$ we simply insert the definition of $\boldsymbol{x}_\downarrow$. Moving the constant term to the right hand side results in

$$\boldsymbol{a}_j^\top \boldsymbol{x} \le 1 + \boldsymbol{a}_j^\top \boldsymbol{x}^*\,,\ \forall\, j = 0, \dots, k.$$

The right hand side can then be simplified according to the following

Lemma 3.2.1. *Let* $\boldsymbol{x}^* := \frac{2}{n-1} \cdot \mathbf{1}$. *Then*

$$\boldsymbol{a}^\top \boldsymbol{x}^* = 0\,,\ \forall\, \boldsymbol{a} \in S_\downarrow.$$

Proof. We remember the matrix $\mathbf{D}$, whose columns are the normalized left hand sides of the degree inequalities. Per construction, each $\boldsymbol{a} \in S_\downarrow$ satisfies (3.1.1). Hence $S_\downarrow$ is a subset of $\ker\left(\mathbf{D}^\top\right)$.

Since $\mathbf{D}$ is equal to the transposed node-edge incidence matrix and due to the fact that each edge has two end nodes, each row of $\mathbf{D}$ has exactly two non-zero entries. As a consequence,

$$\boldsymbol{x}^* := \tfrac{2}{n-1} \cdot \mathbf{1} = \mathbf{D}\left(\tfrac{1}{n-1} \cdot \mathbf{1}\right) \in \mathrm{im}(\mathbf{D}).$$

From Lemma 2.4.2 we can derive

$$\mathrm{im}(\mathbf{D}) = \left(\ker\left(\mathbf{D}^\top\right)\right)^\perp.$$

Now the assertion follows directly from the definition of the orthogonal complement. □

So, the facets of $\mathrm{STSP}(n)$ containing F are defined by the inequalities $\boldsymbol{a}_j^\top \boldsymbol{x} \leq 1$, $j = 0, \ldots, k$. Now we scale each inequality with the least common multiple of the denominators of its left hand side coefficients to assure that the resulting coefficients are integer. Finally, the inequalities are transformed into TT-form and scaled with the reciprocal of the greatest common divisor of the left hand side coefficients and the right hand side. In this form they are also valid for $\mathrm{GTSP}(n)$.

The pseudocode of this whole procedure is given in Algorithm 3.2.1.

Algorithm 3.2.1 Compute the NR-facets containing the good face F

Input: Number of nodes n
Incidence vectors $\boldsymbol{\chi}^H$ of Hamiltonian cycles in F

Output: Inequalities $(\boldsymbol{a}_j, \alpha_j)$ in TT-form defining the NR-facets that contain F

Generate all Hamiltonian cycles. These are exactly the vertices of $\mathrm{STSP}(n)$;
// Translate $\mathrm{STSP}(n)$ such that the relative interior point $\boldsymbol{x}^* := \frac{2}{n-1} \cdot \mathbf{1}$ is mapped
// to the origin $\rightsquigarrow S_\downarrow$. Since $F \subseteq \mathrm{STSP}(n)$ this also yields $F_\downarrow$
for ($\boldsymbol{v} \in \mathrm{vert}(\mathrm{STSP}(n))$) **{**
  $\boldsymbol{v}_\downarrow := \boldsymbol{v} - \boldsymbol{x}^*$;
} // for
// Equalities for the vertices of $F_\downarrow$
for ($\boldsymbol{v}_\downarrow \in \mathrm{vert}(F_\downarrow)$) **{**
  Add $\boldsymbol{v}_\downarrow^\top \boldsymbol{a} = 1$ to outer description;
} // for
// Translated degree equalities
for ($u \in V_n$) **{**
  Add $\boldsymbol{\partial}_u^\top \boldsymbol{a} = 0$ to outer description;
} // for
// Inequalities
for ($\boldsymbol{v}_\downarrow \in \mathrm{vert}(S_\downarrow)$) **{**
  Check redundancy using *CPLEX* (cf. Algorithm 3.1.1);
  if (inequality is not redundant) **{**
    Add $\boldsymbol{v}_\downarrow^\top \boldsymbol{a} \leq 1$ to outer description;
  } // if
} // for
Compute the vertices $\boldsymbol{a}_0, \ldots, \boldsymbol{a}_k$ of $F_\downarrow^\diamond$ using *polymake*;
// Determine the inequalities defining the NR-facets
for ($j = 0, \ldots, k$) **{**
  Generate $\boldsymbol{a}_j^\top \boldsymbol{x} \leq 1$;
  Multiply the inequality by the least common multiple of the denominators of its left hand side coefficients;
  Transform inequality into TT-form and scale it with the reciprocal of the greatest common divisor of the left hand side coefficients and the right hand side;
} // for

Chapter 4

Results

4.1 Utilities

It follows a detailed description of the main utilities implemented in the course of this diploma thesis. Both the corresponding source code and the executable programs can be found on the attached CD in the directory `Utils`.

4.1.1 findFacets

This program is an implementation of Algorithm 3.2.1 for the computation of all NR-facets $(\boldsymbol{a}_j, \alpha_j)$ containing a given good face F of STSP(n). The usage is as follows:

```
./findFacets <n> <poi>
```

Inputs:

`n`	number of nodes in K_n
`poi`	name of the .poi file storing the characteristic vectors of the Hamiltonian cycles that are contained in F

Requirements:

UNIX operating system
CPLEX
polymake

Average running time[1]:

$\approx$ 25s for GTSP(10)
$\approx$ 1h 23m for GTSP(12)

In the preface we shortly mentioned a former implementation for the computation of the NR-facets, whose actual name is `find_face`. Since our program intends to be a replacement, it inherits the output format of `find_face`, thus creating a .ieq file with the following structure:

[1] the running time was measured on an Intel® Xeon™ 2.80 GHz processor with 2 GB of RAM.

```
DIM = <(n * (n - 1)) / 2>
COMMENT <comment>
INEQUALITIES_SECTION
(<count>)   <LHS> >= <RHS>    # <SMAPO number>
END
```

The left hand side (LHS) $\boldsymbol{a}_j^\top \boldsymbol{x}$ is given in sparse format. SMAPO is a library of linear descriptions of low-dimensional 0/1-polytopes connected with small instances of combinatorial optimization problems. In the case of GTSP and STSP the facets are enumerated and partitioned into classes, whose members are equal modulo permutation of nodes and are assigned the number of the class – the so-called SMAPO number.

We now give a small example of such an output file for a face of STSP(10).

```
DIM = 45
COMMENT
INEQUALITIES_SECTION
(   1)  +1x1   +2x2   +2x3   +3x4   +3x5   +2x6   +4x7
        +3x8   +4x9   +3x10  +3x11  +4x12  +4x13  +3x14
        +3x15  +2x16  +3x17  +4x18  +3x19  +5x20  +4x21
        +2x22  +3x23  +4x24  +5x25  +3x26  +4x27  +4x28
        +3x29  +2x30  +2x31  +3x32  +5x33  +4x34  +3x35
        +3x36  +3x37  +4x38  +5x39  +2x40  +1x41  +2x42
        +3x43  +4x44  +3x45  >= 22  # 464
(   2)  +3x1   +7x2   +9x3   +8x4   +11x5  +13x6  +12x7
        +13x8  +13x9  +10x10 +10x11 +11x12 +14x13 +16x14
        +9x15  +10x16 +16x17 +16x18 +9x19  +14x20 +16x21
        +5x22  +16x23 +16x24 +17x25 +12x26 +18x27 +13x28
        +12x29 +6x30  +5x31  +11x32 +14x33 +17x34 +11x35
        +14x36 +9x37  +16x38 +16x39 +11x40 +6x41  +12x42
        +13x43 +15x44 +18x45 >= 80  # 13606
END
```

In addition, an update file for the facet database of GTSP(n) is created just in case the computed NR-facets aren't already included.

4.1.2 tiltingComplex

This program computes the tilting complex $\mathcal{T}(F)$ for a given good face F of STSP(n). It combines Algorithms 2.3.1 and 2.4.1 and has the following usage:

```
./tiltingComplex <n> <ieq>
```

Inputs:

- `n` number of nodes in K_n
- `ieq` name of the .ieq file storing the NR-facets that contain the good face F

Requirements:

UNIX operating system
polymake

Average running time:

$\approx$ 4s for GTSP(10), codim$(F) = 2$
$\approx$ 17s for GTSP(10), codim$(F) = 3$
$\approx$ 19s for GTSP(12), codim$(F) = 3$

For each $u \in V_n$ with $\mathcal{P}_u \neq \mathbb{A}^k$ the program creates three files

(i) u.poly

(ii) subdivisionAtNode.u.poly

(iii) tiltingComplexAtNode.u.poly

containing the descriptions of $\mathcal{P}_u$, $\mathcal{C}_u$ and $\mathcal{T}_u$ respectively. If the dimension of $\mathcal{P}_u$ is appropriate for a visualization, *polymake* is executed with the parameter VISUAL, which displays the polytope using *JavaView*[2].

The first two files are in standard *polymake* file format, whereas the third one is slightly modified as shown below.

```
#
# <comment>
#
VERTICES
1 <vertex coordinates>

VERTICES_IN_FACETS
{<vertices contained in this facet>}

# LHS_RHS_SMAPO
# <LHS> <RHS> <SMAPO>
```

As already mentioned in Section 2.4.2, the vertices are sorted in such a way that the first $k+1$ of them correspond to the unit vectors in reverse lexicographical order, which are then projected onto the vertices of $F^{\diamond}$. Consequently, these vertices also correspond to the NR-facets containing F. In order to allow the identification of these NR-facets, additional information is provided in the LHS_RHS_SMAPO section, which is the only non-standard part of the file. It is therefore commented out to grant compatibility with *polymake.* For $j \in \{0, \ldots, k\}$ the j-th row of the LHS_RHS_SMAPO section provides the left hand side coefficients, the right hand side and the SMAPO number of the NR-facet represented by the vertex in the j-th row of the VERTICES section.

Again, we give a small example for a face of STSP(10).

```
#
# COMMENT
#
VERTICES
1 -0.7071067811865476
1 0.7071067811865476
1 0.4380307494075958
```

[2]See `http://www.javaview.de/` for more information

```
VERTICES_IN_FACETS
{0 2}
{1 2}

# LHS_RHS_SMAPO
# 1  2  2  3  3  2  4  3  4  3  3  4  4  3  3  2  3  4  3
  5  4  2  3  4  5  3  4  4  3  2  2  3  5  4  3  3  3  4
  5  2  1  2  3  4  3  22 464
# 3  7  9  8  11 13 12 13 13 10 10 11 14 16 9  10 16 16 9
  14 16 5  16 16 17 12 18 13 12 6  5  11 14 17 11 14 9  16
  16 11 6  12 13 15 18 80 13606
```

All non-NR facets found during the computation are stored in an update file for the facet database of GTSP(n). Furthermore a LaTeX code is printed that can be redirected to a file. This code contains a visualization of the tilting complex using the package *PSTricks*[3] as well as tabular representations of the involved NR- and non-NR facets. A detailed description and examples are given in Appendices A and B.

4.1.3 Auxiliary functions

Some modules of the main utilities from the previous sections may also be interesting as stand-alone tools and shall therefore be introduced. Each of them is implemented with exact arithmetic using the *GMP* package.

(i) **gaussJordanElim**

This function computes a basis of the kernel of a given matrix by applying a Gauss-Jordan elimination with pivoting. The function call is as follows:

```
vector<mpq_class *> *gaussJordanElim(unsigned c,
                                     vector<mpq_class *> *m)
```

Inputs:
- `c` number of columns
- `m` vector containing the rows of the matrix

Return:
basis vectors of the kernel, or NULL if matrix is regular

Requirements:
GMP package

(ii) **orthogonalProjection**

This function computes the orthogonal projection of a given vector with respect to a certain basis and has the function call

```
mpq_class *orthogonalProjection(unsigned l,
                                unsigned n,
                                vector<mpq_class *> *b,
                                mpq_class *p)
```

[3] See `http://tug.org/PSTricks/main.cgi/` for more information

Inputs:

- `l` length of input vectors
- `n` number of basis vectors
- `b` vector containing the basis vectors
- `p` array containing the vector to be projected

Return:

orthogonal projection of `p` with respect to basis `b`

Requirements:

GMP package

(iii) **gramSchmidt**

Given a generating system, this function computes an orthogonal basis using the Gram-Schmidt procedure.

```
void gramSchmidt(unsigned l, vector<mpq_class *> *g)
```

Inputs:

- `l` length of input vectors
- `g` vector containing the vectors of the generating system

Return:

orthogonal basis

Requirements:

GMP package

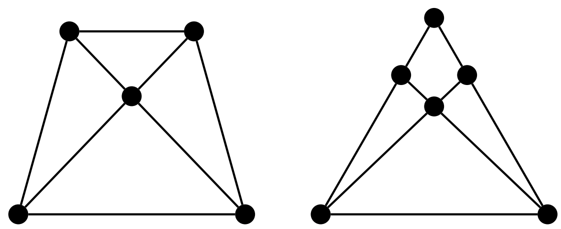
4.2 Computational results

The following conclusions are based on the data for GTSP(10) and GTSP(12) gathered in Appendices A and B. First of all, we would like to emphasize that up to now we have only encountered non-NR facets, whose corresponding face F of STSP(n) has co-dimension 2 or 3. As a consequence, all computed tilting complexes have a dimension of at most 2. Another interesting observation in this context is the fact that the number of non-NR facets with $\operatorname{codim}(F) = 3$ compared to those with $\operatorname{codim}(F) = 2$ is always substantially smaller. For example, in the case of GTSP(10) we have found 221 non-NR facets with $\operatorname{codim}(F) = 2$, but only 6 with $\operatorname{codim}(F) = 3$. For GTSP(12) the ratio is 1653 to 2 so far.

We have computed and visualized the tilting complexes $\mathcal{T}(F)$ for the corresponding STSP faces F of all known non-NR facets of both GTSP(10) and GTSP(12). The results for GTSP(10) are listed in the Sections A.1 and A.2 respectively. In the case of GTSP(12) we only included the most interesting one-dimensional tilting complexes, because otherwise the Section B.1 would have covered approximately 300 pages. The two-dimensional tilting complexes are given in Section B.2. All generated data can also be found on the attached CD in the directory `Data`.

For GTSP(10) the following observations are worth mentioning:

- all one-dimensional tilting complexes have exactly one relative interior vertex.
- for all one-dimensional tilting complexes the two involved NR-facets are TT-disjoint at exactly one node.
- there are only the following two types of two-dimensional tilting complexes:

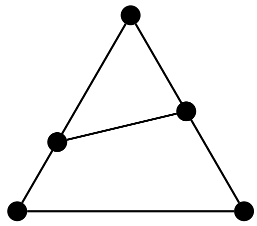


- all two-dimensional tilting complexes have exactly one relative interior vertex, although vertices on the boundary of the conjugate face may occur. In addition, the involved NR-facets are always TT-disjoint at exactly two nodes. However, the significance of this results is arguable considering the small number of examples.
- all two-dimensional tilting complexes are remarkably symmetric. In order to appreciate this fact, one has to take into account that every 0/1-polytope is isomorphic to a face of STSP(n), for n large enough. This was shown by Karp [Kar72] using an analogous result for the *Asymmetric Traveling Salesman Polytope* ATSP(n) by Billera & Sarangarajan [BS96]. This foreshadows the complexity of STSP(n) and indicates that the symmetry of the tilting complexes is anything but self-evident.

The question arises to what extent these patterns can be transferred to the tilting complexes for GTSP(12).

- as in the case of GTSP(10), all one-dimensional tilting complexes have exactly one relative interior vertex.
- for most of the one-dimensional tilting complexes the two involved NR-facets are TT-disjoint at exactly one node. But we found 11 counter-examples, where the NR-facets are TT-disjoint at **two** nodes u, v (see B.1). Particularly, each counter-example has only one relative interior vertex, which means that the tilting complexes at the nodes u and v feature this very vertex. Hence, $\mathcal{T}_u = \mathcal{T}_v = \mathcal{T}$. In conjunction with Theorem 5.8.4 of [The05] this answers an open question in [The05] (cf. paragraph on page 76 after Remark 5.9.1).
- the aforementioned counter-examples are also of importance in the context of *0-node-lifting*. Let G be the non-NR facet of GTSP(12) that corresponds to the relative interior vertex. As a consequence of $\mathcal{T}_u = \mathcal{T}_v = \mathcal{T}$, both $\mathcal{T}_u$ and $\mathcal{T}_v$ represent a complete description of the tilting complex $\mathcal{T}$. Thus, it is legitimate to ask: "Will 0-node-lifting of G at one of the nodes u, v result in a non-NR facet of GTSP(13)?", since the tilting complex at the other node would still carry all the information on $\mathcal{T}$. But it turned out that 0-node-lifting in this situation does not preserve non-NR facets.

- the two-dimensional tilting complexes feature a $\mathcal{T}_u$ of the following structure:



  This is remarkable, since these are the only examples of tilting complexes at a node u, where an edge of $\mathcal{T}_u$ is not incident to at least one of the vertices of $F^\diamond$.

- the two-dimensional tilting complexes have exactly one relative interior vertex and several vertices on the boundary of the conjugate face. Furthermore, the involved NR-facets are TT-disjoint at exactly two nodes. But this is hardly significant due to the small number of only two two-dimensional tilting complexes.

- the two-dimensional tilting complexes completely lack the symmetry of their counterparts for GTSP(10).

Appendix A

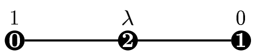
Visualization of tilting complexes for GTSP(10)

At the end of Section 2.4.2 we shortly mentioned the fact that we used the intersections of known non-NR facets with $\mathrm{STSP}(n)$ as good faces F for our computation. We now give an overview of the computational results for $n = 10$.

A.1 $\operatorname{codim}(F) = 2$

A face F with co-dimension 2, i.e. a ridge, has the property that it is contained in exactly two facets. This fact can easily be derived from the face lattice of the polar, where ridges correspond to edges, which contain exactly two vertices (cf. Corollary 1.1.20). As a consequence, $F^\diamond$ has exactly two vertices corresponding to the two NR-facets that contain F.

The results are sorted by right hand sides. The information for each non-NR facet consists of a figure and three tables. The figure shows a visualization of $F^\diamond$ together with the tilting complex and the information, at which node the two NR-facets are TT-disjoint. The number within each vertex of the complex represent its index. Additionally, the vertices are assigned values.

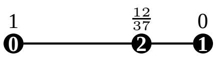


TT-disjoint at node u

Just as $\boldsymbol{v}_2$ is a convex combination of $\boldsymbol{v}_0$ and $\boldsymbol{v}_1$, the inequality defining the corresponding non-NR facet G is a convex combination of the inequalities $(\boldsymbol{a}_0, \alpha_0), (\boldsymbol{a}_1, \alpha_1)$ in standard scaling that define the two NR-facets H_0 and H_1. This convex combination is then transformed into TT-form and scaled with the reciprocal of the greatest common divisor of the left hand side coefficients and the right hand side. The coefficients $\big(\lambda, (1-\lambda)\big)$ of the convex combination can be computed from the actual coordinates of the vertices in $\mathbb{R}^1$. If $\boldsymbol{v}_2 = \lambda \boldsymbol{v}_0 + (1-\lambda)\boldsymbol{v}_1$, then we have $\lambda = \frac{\boldsymbol{v}_2 - \boldsymbol{v}_1}{\boldsymbol{v}_0 - \boldsymbol{v}_1}$. The values assigned to the vertices represent their coordinates with respect to both $\{\, \boldsymbol{v}_0, \boldsymbol{v}_1 \,\}$ and $\{\, (\boldsymbol{a}_0, \alpha_0), (\boldsymbol{a}_1, \alpha_1) \,\}$. For reasons of lucidity only the first coordinate is given, since it directly implies the second one.

The tables represent H_0, H_1 and G in this order, with G defining exactly the non-NR facet, whose intersection with STSP(10) was used as input for the computation. As additional information to the right of the right hand side we have the corresponding SMAPO number for NR-facets and the coefficients $\big(\lambda, (1-\lambda)\big)$ for non-NR facets.

RHS = 16



TT-disjoint at 4

	0	1	2	3	4	5	6	7	8	9
0	-	0	0	0	2	1	1	1	2	2
1	0	-	0	0	2	1	1	1	2	2
2	0	0	-	0	2	1	1	1	2	2
3	0	0	0	-	2	1	1	1	2	2
4	2	2	2	2	-	3	3	1	2	2
5	1	1	1	1	3	-	2	2	1	3
6	1	1	1	1	3	2	-	2	3	1
7	1	1	1	1	1	2	2	-	3	3
8	2	2	2	2	2	1	3	3	-	2
9	2	2	2	2	2	3	1	3	2	-

$\geq$ 10; SMAPO# 7

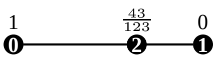
	0	1	2	3	4	5	6	7	8	9
0	-	2	4	4	3	3	5	5	4	4
1	2	-	4	4	5	5	3	3	4	4
2	4	4	-	2	3	5	3	5	4	4
3	4	4	2	-	5	3	5	3	4	4
4	3	5	3	5	-	4	4	2	3	3
5	3	5	5	3	4	-	2	4	1	3
6	5	3	3	5	4	2	-	4	3	1
7	5	3	5	3	2	4	4	-	3	3
8	4	4	4	4	3	1	3	3	-	2
9	4	4	4	4	3	3	1	3	2	-

$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	2	2	3	3	3	3
1	1	-	2	2	3	3	2	2	3	3
2	2	2	-	1	2	3	2	3	3	3
3	2	2	1	-	3	2	3	2	3	3
4	2	3	2	3	-	3	3	1	2	2
5	2	3	3	2	3	-	2	3	1	3
6	3	2	2	3	3	2	-	3	3	1
7	3	2	3	2	1	3	3	-	3	3
8	3	3	3	3	2	1	3	3	-	2
9	3	3	3	3	2	3	1	3	2	-

$\geq 16; (\frac{12}{37}, \frac{25}{37})$

RHS = 27



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	4	3	3	2	4
1	1	-	3	3	4	3	2	4	3	3
2	2	3	-	4	3	2	3	5	4	4
3	2	3	4	-	5	4	3	3	4	2
4	3	4	3	5	-	5	4	2	3	3
5	4	3	2	4	5	-	3	3	2	4
6	3	2	3	3	4	3	-	4	1	3
7	3	4	5	3	2	3	4	-	3	5
8	2	3	4	4	3	2	1	3	-	2
9	4	3	4	2	3	4	3	5	2	-

$\geq$ 22; SMAPO# 464

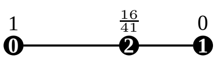
	0	1	2	3	4	5	6	7	8	9
0	-	1	2	5	3	4	5	5	6	6
1	1	-	3	4	4	3	4	6	7	7
2	2	3	-	7	3	2	7	5	6	6
3	5	4	7	-	8	7	6	6	9	3
4	3	4	3	8	-	5	8	2	5	5
5	4	3	2	7	5	-	7	3	4	6
6	5	4	7	6	8	7	-	8	3	9
7	5	6	5	6	2	3	8	-	5	7
8	6	7	6	9	5	4	3	5	-	6
9	6	7	6	3	5	6	9	7	6	-

$\geq$ 34; SMAPO# 2445

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	4	4	4	4	5
1	1	-	3	3	4	3	3	5	5	5
2	2	3	-	5	3	2	5	5	5	5
3	3	3	5	-	6	5	4	4	6	2
4	3	4	3	6	-	5	6	2	4	4
5	4	3	2	5	5	-	5	3	3	5
6	4	3	5	4	6	5	-	6	2	6
7	4	5	5	4	2	3	6	-	4	6
8	4	5	5	6	4	3	2	4	-	4
9	5	5	5	2	4	5	6	6	4	-

$\geq 27; (\frac{43}{123}, \frac{80}{123})$

RHS = 32



TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	1	2	2	1	1	3	3
1	1	-	0	0	3	3	0	0	2	2
2	1	0	-	0	3	3	0	0	2	2
3	1	0	0	-	3	3	0	0	2	2
4	2	3	3	3	-	2	3	3	1	3
5	2	3	3	3	2	-	3	3	3	1
6	1	0	0	0	3	3	-	0	2	2
7	1	0	0	0	3	3	0	-	2	2
8	3	2	2	2	1	3	2	2	-	2
9	3	2	2	2	3	1	2	2	2	-

$\geq$ 10; SMAPO# 51

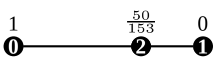
	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	5	5	4	4
1	2	-	5	5	3	3	3	3	4	4
2	3	5	-	4	4	4	2	4	3	5
3	3	5	4	-	4	4	4	2	5	3
4	3	3	4	4	-	2	4	4	1	3
5	3	3	4	4	2	-	4	4	3	1
6	5	3	2	4	4	4	-	4	5	3
7	5	3	4	2	4	4	4	-	3	5
8	4	4	3	5	1	3	5	3	-	2
9	4	4	5	3	3	1	3	5	2	-

$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	4	4	5	5	6	6
1	2	-	5	5	6	6	3	3	6	6
2	3	5	-	4	7	7	2	4	5	7
3	3	5	4	-	7	7	4	2	7	5
4	4	6	7	7	-	4	7	7	2	6
5	4	6	7	7	4	-	7	7	6	2
6	5	3	2	4	7	7	-	4	7	5
7	5	3	4	2	7	7	4	-	5	7
8	6	6	5	7	2	6	7	5	-	4
9	6	6	7	5	6	2	5	7	4	-

$\geq 32; (\frac{16}{41}, \frac{25}{41})$

RHS = 33



TT-disjoint at 6

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	5	5	4	4
1	2	-	3	3	5	5	3	3	4	4
2	3	3	-	2	4	4	4	4	1	3
3	3	3	2	-	4	4	4	4	3	1
4	3	5	4	4	-	4	4	2	5	3
5	3	5	4	4	4	-	2	4	3	5
6	5	3	4	4	4	2	-	4	5	3
7	5	3	4	4	2	4	4	-	3	5
8	4	4	1	3	5	3	5	3	-	2
9	4	4	3	1	3	5	3	5	2	-

$\geq$ 24; SMAPO# 662

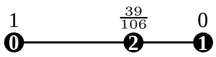
	0	1	2	3	4	5	6	7	8	9
0	-	2	5	5	5	5	8	7	8	8
1	2	-	7	7	7	7	6	5	8	8
2	5	7	-	6	8	10	7	8	3	9
3	5	7	6	-	8	10	9	8	9	3
4	5	7	8	8	-	8	5	2	9	5
5	5	7	10	10	8	-	3	6	7	7
6	8	6	7	9	5	3	-	7	10	6
7	7	5	8	8	2	6	7	-	7	7
8	8	8	3	9	9	7	10	7	-	6
9	8	8	9	3	5	7	6	7	6	-

$\geq$ 44; SMAPO# 5162

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	4	4	4	6	6	6	6
1	2	-	5	5	6	6	4	4	6	6
2	4	5	-	4	6	7	5	6	2	6
3	4	5	4	-	6	7	6	6	6	2
4	4	6	6	6	-	6	4	2	7	4
5	4	6	7	7	6	-	2	5	5	6
6	6	4	5	6	4	2	-	5	7	4
7	6	4	6	6	2	5	5	-	5	6
8	6	6	2	6	7	5	7	5	-	4
9	6	6	6	2	4	6	4	6	4	-

$\geq 33; (\frac{50}{153}, \frac{103}{153})$

RHS = 36



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	0	1	2	2	2	2	3	3	3
1	0	-	1	2	2	2	2	3	3	3
2	1	1	-	3	3	3	3	2	2	2
3	2	2	3	-	2	4	2	3	1	3
4	2	2	3	2	-	2	2	3	1	1
5	2	2	3	4	2	-	4	1	3	3
6	2	2	3	2	2	4	-	3	3	1
7	3	3	2	3	3	1	3	-	2	2
8	3	3	2	1	1	3	3	2	-	2
9	3	3	2	3	1	3	1	2	2	-

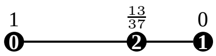
≥ 14; SMAPO# 130

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	4	3	4	5	4	4	5
1	1	-	2	5	4	3	4	5	5	6
2	1	2	-	3	4	5	6	3	5	4
3	4	5	3	-	3	6	3	4	2	5
4	3	4	4	3	-	3	4	5	1	2
5	4	3	5	6	3	-	5	2	4	5
6	5	4	6	3	4	5	-	7	3	2
7	4	5	3	4	5	2	7	-	6	5
8	4	5	5	2	1	4	3	6	-	3
9	5	6	4	5	2	5	2	5	3	-

≥ 24; SMAPO# 786

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	5	5	6	7	7	7	8
1	1	-	3	6	6	5	6	8	8	9
2	2	3	-	5	7	8	9	5	7	6
3	5	6	5	-	4	9	4	6	2	7
4	5	6	7	4	-	5	6	8	2	3
5	6	5	8	9	5	-	9	3	7	8
6	7	6	9	4	6	9	-	10	6	3
7	7	8	5	6	8	3	10	-	8	7
8	7	8	7	2	2	7	6	8	-	5
9	8	9	6	7	3	8	3	7	5	-

≥ 36; $(\frac{39}{106}, \frac{67}{106})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	0	1	2	2	3	2	3	2	3
1	0	-	1	2	2	3	2	3	2	3
2	1	1	-	3	3	2	3	2	3	2
3	2	2	3	-	2	3	2	1	2	1
4	2	2	3	2	-	1	4	3	4	3
5	3	3	2	3	1	-	3	2	3	2
6	2	2	3	2	4	3	-	1	2	3
7	3	3	2	1	3	2	1	-	3	2
8	2	2	3	2	4	3	2	3	-	1
9	3	3	2	1	3	2	3	2	1	-

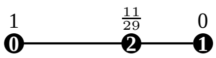
≥ 14; SMAPO# 130

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	4	3	5	4	6	6
1	1	-	3	4	3	4	4	5	5	7
2	2	3	-	5	4	3	3	4	4	6
3	3	4	5	-	3	4	2	1	5	3
4	4	3	4	3	-	1	5	4	6	6
5	3	4	3	4	1	-	6	5	7	5
6	5	4	3	2	5	6	-	1	3	5
7	4	5	4	1	4	5	1	-	4	4
8	6	5	4	5	6	7	3	4	-	2
9	6	7	6	3	6	5	5	4	2	-

≥ 24; SMAPO# 826

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	5	6	6	7	7	8	9
1	1	-	3	6	5	7	6	8	7	10
2	2	3	-	7	6	4	5	5	6	7
3	5	6	7	-	5	7	4	2	7	4
4	6	5	6	5	-	2	9	7	10	9
5	6	7	4	7	2	-	9	7	10	7
6	7	6	5	4	9	9	-	2	5	8
7	7	8	5	2	7	7	2	-	7	6
8	8	7	6	7	10	10	5	7	-	3
9	9	10	7	4	9	7	8	6	3	-

≥ 36; $(\frac{13}{37}, \frac{24}{37})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	0	1	2	2	2	3	3	3	3
1	0	-	1	2	2	2	3	3	3	3
2	1	1	-	3	3	3	2	2	4	2
3	2	2	3	-	4	2	3	1	3	1
4	2	2	3	4	-	4	1	3	3	3
5	2	2	3	2	4	-	3	3	1	3
6	3	3	2	3	1	3	-	2	4	2
7	3	3	2	1	3	3	2	-	2	0
8	3	3	4	3	3	1	4	2	-	2
9	3	3	2	1	3	3	2	0	2	-

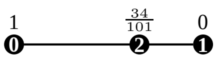
≥ 14; SMAPO# 138

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	4	4	5	4	4	4	5
1	1	-	2	5	3	4	5	5	5	6
2	1	2	-	3	5	6	3	5	5	4
3	4	5	3	-	6	3	4	2	4	3
4	4	3	5	6	-	5	2	4	4	5
5	5	4	6	3	5	-	7	3	1	4
6	4	5	3	4	2	7	-	6	6	5
7	4	5	5	2	4	3	6	-	4	1
8	4	5	5	4	4	1	6	4	-	3
9	5	6	4	3	5	4	5	1	3	-

≥ 24; SMAPO# 804

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	5	6	7	7	7	7	8
1	1	-	3	6	5	6	8	8	8	9
2	2	3	-	5	8	9	5	7	9	6
3	5	6	5	-	9	4	6	2	6	3
4	6	5	8	9	-	9	3	7	7	8
5	7	6	9	4	9	-	10	6	2	7
6	7	8	5	6	3	10	-	8	10	7
7	7	8	7	2	7	6	8	-	6	1
8	7	8	9	6	7	2	10	6	-	5
9	8	9	6	3	8	7	7	1	5	-

≥ 36; $(\frac{11}{29}, \frac{18}{29})$



TT-disjoint at 4

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	2	2	3	3	2	3
1	1	-	2	1	3	1	2	2	3	2
2	1	2	-	3	3	3	2	2	3	2
3	2	1	3	-	2	2	1	3	2	1
4	2	3	3	2	-	2	1	3	2	3
5	2	1	3	2	2	-	3	1	2	3
6	3	2	2	1	1	3	-	2	3	2
7	3	2	2	3	3	1	2	-	3	2
8	2	3	3	2	2	2	3	3	-	1
9	3	2	2	1	3	3	2	2	1	-

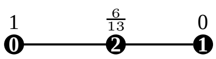
≥ 14; SMAPO# 119

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	4	4	4	4	5	5
1	1	-	2	2	5	3	3	5	4	4
2	1	2	-	4	3	5	5	3	6	4
3	3	2	4	-	3	5	1	5	4	2
4	4	5	3	3	-	6	2	4	3	5
5	4	3	5	5	6	-	6	2	5	7
6	4	3	5	1	2	6	-	6	3	3
7	4	5	3	5	4	2	6	-	7	5
8	5	4	6	4	3	5	3	7	-	2
9	5	4	4	2	5	7	3	5	2	-

≥ 24; SMAPO# 818

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	5	5	6	7	7	7	8
1	2	-	4	3	7	4	5	7	7	6
2	2	4	-	7	5	8	7	5	9	6
3	5	3	7	-	4	7	2	8	6	3
4	5	7	5	4	-	7	2	6	4	7
5	6	4	8	7	7	-	9	3	7	10
6	7	5	7	2	2	9	-	8	6	5
7	7	7	5	8	6	3	8	-	10	7
8	7	7	9	6	4	7	6	10	-	3
9	8	6	6	3	7	10	5	7	3	-

≥ 36; $(\frac{34}{101}, \frac{67}{101})$



TT-disjoint at 1

	0	1	2	3	4	5	6	7	8	9
0	-	2	1	4	4	3	2	3	3	3
1	2	-	3	2	2	3	2	3	3	3
2	1	3	-	3	3	4	3	2	4	2
3	4	2	3	-	2	1	4	3	3	3
4	4	2	3	2	-	3	4	3	1	3
5	3	3	4	1	3	-	3	2	2	4
6	2	2	3	4	4	3	-	3	3	1
7	3	3	2	3	3	2	3	-	2	2
8	3	3	4	3	1	2	3	2	-	4
9	3	3	2	3	3	4	1	2	4	-

≥ 18; SMAPO# 268

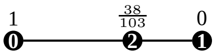
	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	2	3	4	3	3	4
1	1	-	2	3	3	2	3	4	4	5
2	1	2	-	3	3	4	5	2	4	3
3	2	3	3	-	2	1	4	3	3	4
4	2	3	3	2	-	3	4	3	1	4
5	3	2	4	1	3	-	3	2	2	5
6	4	3	5	4	4	3	-	5	5	2
7	3	4	2	3	3	2	5	-	2	3
8	3	4	4	3	1	2	5	2	-	5
9	4	5	3	4	4	5	2	3	5	-

≥ 20; SMAPO# 365

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	6	6	6	6	6	6	7
1	2	-	4	4	4	4	4	6	6	7
2	2	4	-	6	6	8	8	4	8	5
3	6	4	6	-	4	2	8	6	6	7
4	6	4	6	4	-	6	8	6	2	7
5	6	4	8	2	6	-	6	4	4	9
6	6	4	8	8	8	6	-	8	8	3
7	6	6	4	6	6	4	8	-	4	5
8	6	6	8	6	2	4	8	4	-	9
9	7	7	5	7	7	9	3	5	9	-

≥ 36; $(\frac{6}{13}, \frac{7}{13})$

RHS = 40



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	0	1	1	2	3	2	2	3	3
1	0	-	1	1	2	3	2	2	3	3
2	1	1	-	2	3	2	3	3	2	4
3	1	1	2	-	3	2	3	1	2	2
4	2	2	3	3	-	3	2	2	1	3
5	3	3	2	2	3	-	3	1	2	4
6	2	2	3	3	2	3	-	2	3	1
7	2	2	3	1	2	1	2	-	3	3
8	3	3	2	2	1	2	3	3	-	2
9	3	3	4	2	3	4	1	3	2	-

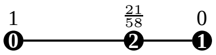
≥ 14; SMAPO# 136

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	3	3	5	5	5	5
1	1	-	3	3	4	4	4	6	6	6
2	2	3	-	2	5	3	5	5	5	5
3	4	3	2	-	5	5	5	3	5	3
4	3	4	5	5	-	6	4	4	2	6
5	3	4	3	5	6	-	6	2	4	4
6	5	4	5	5	4	6	-	6	6	2
7	5	6	5	3	4	2	6	-	6	4
8	5	6	5	5	2	4	6	6	-	4
9	5	6	5	3	6	4	2	4	4	-

≥ 28; SMAPO# 1236

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	5	5	6	7	7	8	8
1	1	-	3	4	6	7	6	8	9	9
2	2	3	-	3	7	4	7	7	6	8
3	5	4	3	-	8	7	8	4	7	5
4	5	6	7	8	-	9	6	6	3	9
5	6	7	4	7	9	-	9	3	6	8
6	7	6	7	8	6	9	-	8	9	3
7	7	8	7	4	6	3	8	-	9	7
8	8	9	6	7	3	6	9	9	-	6
9	8	9	8	5	9	8	3	7	6	-

$\geq 40; (\frac{38}{103}, \frac{65}{103})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	3	3	3	2	2	3
1	1	-	2	3	2	4	2	3	3	4
2	1	2	-	3	4	2	4	3	3	2
3	2	3	3	-	1	1	3	2	2	3
4	3	2	4	1	-	2	2	3	3	4
5	3	4	2	1	2	-	4	3	3	2
6	3	2	4	3	2	4	-	1	3	2
7	2	3	3	2	3	3	1	-	2	3
8	2	3	3	2	3	3	3	2	-	1
9	3	4	2	3	4	2	2	3	1	-

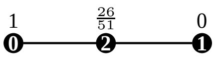
≥ 16; SMAPO# 199

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	4	6	6	6
1	1	-	3	3	2	4	3	5	5	7
2	2	3	-	4	5	3	6	4	4	6
3	2	3	4	-	1	1	4	6	6	6
4	3	2	5	1	-	2	3	5	5	7
5	3	4	3	1	2	-	5	7	7	5
6	4	3	6	4	3	5	-	2	6	4
7	6	5	4	6	5	7	2	-	4	6
8	6	5	4	6	5	7	6	4	-	2
9	6	7	6	6	7	5	4	6	2	-

≥ 26; SMAPO# 1120

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	4	6	6	7	8	8	9
1	2	-	4	6	4	8	5	8	8	11
2	2	4	-	6	8	4	9	6	6	7
3	4	6	6	-	2	2	7	8	8	9
4	6	4	8	2	-	4	5	8	8	11
5	6	8	4	2	4	-	9	10	10	7
6	7	5	9	7	5	9	-	3	9	6
7	8	8	6	8	8	10	3	-	6	9
8	8	8	6	8	8	10	9	6	-	3
9	9	11	7	9	11	7	6	9	3	-

$\geq 40; (\frac{21}{58}, \frac{37}{58})$



TT-disjoint at 4

	0	1	2	3	4	5	6	7	8	9
0	-	0	0	0	2	1	3	3	5	5
1	0	-	0	0	2	1	3	3	5	5
2	0	0	-	0	2	1	3	3	5	5
3	0	0	0	-	2	1	3	3	5	5
4	2	2	2	2	-	1	5	5	3	3
5	1	1	1	1	1	-	4	4	4	4
6	3	3	3	3	5	4	-	4	2	6
7	3	3	3	3	5	4	4	-	6	2
8	5	5	5	5	3	4	2	6	-	4
9	5	5	5	5	3	4	6	2	4	-

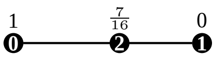
≥ 18; SMAPO# 313

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	4	3	5	3	5	4	4
1	2	-	4	4	5	3	5	3	4	4
2	4	4	-	2	3	5	5	3	4	4
3	4	4	2	-	5	3	3	5	4	4
4	3	5	3	5	-	2	4	4	3	3
5	5	3	5	3	2	-	4	4	3	3
6	3	5	5	3	4	4	-	2	1	3
7	5	3	3	5	4	4	2	-	3	1
8	4	4	4	4	3	3	1	3	-	2
9	4	4	4	4	3	3	3	1	2	-

≥ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	4	4	6	6	8	9	9
1	2	-	4	4	6	4	8	6	9	9
2	4	4	-	2	4	6	8	6	9	9
3	4	4	2	-	6	4	6	8	9	9
4	4	6	4	6	-	2	8	8	5	5
5	6	4	6	4	2	-	8	8	7	7
6	6	8	8	6	8	8	-	6	3	9
7	8	6	6	8	8	8	6	-	9	3
8	9	9	9	9	5	7	3	9	-	6
9	9	9	9	9	5	7	9	3	6	-

$\geq 40; (\frac{26}{51}, \frac{25}{51})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	2	2	3	3	3	4	4
1	1	-	2	3	3	2	4	4	3	3
2	3	2	-	3	3	2	2	2	3	3
3	2	3	3	-	2	1	3	3	4	4
4	2	3	3	2	-	3	3	3	2	2
5	3	2	2	1	3	-	4	4	3	3
6	3	4	2	3	3	4	-	2	1	3
7	3	4	2	3	3	4	2	-	3	1
8	4	3	3	4	2	3	1	3	-	2
9	4	3	3	4	2	3	3	1	2	-

≥ 18; SMAPO# 268

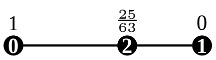
	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	2	3	5	5	5	5
1	1	-	3	3	3	2	4	4	4	4
2	2	3	-	2	2	3	3	3	5	5
3	2	3	2	-	2	1	5	5	5	5
4	2	3	2	2	-	3	5	5	3	3
5	3	2	3	1	3	-	4	4	4	4
6	5	4	3	5	5	4	-	4	2	6
7	5	4	3	5	5	4	4	-	6	2
8	5	4	5	5	3	4	2	6	-	4
9	5	4	5	5	3	4	6	2	4	-

≥ 24; SMAPO# 667

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	4	4	6	8	8	9	9
1	2	-	4	6	6	4	8	8	7	7
2	4	4	-	4	4	4	4	4	7	7
3	4	6	4	-	4	2	8	8	9	9
4	4	6	4	4	-	6	8	8	5	5
5	6	4	4	2	6	-	8	8	7	7
6	8	8	4	8	8	8	-	6	3	9
7	8	8	4	8	8	8	6	-	9	3
8	9	7	7	9	5	7	3	9	-	6
9	9	7	7	9	5	7	9	3	6	-

$\geq 40; (\frac{7}{16}, \frac{9}{16})$

RHS = 42



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	0	1	2	2	3	3	3	4	5
1	0	-	1	2	2	3	3	3	4	5
2	1	1	-	3	3	2	2	4	3	4
3	2	2	3	-	4	3	1	3	2	5
4	2	2	3	4	-	1	3	5	2	3
5	3	3	2	3	1	-	2	4	3	4
6	3	3	2	1	3	2	-	4	1	4
7	3	3	4	3	5	4	4	-	5	2
8	4	4	3	2	2	3	1	5	-	3
9	5	5	4	5	3	4	4	2	3	-

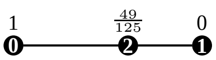
≥ 18; SMAPO# 292

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	4	4	4	4	6	5	6
1	1	-	2	5	3	5	5	5	6	7
2	1	2	-	3	5	3	5	7	4	5
3	4	5	3	-	6	4	2	4	3	6
4	4	3	5	6	-	2	4	6	3	4
5	4	5	3	4	2	-	6	8	5	6
6	4	5	5	2	4	6	-	4	1	4
7	6	5	7	4	6	8	4	-	5	2
8	5	6	4	3	3	5	1	5	-	3
9	6	7	5	6	4	6	4	2	3	-

≥ 26; SMAPO# 1162

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	5	6	7	7	9	9	11
1	1	-	3	6	5	8	8	8	10	12
2	2	3	-	5	8	5	7	11	7	9
3	5	6	5	-	9	6	2	6	4	10
4	6	5	8	9	-	3	7	11	5	7
5	7	8	5	6	3	-	8	12	8	10
6	7	8	7	2	7	8	-	8	2	8
7	9	8	11	6	11	12	8	-	10	4
8	9	10	7	4	5	8	2	10	-	6
9	11	12	9	10	7	10	8	4	6	-

$\geq 42; (\frac{25}{63}, \frac{38}{63})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	0	1	3	2	2	3	3	4	4
1	0	-	1	3	2	2	3	3	4	4
2	1	1	-	2	3	3	2	4	5	3
3	3	3	2	-	3	1	4	2	5	3
4	2	2	3	3	-	2	1	3	4	4
5	2	2	3	1	2	-	3	1	4	4
6	3	3	2	4	1	3	-	2	3	5
7	3	3	4	2	3	1	2	-	3	5
8	4	4	5	5	4	4	3	3	-	2
9	4	4	3	3	4	4	5	5	2	-

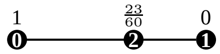
≥ 18; SMAPO# 291

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	4	4	4	4	5	6	6
1	1	-	2	3	3	5	5	4	5	7
2	1	2	-	5	5	5	3	6	7	5
3	4	3	5	-	4	2	6	3	6	4
4	4	3	5	4	-	6	2	5	6	8
5	4	5	5	2	6	-	4	1	4	4
6	4	5	3	6	2	4	-	3	4	6
7	5	4	6	3	5	1	3	-	3	5
8	6	5	7	6	6	4	4	3	-	2
9	6	7	5	4	8	4	6	5	2	-

≥ 26; SMAPO# 1162

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	6	6	6	7	8	10	10
1	1	-	3	5	5	7	8	7	9	11
2	2	3	-	6	8	8	5	10	12	8
3	6	5	6	-	6	2	9	4	10	6
4	6	5	8	6	-	8	3	8	10	12
5	6	7	8	2	8	-	7	2	8	8
6	7	8	5	9	3	7	-	5	7	11
7	8	7	10	4	8	2	5	-	6	10
8	10	9	12	10	10	8	7	6	-	4
9	10	11	8	6	12	8	11	10	4	-

$\geq 42; (\frac{49}{125}, \frac{76}{125})$



TT-disjoint at 1

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	2	2	3	3
1	1	-	3	3	2	2	3	3	4	4
2	2	3	-	4	1	3	4	4	3	5
3	2	3	4	-	3	1	4	4	3	3
4	3	2	1	3	-	2	3	3	4	4
5	3	2	3	1	2	-	3	3	2	4
6	2	3	4	4	3	3	-	2	1	3
7	2	3	4	4	3	3	2	-	3	1
8	3	4	3	3	4	2	1	3	-	2
9	3	4	5	3	4	4	3	1	2	-

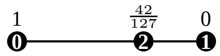
≥ 18; SMAPO# 276

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	2	3	5	5	5	5
1	1	-	2	3	3	4	4	4	6	6
2	1	2	-	3	1	4	6	6	4	6
3	2	3	3	-	4	1	7	7	5	5
4	2	3	1	4	-	3	5	5	5	7
5	3	4	4	1	3	-	6	6	4	6
6	5	4	6	7	5	6	-	4	2	6
7	5	4	6	7	5	6	4	-	6	2
8	5	6	4	5	5	4	2	6	-	4
9	5	6	6	5	7	6	6	2	4	-

≥ 26; SMAPO# 1027

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	4	5	6	7	7	8	8
1	1	-	4	5	4	5	6	6	9	9
2	3	4	-	7	2	7	10	10	7	11
3	4	5	7	-	7	2	11	11	8	8
4	5	4	2	7	-	5	8	8	9	11
5	6	5	7	2	5	-	9	9	6	10
6	7	6	10	11	8	9	-	6	3	9
7	7	6	10	11	8	9	6	-	9	3
8	8	9	7	8	9	6	3	9	-	6
9	8	9	11	8	11	10	9	3	6	-

≥ 42; $(\frac{23}{60}, \frac{37}{60})$



TT-disjoint at 6

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	2	3	3	2	2	3
1	1	-	2	2	3	2	4	3	3	4
2	1	2	-	4	3	4	2	3	3	2
3	3	2	4	-	1	2	2	3	3	4
4	2	3	3	1	-	3	1	2	2	3
5	3	2	4	2	3	-	4	1	3	2
6	3	4	2	2	1	4	-	3	3	2
7	2	3	3	3	2	1	3	-	2	3
8	2	3	3	3	2	3	3	2	-	1
9	3	4	2	4	3	2	2	3	1	-

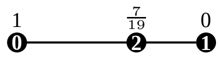
≥ 16; SMAPO# 199

	0	1	2	3	4	5	6	7	8	9
0	-	0	2	3	4	4	6	6	6	8
1	0	-	2	3	4	4	6	6	6	8
2	2	2	-	5	4	6	4	8	8	6
3	3	3	5	-	1	3	3	5	5	7
4	4	4	4	1	-	4	2	6	6	6
5	4	4	6	3	4	-	6	2	6	4
6	6	6	4	3	2	6	-	4	4	6
7	6	6	8	5	6	2	4	-	4	6
8	6	6	8	5	6	6	4	4	-	2
9	8	8	6	7	6	4	6	6	2	-

≥ 28; SMAPO# 1417

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	6	6	7	8	8	8	11
1	1	-	4	5	7	6	9	9	9	12
2	3	4	-	9	7	10	5	11	11	8
3	6	5	9	-	2	5	4	8	8	11
4	6	7	7	2	-	7	2	8	8	9
5	7	6	10	5	7	-	9	3	9	6
6	8	9	5	4	2	9	-	6	6	7
7	8	9	11	8	8	3	6	-	6	9
8	8	9	11	8	8	9	6	6	-	3
9	11	12	8	11	9	6	7	9	3	-

≥ 42; $(\frac{42}{127}, \frac{85}{127})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	3	3	3	2	2	3
1	1	-	2	3	2	4	2	3	3	4
2	1	2	-	3	4	2	4	3	3	2
3	2	3	3	-	1	1	3	2	2	3
4	3	2	4	1	-	2	2	3	3	4
5	3	4	2	1	2	-	4	3	3	2
6	3	2	4	3	2	4	-	1	3	2
7	2	3	3	2	3	3	1	-	2	3
8	2	3	3	2	3	3	3	2	-	1
9	3	4	2	3	4	2	2	3	1	-

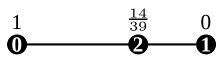
≥ 16; SMAPO# 199

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	3	4	6	6	6
1	1	-	3	4	2	4	3	5	5	7
2	2	3	-	5	5	3	6	4	4	6
3	3	4	5	-	2	2	5	5	5	7
4	3	2	5	2	-	4	3	5	5	7
5	3	4	3	2	4	-	5	7	7	5
6	4	3	6	5	3	5	-	2	6	4
7	6	5	4	5	5	7	2	-	4	6
8	6	5	4	5	5	7	6	4	-	2
9	6	7	6	7	7	5	4	6	2	-

≥ 28; SMAPO# 1382

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	5	6	6	7	8	8	9
1	2	-	4	7	4	8	5	8	8	11
2	2	4	-	7	8	4	9	6	6	7
3	5	7	7	-	3	3	8	7	7	10
4	6	4	8	3	-	6	5	8	8	11
5	6	8	4	3	6	-	9	10	10	7
6	7	5	9	8	5	9	-	3	9	6
7	8	8	6	7	8	10	3	-	6	9
8	8	8	6	7	8	10	9	6	-	3
9	9	11	7	10	11	7	6	9	3	-

≥ 42; $(\frac{7}{19}, \frac{12}{19})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	2	3	3	2	2	3
1	1	-	2	2	3	2	4	3	3	4
2	1	2	-	4	3	4	2	3	3	2
3	3	2	4	-	1	2	2	3	3	4
4	2	3	3	1	-	3	1	2	2	3
5	3	2	4	2	3	-	4	1	3	2
6	3	4	2	2	1	4	-	3	3	2
7	2	3	3	3	2	1	3	-	2	3
8	2	3	3	3	2	3	3	2	-	1
9	3	4	2	4	3	2	2	3	1	-

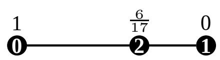
≥ 16; SMAPO# 199

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	4	4	4	6	6	6
1	1	-	3	1	3	3	5	5	5	7
2	2	3	-	4	6	6	4	4	4	6
3	2	1	4	-	2	2	4	4	4	6
4	4	3	6	2	-	4	2	6	6	8
5	4	3	6	2	4	-	6	2	6	4
6	4	5	4	4	2	6	-	8	8	6
7	6	5	4	4	6	2	8	-	4	6
8	6	5	4	4	6	6	8	4	-	2
9	6	7	6	6	8	4	6	6	2	-

≥ 28; SMAPO# 1405

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	5	6	7	7	8	8	9
1	2	-	4	3	6	5	9	8	8	11
2	2	4	-	7	8	9	5	6	6	7
3	5	3	7	-	3	4	6	7	7	10
4	6	6	8	3	-	7	3	8	8	11
5	7	5	9	4	7	-	10	3	9	6
6	7	9	5	6	3	10	-	11	11	8
7	8	8	6	7	8	3	11	-	6	9
8	8	8	6	7	8	9	11	6	-	3
9	9	11	7	10	11	6	8	9	3	-

≥ 42; $(\frac{14}{39}, \frac{25}{39})$



TT-disjoint at 1

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	3	3	3	2	2	3
1	1	-	2	3	2	4	4	3	3	2
2	1	2	-	3	4	2	2	3	3	4
3	2	3	3	-	1	1	3	2	2	3
4	3	2	4	1	-	2	4	3	3	2
5	3	4	2	1	2	-	2	3	3	4
6	3	4	2	3	4	2	-	1	3	2
7	2	3	3	2	3	3	1	-	2	3
8	2	3	3	2	3	3	3	2	-	1
9	3	2	4	3	2	4	2	3	1	-

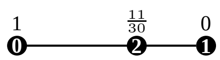
≥ 16; SMAPO# 199

	0	1	2	3	4	5	6	7	8	9
0	-	2	1	3	3	4	4	6	6	6
1	2	-	3	5	3	6	6	4	4	6
2	1	3	-	4	4	3	3	5	5	7
3	3	5	4	-	2	1	5	5	5	7
4	3	3	4	2	-	3	5	7	7	5
5	4	6	3	1	3	-	4	6	6	8
6	4	6	3	5	5	4	-	2	6	4
7	6	4	5	5	7	6	2	-	4	6
8	6	4	5	5	7	6	6	4	-	2
9	6	6	7	7	5	8	4	6	2	-

≥ 28; SMAPO# 1398

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	5	6	7	7	8	8	9
1	2	-	4	7	4	9	9	6	6	7
2	2	4	-	7	8	5	5	8	8	11
3	5	7	7	-	3	2	8	7	7	10
4	6	4	8	3	-	5	9	10	10	7
5	7	9	5	2	5	-	6	9	9	12
6	7	9	5	8	9	6	-	3	9	6
7	8	6	8	7	10	9	3	-	6	9
8	8	6	8	7	10	9	9	6	-	3
9	9	7	11	10	7	12	6	9	3	-

≥ 42; $(\frac{6}{17}, \frac{11}{17})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	2	2	3	3	4	4
1	1	-	2	2	1	1	2	2	3	3
2	1	2	-	2	3	3	2	4	5	3
3	3	2	2	-	3	1	4	2	5	3
4	2	1	3	3	-	2	1	3	4	4
5	2	1	3	1	2	-	3	1	4	4
6	3	2	2	4	1	3	-	2	3	5
7	3	2	4	2	3	1	2	-	3	5
8	4	3	5	5	4	4	3	3	-	2
9	4	3	3	3	4	4	5	5	2	-

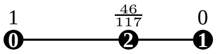
≥ 18; SMAPO# 287

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	4	4	4	4	5	6	6
1	1	-	2	3	3	5	5	4	5	7
2	1	2	-	5	5	5	3	6	7	5
3	4	3	5	-	4	2	6	3	6	4
4	4	3	5	4	-	6	2	5	6	8
5	4	5	5	2	6	-	4	1	4	4
6	4	5	3	6	2	4	-	3	4	6
7	5	4	6	3	5	1	3	-	3	5
8	6	5	7	6	6	4	4	3	-	2
9	6	7	5	4	8	4	6	5	2	-

≥ 26; SMAPO# 1162

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	6	6	6	7	8	10	10
1	2	-	4	4	4	6	7	6	8	10
2	2	4	-	6	8	8	5	10	12	8
3	6	4	6	-	6	2	9	4	10	6
4	6	4	8	6	-	8	3	8	10	12
5	6	6	8	2	8	-	7	2	8	8
6	7	7	5	9	3	7	-	5	7	11
7	8	6	10	4	8	2	5	-	6	10
8	10	8	12	10	10	8	7	6	-	4
9	10	10	8	6	12	8	11	10	4	-

≥ 42; $(\frac{11}{30}, \frac{19}{30})$



TT-disjoint at 1

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	2	2	3	3
1	1	-	3	3	2	2	3	3	4	4
2	2	3	-	4	1	3	4	4	5	3
3	2	3	4	-	3	1	4	4	3	3
4	3	2	1	3	-	2	3	3	4	4
5	3	2	3	1	2	-	3	3	4	2
6	2	3	4	4	3	3	-	2	1	3
7	2	3	4	4	3	3	2	-	3	1
8	3	4	5	3	4	4	1	3	-	2
9	3	4	3	3	4	2	3	1	2	-

$\geq$ 18; SMAPO# 276

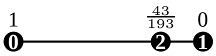
	0	1	2	3	4	5	6	7	8	9
0	-	2	1	2	2	3	4	4	4	6
1	2	-	3	4	2	3	4	4	6	6
2	1	3	-	3	1	4	5	5	5	5
3	2	4	3	-	4	1	6	6	4	6
4	2	2	1	4	-	3	6	6	4	6
5	3	3	4	1	3	-	7	7	5	5
6	4	4	5	6	6	7	-	4	2	6
7	4	4	5	6	6	7	4	-	6	2
8	4	6	5	4	4	5	2	6	-	4
9	6	6	5	6	6	5	6	2	4	-

$\geq$ 26; SMAPO# 1036

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	4	5	6	6	6	7	9
1	2	-	5	6	3	4	6	6	9	9
2	3	5	-	7	2	7	9	9	10	8
3	4	6	7	-	7	2	10	10	7	9
4	5	3	2	7	-	5	9	9	8	10
5	6	4	7	2	5	-	10	10	9	7
6	6	6	9	10	9	10	-	6	3	9
7	6	6	9	10	9	10	6	-	9	3
8	7	9	10	7	8	9	3	9	-	6
9	9	9	8	9	10	7	9	3	6	-

$\geq 42; (\frac{46}{117}, \frac{71}{117})$

RHS $= 43$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	2	3	4	4
1	1	-	3	3	4	4	3	2	3	3
2	2	3	-	4	5	3	4	3	4	2
3	2	3	4	-	3	5	4	3	2	4
4	3	4	5	3	-	2	3	4	5	3
5	3	4	3	5	2	-	3	4	3	5
6	2	3	4	4	3	3	-	1	2	2
7	3	2	3	3	4	4	1	-	3	3
8	4	3	4	2	5	3	2	3	-	4
9	4	3	2	4	3	5	2	3	4	-

$\geq$ 22; SMAPO# 464

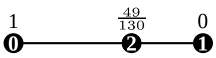
	0	1	2	3	4	5	6	7	8	9
0	-	3	7	6	7	9	10	11	10	10
1	3	-	8	9	10	10	13	8	7	13
2	7	8	-	13	14	10	15	10	11	5
3	6	9	13	-	7	11	14	13	4	14
4	7	10	14	7	-	4	9	14	11	9
5	9	10	10	11	4	-	11	14	7	13
6	10	13	15	14	9	11	-	5	10	10
7	11	8	10	13	14	14	5	-	11	15
8	10	7	11	4	11	7	10	11	-	12
9	10	13	5	14	9	13	10	15	12	-

$\geq$ 66; SMAPO# 11491

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	4	5	6	6	7	7	7
1	2	-	5	6	7	7	8	5	5	8
2	4	5	-	8	9	6	9	6	7	3
3	4	6	8	-	5	8	9	8	3	9
4	5	7	9	5	-	3	6	9	8	6
5	6	7	6	8	3	-	7	9	5	9
6	6	8	9	9	6	7	-	3	6	6
7	7	5	6	8	9	9	3	-	7	9
8	7	5	7	3	8	5	6	7	-	8
9	7	8	3	9	6	9	6	9	8	-

$\geq 43; (\frac{43}{193}, \frac{150}{193})$

RHS $= 44$



TT-disjoint at 7

	0	1	2	3	4	5	6	7	8	9
0	-	1	0	3	2	3	4	3	2	4
1	1	-	1	2	3	2	3	4	3	5
2	0	1	-	3	2	3	4	3	2	4
3	3	2	3	-	1	4	3	2	3	5
4	2	3	2	1	-	3	4	1	2	4
5	3	2	3	4	3	-	5	2	1	3
6	4	3	4	3	4	5	-	5	4	2
7	3	4	3	2	1	2	5	-	3	3
8	2	3	2	3	2	1	4	3	-	4
9	4	5	4	5	4	3	2	3	4	-

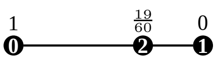
$\geq$ 18; SMAPO# 291

	0	1	2	3	4	5	6	7	8	9
0	-	0	2	3	4	4	4	6	6	6
1	0	-	2	3	4	4	4	6	6	6
2	2	2	-	5	6	6	6	4	4	8
3	3	3	5	-	1	7	3	3	5	5
4	4	4	6	1	-	6	4	2	6	4
5	4	4	6	7	6	-	8	6	2	6
6	4	4	6	3	4	8	-	6	6	2
7	6	6	4	3	2	6	6	-	4	4
8	6	6	4	5	6	2	6	4	-	8
9	6	6	8	5	4	6	2	4	8	-

$\geq$ 28; SMAPO# 1414

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	6	6	7	8	8	8	10
1	1	-	3	5	7	6	7	9	9	11
2	2	3	-	8	8	9	10	6	6	12
3	6	5	8	-	2	11	6	4	8	10
4	6	7	8	2	-	9	8	2	8	8
5	7	6	9	11	9	-	13	7	3	9
6	8	7	10	6	8	13	-	10	10	4
7	8	9	6	4	2	7	10	-	6	6
8	8	9	6	8	8	3	10	6	-	12
9	10	11	12	10	8	9	4	6	12	-

$\geq 44; (\frac{49}{130}, \frac{81}{130})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	0	1	1	3	2	2	3	2	3
1	0	-	1	1	3	2	2	3	2	3
2	1	1	-	2	2	3	3	2	3	4
3	1	1	2	-	2	3	1	2	3	2
4	3	3	2	2	-	3	1	2	3	4
5	2	2	3	3	3	-	2	3	2	1
6	2	2	3	1	1	2	-	3	2	3
7	3	3	2	2	2	3	3	-	1	2
8	2	2	3	3	3	2	2	1	-	3
9	3	3	4	2	4	1	3	2	3	-

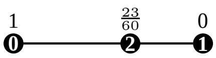
$\geq$ 14; SMAPO# 136

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	5	4	5	6	5	7	7
1	2	-	5	3	6	7	6	7	5	7
2	3	5	-	2	3	6	5	4	6	6
3	5	3	2	-	5	6	3	4	6	4
4	4	6	3	5	-	7	2	5	5	7
5	5	7	6	6	7	-	5	6	4	2
6	6	6	5	3	2	5	-	7	7	7
7	5	7	4	4	5	6	7	-	2	4
8	7	5	6	6	5	4	7	2	-	6
9	7	7	6	4	7	2	7	4	6	-

$\geq$ 32; SMAPO# 1815

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	6	7	7	8	8	9	10
1	2	-	5	4	9	9	8	10	7	10
2	3	5	-	3	4	8	7	5	8	9
3	6	4	3	-	7	9	4	6	9	6
4	7	9	4	7	-	10	3	7	8	11
5	7	9	8	9	10	-	7	9	6	3
6	8	8	7	4	3	7	-	10	9	10
7	8	10	5	6	7	9	10	-	3	6
8	9	7	8	9	8	6	9	3	-	9
9	10	10	9	6	11	3	10	6	9	-

$\geq 44; (\frac{19}{60}, \frac{41}{60})$



TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	2	3	3	3	4	4
1	1	-	3	2	3	2	2	2	3	3
2	2	3	-	3	2	1	3	3	4	4
3	3	2	3	-	1	4	4	4	3	3
4	2	3	2	1	-	3	3	3	2	4
5	3	2	1	4	3	-	4	4	3	5
6	3	2	3	4	3	4	-	2	1	3
7	3	2	3	4	3	4	2	-	3	1
8	4	3	4	3	2	3	1	3	-	2
9	4	3	4	3	4	5	3	1	2	-

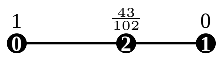
$\geq$ 18; SMAPO# 276

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	4	4	4	4	6	6
1	2	-	3	3	4	2	6	6	4	6
2	3	3	-	4	3	1	5	5	5	7
3	3	3	4	-	1	3	7	7	5	5
4	4	4	3	1	-	4	6	6	4	6
5	4	2	1	3	4	-	6	6	4	6
6	4	6	5	7	6	6	-	4	2	6
7	4	6	5	7	6	6	4	-	6	2
8	6	4	5	5	4	4	2	6	-	4
9	6	6	7	5	6	6	6	2	4	-

$\geq$ 28; SMAPO# 1289

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	5	5	6	6	6	9	9
1	2	-	6	5	7	4	8	8	7	9
2	4	6	-	7	5	2	8	8	9	11
3	5	5	7	-	2	7	11	11	8	8
4	5	7	5	2	-	7	9	9	6	10
5	6	4	2	7	7	-	10	10	7	11
6	6	8	8	11	9	10	-	6	3	9
7	6	8	8	11	9	10	6	-	9	3
8	9	7	9	8	6	7	3	9	-	6
9	9	9	11	8	10	11	9	3	6	-

$\geq 44; (\frac{23}{60}, \frac{37}{60})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	2	4	3	4
1	1	-	3	3	4	4	3	3	2	3
2	2	3	-	4	3	5	4	2	3	4
3	2	3	4	-	5	3	4	4	3	2
4	3	4	3	5	-	2	3	5	4	3
5	3	4	5	3	2	-	3	3	4	5
6	2	3	4	4	3	3	-	2	1	2
7	4	3	2	4	5	3	2	-	3	4
8	3	2	3	3	4	4	1	3	-	3
9	4	3	4	2	3	5	2	4	3	-

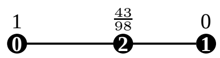
≥ 22; SMAPO# 464

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	2	3	4	3	4	4
1	1	-	3	3	3	4	5	2	3	5
2	2	3	-	6	2	3	4	1	6	4
3	4	3	6	-	6	5	6	5	4	2
4	2	3	2	6	-	1	4	3	6	4
5	3	4	3	5	1	-	5	2	5	5
6	4	5	4	6	4	5	-	3	2	4
7	3	2	1	5	3	2	3	-	5	3
8	4	3	6	4	6	5	2	5	-	6
9	4	5	4	2	4	5	4	3	6	-

≥ 24; SMAPO# 724

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	5	5	6	6	7	7	8
1	2	-	6	5	7	8	8	5	5	8
2	4	6	-	9	5	8	8	3	9	8
3	5	5	9	-	10	7	9	8	6	3
4	5	7	5	10	-	3	7	8	10	7
5	6	8	8	7	3	-	8	5	9	10
6	6	8	8	9	7	8	-	5	3	6
7	7	5	3	8	8	5	5	-	8	7
8	7	5	9	6	10	9	3	8	-	9
9	8	8	8	3	7	10	6	7	9	-

$\geq 44; (\frac{43}{102}, \frac{59}{102})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	2	4	3	3	4	3	3
1	1	-	2	3	3	4	2	3	2	4
2	3	2	-	3	5	4	4	3	4	2
3	2	3	3	-	4	3	3	4	1	3
4	4	3	5	4	-	5	3	2	3	3
5	3	4	4	3	5	-	2	3	2	4
6	3	2	4	3	3	2	-	5	4	4
7	4	3	3	4	2	3	5	-	3	5
8	3	2	4	1	3	2	4	3	-	2
9	3	4	2	3	3	4	4	5	2	-

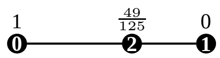
≥ 22; SMAPO# 464

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	2	3	4	3	5	5
1	1	-	4	4	3	4	3	4	4	4
2	3	4	-	4	5	4	5	4	6	2
3	3	4	4	-	5	4	5	4	2	6
4	2	3	5	5	-	3	2	1	3	3
5	3	4	4	4	3	-	1	2	4	4
6	4	3	5	5	2	1	-	3	5	5
7	3	4	4	4	1	2	3	-	4	4
8	5	4	6	2	3	4	5	4	-	4
9	5	4	2	6	3	4	5	4	4	-

≥ 24; SMAPO# 665

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	5	6	6	7	7	8	8
1	2	-	5	7	6	8	5	7	6	8
2	5	5	-	6	9	7	8	6	9	3
3	5	7	6	-	9	7	8	8	3	9
4	6	6	9	9	-	8	5	3	6	6
5	6	8	7	7	8	-	3	5	6	8
6	7	5	8	8	5	3	-	8	9	9
7	7	7	6	8	3	5	8	-	7	9
8	8	6	9	3	6	6	9	7	-	6
9	8	8	3	9	6	8	9	9	6	-

$\geq 44; (\frac{43}{98}, \frac{55}{98})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	0	1	3	2	2	3	3	4	4
1	0	-	1	3	2	2	3	3	4	4
2	1	1	-	4	3	3	2	2	3	5
3	3	3	4	-	3	1	2	2	5	3
4	2	2	3	3	-	2	1	3	4	4
5	2	2	3	1	2	-	3	1	4	4
6	3	3	2	2	1	3	-	4	5	3
7	3	3	2	2	3	1	4	-	3	5
8	4	4	3	5	4	4	5	3	-	2
9	4	4	5	3	4	4	3	5	2	-

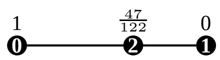
≥ 18; SMAPO# 291

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	4	4	5	6	6
1	1	-	3	3	3	5	5	4	5	7
2	2	3	-	6	6	4	4	3	4	6
3	4	3	6	-	4	2	6	3	6	4
4	4	3	6	4	-	6	2	5	6	8
5	4	5	4	2	6	-	6	1	4	4
6	4	5	4	6	2	6	-	7	8	6
7	5	4	3	3	5	1	7	-	3	5
8	6	5	4	6	6	4	8	3	-	2
9	6	7	6	4	8	4	6	5	2	-

≥ 28; SMAPO# 1389

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	6	6	6	7	8	10	10
1	1	-	4	5	5	7	8	7	9	11
2	3	4	-	9	9	7	6	5	7	11
3	6	5	9	-	6	2	7	4	10	6
4	6	5	9	6	-	8	3	8	10	12
5	6	7	7	2	8	-	9	2	8	8
6	7	8	6	7	3	9	-	11	13	9
7	8	7	5	4	8	2	11	-	6	10
8	10	9	7	10	10	8	13	6	-	4
9	10	11	11	6	12	8	9	10	4	-

$\geq 44; (\frac{49}{125}, \frac{76}{125})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	2	3	3	3	5	4
1	1	-	2	1	3	4	2	2	4	3
2	1	2	-	3	1	2	2	2	4	3
3	2	1	3	-	4	5	3	3	3	2
4	2	3	1	4	-	3	1	3	5	2
5	3	4	2	5	3	-	4	4	2	5
6	3	2	2	3	1	4	-	2	4	3
7	3	2	2	3	3	4	2	-	4	1
8	5	4	4	3	5	2	4	4	-	3
9	4	3	3	2	2	5	3	1	3	-

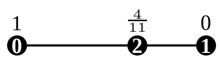
≥ 18; SMAPO# 289

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	3	3	5	5	5	7
1	1	-	2	2	4	4	6	6	4	6
2	1	2	-	4	2	2	4	4	4	6
3	3	2	4	-	6	6	4	4	4	6
4	3	4	2	6	-	4	2	6	6	4
5	3	4	2	6	4	-	6	6	2	8
6	5	6	4	4	2	6	-	4	8	6
7	5	6	4	4	6	6	4	-	8	2
8	5	4	4	4	6	2	8	8	-	6
9	7	6	6	6	4	8	6	2	6	-

≥ 28; SMAPO# 1405

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	4	5	6	8	8	10	11
1	2	-	4	2	7	8	8	8	8	9
2	2	4	-	6	3	4	6	6	8	9
3	4	2	6	-	9	10	6	6	6	7
4	5	7	3	9	-	7	3	9	11	6
5	6	8	4	10	7	-	10	10	4	13
6	8	8	6	6	3	10	-	6	12	9
7	8	8	6	6	9	10	6	-	12	3
8	10	8	8	6	11	4	12	12	-	9
9	11	9	9	7	6	13	9	3	9	-

$\geq 44; (\frac{47}{122}, \frac{75}{122})$



TT-disjoint at 1

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	1	3	2	2	4	4	3
1	1	-	2	2	4	3	3	3	5	2
2	1	2	-	2	2	3	3	5	3	4
3	1	2	2	-	2	1	1	3	3	2
4	3	4	2	2	-	1	3	5	3	2
5	2	3	3	1	1	-	2	4	4	3
6	2	3	3	1	3	2	-	4	4	1
7	4	3	5	3	5	4	4	-	2	3
8	4	5	3	3	3	4	4	2	-	5
9	3	2	4	2	2	3	1	3	5	-

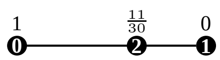
≥ 18; SMAPO# 287

	0	1	2	3	4	5	6	7	8	9
0	-	2	1	4	4	6	6	4	4	6
1	2	-	3	2	6	4	4	4	6	6
2	1	3	-	3	3	5	5	5	3	7
3	4	2	3	-	4	2	2	6	4	4
4	4	6	3	4	-	2	6	6	4	4
5	6	4	5	2	2	-	4	8	6	6
6	6	4	5	2	6	4	-	8	6	2
7	4	4	5	6	6	8	8	-	2	6
8	4	6	3	4	4	6	6	2	-	8
9	6	6	7	4	4	6	2	6	8	-

≥ 28; SMAPO# 1413

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	5	7	8	8	8	8	9
1	2	-	4	3	9	6	6	6	10	7
2	2	4	-	5	5	8	8	10	6	11
3	5	3	5	-	6	3	3	9	7	6
4	7	9	5	6	-	3	9	11	7	6
5	8	6	8	3	3	-	6	12	10	9
6	8	6	8	3	9	6	-	12	10	3
7	8	6	10	9	11	12	12	-	4	9
8	8	10	6	7	7	10	10	4	-	13
9	9	7	11	6	6	9	3	9	13	-

$\geq 44; (\frac{4}{11}, \frac{7}{11})$



TT-disjoint at 1

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	1	2	3	2	4	4	3
1	1	-	2	2	3	4	3	3	5	2
2	1	2	-	2	3	2	3	5	3	4
3	1	2	2	-	1	2	1	3	3	2
4	2	3	3	1	-	1	2	4	4	3
5	3	4	2	2	1	-	3	5	3	2
6	2	3	3	1	2	3	-	4	4	1
7	4	3	5	3	4	5	4	-	2	3
8	4	5	3	3	4	3	4	2	-	5
9	3	2	4	2	3	2	1	3	5	-

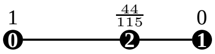
≥ 18; SMAPO# 287

	0	1	2	3	4	5	6	7	8	9
0	-	2	1	5	4	4	6	4	4	6
1	2	-	3	3	4	6	4	4	6	6
2	1	3	-	4	5	3	5	5	3	7
3	5	3	4	-	1	3	3	7	5	5
4	4	4	5	1	-	2	4	6	6	4
5	4	6	3	3	2	-	6	6	4	4
6	6	4	5	3	4	6	-	8	6	2
7	4	4	5	7	6	6	8	-	2	6
8	4	6	3	5	6	4	6	2	-	8
9	6	6	7	5	4	4	2	6	8	-

≥ 28; SMAPO# 1389

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	6	6	7	8	8	8	9
1	2	-	4	4	6	9	6	6	10	7
2	2	4	-	6	8	5	8	10	6	11
3	6	4	6	-	2	5	4	10	8	7
4	6	6	8	2	-	3	6	10	10	7
5	7	9	5	5	3	-	9	11	7	6
6	8	6	8	4	6	9	-	12	10	3
7	8	6	10	10	10	11	12	-	4	9
8	8	10	6	8	10	7	10	4	-	13
9	9	7	11	7	7	6	3	9	13	-

$\geq 44; (\frac{11}{30}, \frac{19}{30})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	3	1	3	4	2	4
1	1	-	2	3	2	2	4	3	3	5
2	1	2	-	3	4	2	2	5	3	3
3	2	3	3	-	3	1	1	4	2	4
4	3	2	4	3	-	2	2	3	1	5
5	1	2	2	1	2	-	2	3	1	3
6	3	4	2	1	2	2	-	5	3	3
7	4	3	5	4	3	3	5	-	4	2
8	2	3	3	2	1	1	3	4	-	4
9	4	5	3	4	5	3	3	2	4	-

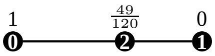
$\geq$ 18; SMAPO# 287

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	6	4	4	6	4
1	1	-	3	3	3	5	5	3	5	5
2	2	3	-	6	6	4	4	6	4	4
3	4	3	6	-	4	2	2	4	6	6
4	4	3	6	4	-	6	2	4	2	6
5	6	5	4	2	6	-	4	6	4	8
6	4	5	4	2	2	4	-	6	4	4
7	4	3	6	4	4	6	6	-	6	2
8	6	5	4	6	2	4	4	6	-	8
9	4	5	4	6	6	8	4	2	8	-

$\geq$ 28; SMAPO# 1404

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	6	7	7	7	8	8	8
1	2	-	4	6	5	7	9	6	8	10
2	2	4	-	8	9	5	5	10	6	6
3	6	6	8	-	7	3	3	8	8	10
4	7	5	9	7	-	8	4	7	3	11
5	7	7	5	3	8	-	6	9	5	11
6	7	9	5	3	4	6	-	11	7	7
7	8	6	10	8	7	9	11	-	10	4
8	8	8	6	8	3	5	7	10	-	12
9	8	10	6	10	11	11	7	4	12	-

$\geq 44; (\frac{44}{115}, \frac{71}{115})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	3	3	4	2	2	4
1	1	-	2	3	2	4	3	3	3	5
2	1	2	-	3	4	2	5	3	3	3
3	2	3	3	-	3	1	4	0	2	4
4	3	2	4	3	-	2	3	3	1	5
5	3	4	2	1	2	-	5	1	3	3
6	4	3	5	4	3	5	-	4	4	2
7	2	3	3	0	3	1	4	-	2	4
8	2	3	3	2	1	3	4	2	-	4
9	4	5	3	4	5	3	2	4	4	-

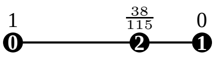
$\geq$ 18; SMAPO# 291

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	4	4	6	6	4
1	1	-	3	3	3	5	3	5	5	5
2	2	3	-	6	6	4	6	4	4	4
3	4	3	6	-	4	2	4	2	6	6
4	4	3	6	4	-	2	4	6	2	6
5	4	5	4	2	2	-	6	4	4	4
6	4	3	6	4	4	6	-	6	6	2
7	6	5	4	2	6	4	6	-	4	8
8	6	5	4	6	2	4	6	4	-	8
9	4	5	4	6	6	4	2	8	8	-

$\geq$ 28; SMAPO# 1404

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	6	7	7	8	8	8	8
1	2	-	4	6	5	9	6	8	8	10
2	2	4	-	8	9	5	10	6	6	6
3	6	6	8	-	7	3	8	2	8	10
4	7	5	9	7	-	4	7	9	3	11
5	7	9	5	3	4	-	11	5	7	7
6	8	6	10	8	7	11	-	10	10	4
7	8	8	6	2	9	5	10	-	6	12
8	8	8	6	8	3	7	10	6	-	12
9	8	10	6	10	11	7	4	12	12	-

$\geq 44; (\frac{49}{120}, \frac{71}{120})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	2	3	3	4	4	3
1	1	-	3	1	3	2	2	3	3	2
2	2	3	-	2	2	3	3	2	2	3
3	2	1	2	-	2	3	1	2	2	3
4	2	3	2	2	-	3	1	4	4	3
5	3	2	3	3	3	-	2	1	3	2
6	3	2	3	1	1	2	-	3	3	2
7	4	3	2	2	4	1	3	-	2	3
8	4	3	2	2	4	3	3	2	-	1
9	3	2	3	3	3	2	2	3	1	-

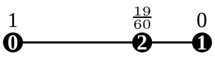
$\geq$ 16; SMAPO# 200

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	5	5	5	5	7
1	1	-	3	2	4	4	4	6	6	6
2	2	3	-	5	3	3	5	5	5	5
3	3	2	5	-	6	6	4	4	4	6
4	3	4	3	6	-	6	2	6	6	8
5	5	4	3	6	6	-	6	2	6	4
6	5	4	5	4	2	6	-	8	8	6
7	5	6	5	4	6	2	8	-	4	6
8	5	6	5	4	6	6	8	4	-	2
9	7	6	5	6	8	4	6	6	2	-

$\geq$ 30; SMAPO# 1617

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	5	5	8	8	9	9	10
1	2	-	5	3	7	6	6	9	9	8
2	3	5	-	6	4	5	7	6	6	7
3	5	3	6	-	8	9	5	6	6	9
4	5	7	4	8	-	9	3	10	10	11
5	8	6	5	9	9	-	8	3	9	6
6	8	6	7	5	3	8	-	11	11	8
7	9	9	6	6	10	3	11	-	6	9
8	9	9	6	6	10	9	11	6	-	3
9	10	8	7	9	11	6	8	9	3	-

$\geq 44; (\frac{38}{115}, \frac{77}{115})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	0	1	1	3	2	2	3	2	3
1	0	-	1	1	3	2	2	3	2	3
2	1	1	-	2	2	3	3	2	3	4
3	1	1	2	-	2	3	1	2	3	2
4	3	3	2	2	-	3	1	2	3	4
5	2	2	3	3	3	-	2	1	2	3
6	2	2	3	1	1	2	-	3	2	3
7	3	3	2	2	2	1	3	-	3	2
8	2	2	3	3	3	2	2	3	-	1
9	3	3	4	2	4	3	3	2	1	-

$\geq$ 14; SMAPO# 136

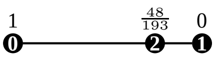
	0	1	2	3	4	5	6	7	8	9
0	-	2	3	5	4	5	6	5	7	7
1	2	-	5	3	6	7	6	7	5	7
2	3	5	-	2	3	6	5	4	6	6
3	5	3	2	-	5	6	3	4	6	4
4	4	6	3	5	-	7	2	5	5	7
5	5	7	6	6	7	-	5	2	4	6
6	6	6	5	3	2	5	-	7	7	7
7	5	7	4	4	5	2	7	-	6	4
8	7	5	6	6	5	4	7	6	-	2
9	7	7	6	4	7	6	7	4	2	-

$\geq$ 32; SMAPO# 1815

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	6	7	7	8	8	9	10
1	2	-	5	4	9	9	8	10	7	10
2	3	5	-	3	4	8	7	5	8	9
3	6	4	3	-	7	9	4	6	9	6
4	7	9	4	7	-	10	3	7	8	11
5	7	9	8	9	10	-	7	3	6	9
6	8	8	7	4	3	7	-	10	9	10
7	8	10	5	6	7	3	10	-	9	6
8	9	7	8	9	8	6	9	9	-	3
9	10	10	9	6	11	9	10	6	3	-

$\geq 44; (\frac{19}{60}, \frac{41}{60})$

RHS = 45



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	2	2	3	3	3	4	5
1	1	-	4	3	3	2	2	2	3	4
2	3	4	-	3	5	4	4	4	5	2
3	2	3	3	-	4	3	3	1	2	5
4	2	3	5	4	-	3	1	3	2	3
5	3	2	4	3	3	-	4	2	1	4
6	3	2	4	3	1	4	-	4	3	4
7	3	2	4	1	3	2	4	-	3	4
8	4	3	5	2	2	1	3	3	-	3
9	5	4	2	5	3	4	4	4	3	-

$\geq$ 20; SMAPO# 363

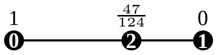
	0	1	2	3	4	5	6	7	8	9
0	-	5	8	8	8	11	13	13	12	11
1	5	-	13	13	13	10	8	8	11	12
2	8	13	-	10	14	9	9	11	14	5
3	8	13	10	-	14	13	11	5	8	13
4	8	13	14	14	-	11	5	11	10	9
5	11	10	9	13	11	-	14	8	5	14
6	13	8	9	11	5	14	-	12	9	14
7	13	8	11	5	11	8	12	-	13	12
8	12	11	14	8	10	5	9	13	-	9
9	11	12	5	13	9	14	14	12	9	-

$\geq$ 72; SMAPO# 12367

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	5	5	7	8	8	8	8
1	3	-	8	8	8	6	5	5	7	8
2	5	8	-	6	9	6	6	7	9	3
3	5	8	6	-	9	8	7	3	5	9
4	5	8	9	9	-	7	3	7	6	6
5	7	6	6	8	7	-	9	5	3	9
6	8	5	6	7	3	9	-	8	6	9
7	8	5	7	3	7	5	8	-	8	8
8	8	7	9	5	6	3	6	8	-	6
9	8	8	3	9	6	9	9	8	6	-

$\geq 45; (\frac{48}{193}, \frac{145}{193})$

RHS $= 46$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	0	1	1	3	2	2	4	4	3
1	0	-	1	1	3	2	2	4	4	3
2	1	1	-	2	4	3	3	3	5	2
3	1	1	2	-	2	3	3	5	3	4
4	3	3	4	2	-	1	3	5	3	2
5	2	2	3	3	1	-	2	4	4	3
6	2	2	3	3	3	2	-	4	4	1
7	4	4	3	5	5	4	4	-	2	3
8	4	4	5	3	3	4	4	2	-	5
9	3	3	2	4	2	3	1	3	5	-

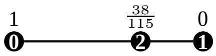
$\geq$ 18; SMAPO# 290

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	4	6	6	4	4	6
1	1	-	3	2	5	5	5	5	5	7
2	2	3	-	3	6	4	4	4	6	6
3	3	2	3	-	3	5	5	5	3	7
4	4	5	6	3	-	2	6	6	4	4
5	6	5	4	5	2	-	4	8	6	6
6	6	5	4	5	6	4	-	8	6	2
7	4	5	4	5	6	8	8	-	2	6
8	4	5	6	3	4	6	6	2	-	8
9	6	7	6	7	4	6	2	6	8	-

$\geq$ 30; SMAPO# 1618

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	7	8	8	8	8	9
1	1	-	3	3	8	7	7	9	9	10
2	2	3	-	4	9	6	6	6	10	7
3	4	3	4	-	5	8	8	10	6	11
4	7	8	9	5	-	3	9	11	7	6
5	8	7	6	8	3	-	6	12	10	9
6	8	7	6	8	9	6	-	12	10	3
7	8	9	6	10	11	12	12	-	4	9
8	8	9	10	6	7	10	10	4	-	13
9	9	10	7	11	6	9	3	9	13	-

$\geq 46; (\frac{47}{124}, \frac{77}{124})$



TT-disjoint at 4

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	2	2	2	2	3	3
1	1	-	2	1	3	3	3	3	2	2
2	1	2	-	3	1	3	3	3	2	2
3	2	1	3	-	2	2	4	4	3	3
4	2	3	1	2	-	2	4	4	3	3
5	2	3	3	2	2	-	2	2	3	3
6	2	3	3	4	4	2	-	2	1	3
7	2	3	3	4	4	2	2	-	3	1
8	3	2	2	3	3	3	1	3	-	2
9	3	2	2	3	3	3	3	1	2	-

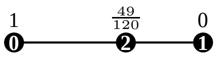
$\geq$ 16; SMAPO# 200

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	3	4	5	5	6	6
1	1	-	2	1	4	3	6	6	5	5
2	1	2	-	3	2	3	6	6	7	7
3	2	1	3	-	3	2	7	7	6	6
4	3	4	2	3	-	3	8	8	5	5
5	4	3	3	2	3	-	5	5	8	8
6	5	6	6	7	8	5	-	6	3	9
7	5	6	6	7	8	5	6	-	9	3
8	6	5	7	6	5	8	3	9	-	6
9	6	5	7	6	5	8	9	3	6	-

$\geq$ 32; SMAPO# 1961

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	4	4	6	7	7	9	9
1	2	-	4	2	6	6	9	9	7	7
2	2	4	-	6	2	6	9	9	9	9
3	4	2	6	-	4	4	11	11	9	9
4	4	6	2	4	-	4	11	11	7	7
5	6	6	6	4	4	-	7	7	11	11
6	7	9	9	11	11	7	-	8	4	12
7	7	9	9	11	11	7	8	-	12	4
8	9	7	9	9	7	11	4	12	-	8
9	9	7	9	9	7	11	12	4	8	-

$\geq 46; (\frac{38}{115}, \frac{77}{115})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	3	3	4	2	2	4
1	1	-	2	2	4	4	3	3	3	5
2	1	2	-	4	4	2	5	3	3	3
3	3	2	4	-	2	2	3	1	3	5
4	3	4	4	2	-	2	5	3	1	5
5	3	4	2	2	2	-	5	3	3	3
6	4	3	5	3	5	5	-	4	4	2
7	2	3	3	1	3	3	4	-	2	4
8	2	3	3	3	1	3	4	2	-	4
9	4	5	3	5	5	3	2	4	4	-

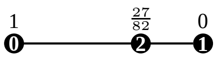
$\geq$ 20; SMAPO# 365

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	4	4	6	6	4
1	1	-	3	3	3	5	3	5	5	5
2	2	3	-	6	6	4	6	4	4	4
3	4	3	6	-	4	2	4	2	6	6
4	4	3	6	4	-	2	4	6	2	6
5	4	5	4	2	2	-	6	4	4	4
6	4	3	6	4	4	6	-	6	6	2
7	6	5	4	2	6	4	6	-	4	8
8	6	5	4	6	2	4	6	4	-	8
9	4	5	4	6	6	4	2	8	8	-

$\geq$ 28; SMAPO# 1404

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	7	7	7	8	8	8	8
1	2	-	4	5	7	9	6	8	8	10
2	2	4	-	9	9	5	10	6	6	6
3	7	5	9	-	6	4	7	3	9	11
4	7	7	9	6	-	4	9	9	3	11
5	7	9	5	4	4	-	11	7	7	7
6	8	6	10	7	9	11	-	10	10	4
7	8	8	6	3	9	7	10	-	6	12
8	8	8	6	9	3	7	10	6	-	12
9	8	10	6	11	11	7	4	12	12	-

$\geq 46; (\frac{49}{120}, \frac{71}{120})$



TT-disjoint at 1

	0	1	2	3	4	5	6	7	8	9
0	-	2	1	6	6	4	6	4	4	6
1	2	-	3	4	6	6	4	4	6	6
2	1	3	-	5	7	3	5	5	3	7
3	6	4	5	-	2	6	4	8	6	6
4	6	6	7	2	-	4	6	6	8	4
5	4	6	3	6	4	-	6	6	4	4
6	6	4	5	4	6	6	-	8	6	2
7	4	4	5	8	6	6	8	-	2	6
8	4	6	3	6	8	4	6	2	-	8
9	6	6	7	6	4	4	2	6	8	-

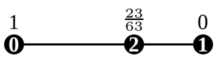
$\geq$ 32; SMAPO# 1859

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	8	8	10	10	12	12	12
1	3	-	6	7	9	13	9	9	15	9
2	3	6	-	9	11	7	11	15	9	15
3	8	7	9	-	2	6	6	14	12	10
4	8	9	11	2	-	4	8	12	14	8
5	10	13	7	6	4	-	12	16	10	8
6	10	9	11	6	8	12	-	16	14	4
7	12	9	15	14	12	16	16	-	6	12
8	12	15	9	12	14	10	14	6	-	18
9	12	9	15	10	8	8	4	12	18	-

$\geq$ 62; SMAPO# 10896

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	7	7	7	8	8	8	9
1	2	-	4	5	7	9	6	6	10	7
2	2	4	-	7	9	5	8	10	6	11
3	7	5	7	-	2	6	5	11	9	8
4	7	7	9	2	-	4	7	9	11	6
5	7	9	5	6	4	-	9	11	7	6
6	8	6	8	5	7	9	-	12	10	3
7	8	6	10	11	9	11	12	-	4	9
8	8	10	6	9	11	7	10	4	-	13
9	9	7	11	8	6	6	3	9	13	-

$\geq 46; (\frac{27}{82}, \frac{55}{82})$



TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	2	2	3	3	3	4	4
1	1	-	2	3	3	2	2	2	3	3
2	3	2	-	1	3	4	4	4	5	3
3	2	3	1	-	2	3	3	3	4	4
4	2	3	3	2	-	3	3	1	4	2
5	3	2	4	3	3	-	2	4	1	3
6	3	2	4	3	3	2	-	4	3	1
7	3	2	4	3	1	4	4	-	3	3
8	4	3	5	4	4	1	3	3	-	2
9	4	3	3	4	2	3	1	3	2	-

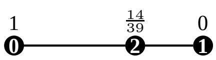
$\geq$ 18; SMAPO# 276

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	3	5	4	4	5	6	6
1	2	-	2	3	5	6	6	3	4	6
2	2	2	-	1	5	6	6	5	6	4
3	3	3	1	-	4	5	5	6	7	5
4	5	5	5	4	-	7	7	2	7	5
5	4	6	6	5	7	-	4	7	2	6
6	4	6	6	5	7	4	-	7	6	2
7	5	3	5	6	2	7	7	-	5	7
8	6	4	6	7	7	2	6	5	-	4
9	6	6	4	5	5	6	2	7	4	-

$\geq$ 30; SMAPO# 1559

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	4	6	6	6	7	9	9
1	2	-	4	6	8	8	8	5	7	9
2	4	4	-	2	8	10	10	9	11	7
3	4	6	2	-	6	8	8	9	11	9
4	6	8	8	6	-	10	10	3	11	7
5	6	8	10	8	10	-	6	11	3	9
6	6	8	10	8	10	6	-	11	9	3
7	7	5	9	9	3	11	11	-	8	10
8	9	7	11	11	11	3	9	8	-	6
9	9	9	7	9	7	9	3	10	6	-

$\geq 46; (\frac{23}{63}, \frac{40}{63})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	4	3	4	3
1	1	-	3	3	2	2	3	4	3	4
2	2	3	-	2	3	1	4	3	4	3
3	2	3	2	-	3	3	2	3	2	3
4	3	2	3	3	-	2	3	2	3	2
5	3	2	1	3	2	-	3	4	3	4
6	4	3	4	2	3	3	-	1	2	3
7	3	4	3	3	2	4	1	-	3	2
8	4	3	4	2	3	3	2	3	-	1
9	3	4	3	3	2	4	3	2	1	-

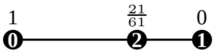
$\geq$ 18; SMAPO# 268

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	4	3	5	5	6
1	1	-	3	4	2	3	4	6	6	7
2	2	3	-	5	5	2	5	5	7	6
3	3	4	5	-	6	3	2	4	4	7
4	3	2	5	6	-	3	6	4	8	5
5	4	3	2	3	3	-	5	7	5	8
6	3	4	5	2	6	5	-	2	6	7
7	5	6	5	4	4	7	2	-	8	5
8	5	6	7	4	8	5	6	8	-	3
9	6	7	6	7	5	8	7	5	3	-

$\geq$ 30; SMAPO# 1627

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	4	6	7	7	8	9	9
1	2	-	6	6	4	5	7	10	9	11
2	4	6	-	6	8	3	9	8	11	9
3	4	6	6	-	8	5	3	6	5	9
4	6	4	8	8	-	5	9	6	11	7
5	7	5	3	5	5	-	8	11	8	12
6	7	7	9	3	9	8	-	3	8	10
7	8	10	8	6	6	11	3	-	11	7
8	9	9	11	5	11	8	8	11	-	4
9	9	11	9	9	7	12	10	7	4	-

$\geq 46; (\frac{14}{39}, \frac{25}{39})$



TT-disjoint at 4

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	3	3	4	4
1	1	-	3	3	2	2	4	4	3	3
2	2	3	-	2	3	3	3	3	2	2
3	2	3	2	-	3	1	3	3	4	4
4	3	2	3	3	-	2	2	2	3	3
5	3	2	3	1	2	-	4	4	3	3
6	3	4	3	3	2	4	-	2	1	3
7	3	4	3	3	2	4	2	-	3	1
8	4	3	2	4	3	3	1	3	-	2
9	4	3	2	4	3	3	3	1	2	-

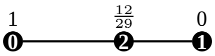
$\geq$ 18; SMAPO# 268

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	4	4	4	6	6	7
1	1	-	3	4	5	3	5	7	5	6
2	2	3	-	3	6	4	6	8	4	5
3	3	4	3	-	3	1	5	5	7	8
4	4	5	6	3	-	4	2	4	4	7
5	4	3	4	1	4	-	4	6	6	7
6	4	5	6	5	2	4	-	6	2	7
7	6	7	8	5	4	6	6	-	8	3
8	6	5	4	7	4	6	2	8	-	5
9	7	6	5	8	7	7	7	3	5	-

$\geq$ 30; SMAPO# 1631

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	5	6	7	7	9	10	11
1	2	-	6	7	6	5	9	11	8	9
2	4	6	-	5	8	7	9	11	6	7
3	5	7	5	-	5	2	8	8	11	12
4	6	6	8	5	-	5	3	5	6	9
5	7	5	7	2	5	-	8	10	9	10
6	7	9	9	8	3	8	-	8	3	10
7	9	11	11	8	5	10	8	-	11	4
8	10	8	6	11	6	9	3	11	-	7
9	11	9	7	12	9	10	10	4	7	-

$\geq 46; (\frac{21}{61}, \frac{40}{61})$



TT-disjoint at 6

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	2	3	4	4	3	3
1	1	-	3	3	3	2	5	3	2	4
2	2	3	-	4	4	1	4	4	3	3
3	2	3	4	-	2	3	4	4	1	3
4	2	3	4	2	-	3	4	4	3	1
5	3	2	1	3	3	-	3	5	4	2
6	4	5	4	4	4	3	-	2	5	3
7	4	3	4	4	4	5	2	-	3	5
8	3	2	3	1	3	4	5	3	-	2
9	3	4	3	3	1	2	3	5	2	-

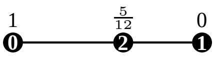
$\geq$ 20; SMAPO# 363

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	4	4	4	4	4	6	6
1	1	-	4	5	5	3	5	3	5	5
2	3	4	-	5	5	1	3	5	5	5
3	4	5	5	-	4	6	4	6	2	6
4	4	5	5	4	-	6	4	6	6	2
5	4	3	1	6	6	-	4	4	6	4
6	4	5	3	4	4	4	-	2	6	4
7	4	3	5	6	6	4	2	-	4	4
8	6	5	5	2	6	6	6	4	-	4
9	6	5	5	6	2	4	4	4	4	-

$\geq$ 28; SMAPO# 1246

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	6	6	7	7	8	9	9
1	2	-	7	8	8	5	9	6	7	9
2	5	7	-	9	9	2	6	9	8	8
3	6	8	9	-	6	9	7	10	3	9
4	6	8	9	6	-	9	7	10	9	3
5	7	5	2	9	9	-	6	9	10	6
6	7	9	6	7	7	6	-	3	10	6
7	8	6	9	10	10	9	3	-	7	9
8	9	7	8	3	9	10	10	7	-	6
9	9	9	8	9	3	6	6	9	6	-

$\geq 46; (\frac{12}{29}, \frac{17}{29})$



TT-disjoint at 6

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	3	4	4	4	4
1	1	-	3	4	2	4	5	3	3	5
2	2	3	-	3	1	3	4	4	4	4
3	3	4	3	-	4	2	5	3	5	3
4	3	2	1	4	-	4	3	5	5	3
5	3	4	3	2	4	-	3	5	3	5
6	4	5	4	5	3	3	-	2	4	4
7	4	3	4	3	5	5	2	-	4	4
8	4	3	4	5	5	3	4	4	-	2
9	4	5	4	3	3	5	4	4	2	-

$\geq$ 24; SMAPO# 662

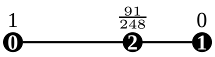
	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	4	4	5	5	5	6
1	1	-	4	4	3	5	6	4	4	5
2	3	4	-	4	1	5	4	6	6	5
3	3	4	4	-	5	1	4	2	4	3
4	4	3	1	5	-	6	5	5	5	4
5	4	5	5	1	6	-	3	3	3	4
6	5	6	4	4	5	3	-	2	4	3
7	5	4	6	2	5	3	2	-	4	5
8	5	4	6	4	5	3	4	4	-	1
9	6	5	5	3	4	4	3	5	1	-

$\geq$ 24; SMAPO# 676

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	6	7	7	8	9	9	10
1	2	-	7	8	5	9	10	7	7	10
2	5	7	-	7	2	8	7	10	10	9
3	6	8	7	-	9	3	8	5	9	6
4	7	5	2	9	-	10	7	10	10	7
5	7	9	8	3	10	-	5	8	6	9
6	8	10	7	8	7	5	-	3	7	6
7	9	7	10	5	10	8	3	-	8	9
8	9	7	10	9	10	6	7	8	-	3
9	10	10	9	6	7	9	6	9	3	-

$\geq 46; (\frac{5}{12}, \frac{7}{12})$

RHS = 48



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	0	2	2	6	6	5	6	7	7
1	0	-	2	2	6	6	5	6	7	7
2	2	2	-	4	4	4	7	6	7	7
3	2	2	4	-	8	8	5	6	5	5
4	6	6	4	8	-	6	3	8	9	5
5	6	6	4	8	6	-	7	6	3	9
6	5	5	7	5	3	7	-	5	8	8
7	6	6	6	6	8	6	5	-	9	3
8	7	7	7	5	9	3	8	9	-	6
9	7	7	7	5	5	9	8	3	6	-

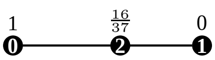
$\geq$ 36; SMAPO# 2949

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	5	8	8	9	10	11	13
1	2	-	4	3	10	10	11	12	9	11
2	2	4	-	7	6	6	11	10	11	13
3	5	3	7	-	13	13	8	9	8	10
4	8	10	6	13	-	10	5	14	15	9
5	8	10	6	13	10	-	13	12	5	17
6	9	11	11	8	5	13	-	9	16	14
7	10	12	10	9	14	12	9	-	17	5
8	11	9	11	8	15	5	16	17	-	12
9	13	11	13	10	9	17	14	5	12	-

$\geq$ 62; SMAPO# 10880

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	7	7	7	8	9	10
1	1	-	3	2	8	8	8	9	8	9
2	2	3	-	5	5	5	9	8	9	10
3	3	2	5	-	10	10	6	7	6	7
4	7	8	5	10	-	8	4	11	12	7
5	7	8	5	10	8	-	10	9	4	13
6	7	8	9	6	4	10	-	7	12	11
7	8	9	8	7	11	9	7	-	13	4
8	9	8	9	6	12	4	12	13	-	9
9	10	9	10	7	7	13	11	4	9	-

$\geq 48; (\frac{91}{248}, \frac{157}{248})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	2	3	3	3	5	4
1	1	-	3	4	3	2	2	2	4	3
2	2	3	-	5	4	3	1	3	3	2
3	3	4	5	-	3	4	4	4	2	5
4	2	3	4	3	-	3	3	1	5	2
5	3	2	3	4	3	-	4	2	4	1
6	3	2	1	4	3	4	-	4	4	3
7	3	2	3	4	1	2	4	-	4	3
8	5	4	3	2	5	4	4	4	-	3
9	4	3	2	5	2	1	3	3	3	-

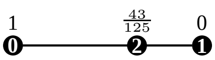
$\geq$ 20; SMAPO# 363

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	4	4	4	5	6	4	6
1	2	-	5	6	6	4	3	4	4	6
2	3	5	-	5	5	5	2	5	3	5
3	4	6	5	-	4	4	3	6	2	6
4	4	6	5	4	-	6	3	2	4	4
5	4	4	5	4	6	-	5	4	6	2
6	5	3	2	3	3	5	-	5	5	5
7	6	4	5	6	2	4	5	-	6	6
8	4	4	3	2	4	6	5	6	-	4
9	6	6	5	6	4	2	5	6	4	-

$\geq$ 30; SMAPO# 1467

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	6	6	7	8	9	9	10
1	3	-	8	9	9	6	5	6	8	9
2	5	8	-	9	9	8	3	8	6	7
3	6	9	9	-	6	7	6	9	3	10
4	6	9	9	6	-	9	6	3	9	6
5	7	6	8	7	9	-	9	6	10	3
6	8	5	3	6	6	9	-	9	9	8
7	9	6	8	9	3	6	9	-	10	9
8	9	8	6	3	9	10	9	10	-	7
9	10	9	7	10	6	3	8	9	7	-

$\geq 48; (\frac{16}{37}, \frac{21}{37})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	0	1	2	3	3	2	2	3	3
1	0	-	1	2	3	3	2	2	3	3
2	1	1	-	3	2	2	3	3	4	4
3	2	2	3	-	1	3	4	4	3	3
4	3	3	2	1	-	2	3	3	4	4
5	3	3	2	3	2	-	3	3	2	2
6	2	2	3	4	3	3	-	2	1	3
7	2	2	3	4	3	3	2	-	3	1
8	3	3	4	3	4	2	1	3	-	2
9	3	3	4	3	4	2	3	1	2	-

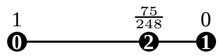
$\geq$ 16; SMAPO# 206

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	3	3	7	7	6	6
1	1	-	3	3	4	4	6	6	7	7
2	2	3	-	4	3	3	5	5	8	8
3	4	3	4	-	1	3	7	7	6	6
4	3	4	3	1	-	2	6	6	7	7
5	3	4	3	3	2	-	8	8	5	5
6	7	6	5	7	6	8	-	6	3	9
7	7	6	5	7	6	8	6	-	9	3
8	6	7	8	6	7	5	3	9	-	6
9	6	7	8	6	7	5	9	3	6	-

$\geq$ 34; SMAPO# 2395

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	6	6	6	9	9	9	9
1	1	-	3	5	7	7	8	8	10	10
2	2	3	-	6	4	4	7	7	11	11
3	6	5	6	-	2	6	11	11	9	9
4	6	7	4	2	-	4	9	9	11	11
5	6	7	4	6	4	-	11	11	7	7
6	9	8	7	11	9	11	-	8	4	12
7	9	8	7	11	9	11	8	-	12	4
8	9	10	11	9	11	7	4	12	-	8
9	9	10	11	9	11	7	12	4	8	-

$\geq 48; (\frac{43}{125}, \frac{82}{125})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	3	3	5	5	5	7
1	1	-	2	2	4	4	6	6	4	6
2	1	2	-	4	2	2	4	4	4	6
3	3	2	4	-	6	6	4	4	4	6
4	3	4	2	6	-	4	2	6	6	4
5	3	4	2	6	4	-	6	6	2	8
6	5	6	4	4	2	6	-	4	8	6
7	5	6	4	4	6	6	4	-	8	2
8	5	4	4	4	6	2	8	8	-	6
9	7	6	6	6	4	8	6	2	6	-

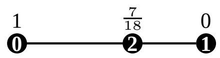
$\geq$ 28; SMAPO# 1405

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	6	9	9	11	13	15	15
1	3	-	6	3	12	12	10	12	12	12
2	3	6	-	9	6	6	12	10	12	12
3	6	3	9	-	15	15	9	11	9	9
4	9	12	6	15	-	12	6	16	18	10
5	9	12	6	15	12	-	14	12	6	18
6	11	10	12	9	6	14	-	10	16	16
7	13	12	10	11	16	12	10	-	18	6
8	15	12	12	9	18	6	16	18	-	12
9	15	12	12	9	10	18	16	6	12	-

$\geq$ 70; SMAPO# 12312

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	4	6	6	8	9	10	11
1	2	-	4	2	8	8	8	9	8	9
2	2	4	-	6	4	4	8	7	8	9
3	4	2	6	-	10	10	6	7	6	7
4	6	8	4	10	-	8	4	11	12	7
5	6	8	4	10	8	-	10	9	4	13
6	8	8	8	6	4	10	-	7	12	11
7	9	9	7	7	11	9	7	-	13	4
8	10	8	8	6	12	4	12	13	-	9
9	11	9	9	7	7	13	11	4	9	-

$\geq 48; (\frac{75}{248}, \frac{173}{248})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	3	4	2	2	4	3
1	1	-	2	2	4	3	3	3	5	4
2	1	2	-	4	4	5	3	3	3	2
3	3	2	4	-	2	3	3	3	5	2
4	3	4	4	2	-	5	1	3	5	2
5	4	3	5	3	5	-	4	4	2	5
6	2	3	3	3	1	4	-	2	4	3
7	2	3	3	3	3	4	2	-	4	1
8	4	5	3	5	5	2	4	4	-	3
9	3	4	2	2	2	5	3	1	3	-

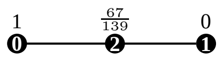
$\geq$ 20; SMAPO# 365

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	4	4	6	6	4	6
1	1	-	3	2	5	3	5	5	5	7
2	2	3	-	5	6	6	4	4	4	6
3	3	2	5	-	3	3	5	5	5	5
4	4	5	6	3	-	6	2	6	6	4
5	4	3	6	3	6	-	6	6	2	8
6	6	5	4	5	2	6	-	4	8	6
7	6	5	4	5	6	6	4	-	8	2
8	4	5	4	5	6	2	8	8	-	6
9	6	7	6	5	4	8	6	2	6	-

$\geq$ 30; SMAPO# 1617

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	6	7	8	8	8	8	9
1	2	-	4	4	9	6	8	8	10	11
2	2	4	-	8	9	10	6	6	6	7
3	6	4	8	-	5	6	8	8	10	7
4	7	9	9	5	-	11	3	9	11	6
5	8	6	10	6	11	-	10	10	4	13
6	8	8	6	8	3	10	-	6	12	9
7	8	8	6	8	9	10	6	-	12	3
8	8	10	6	10	11	4	12	12	-	9
9	9	11	7	7	6	13	9	3	9	-

$\geq 48; (\frac{7}{18}, \frac{11}{18})$



TT-disjoint at 5

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	4	4	4	4	5	5
1	1	-	2	2	3	5	5	3	4	4
2	1	2	-	4	5	3	3	5	4	6
3	3	2	4	-	5	3	5	1	2	4
4	4	3	5	5	-	6	2	6	7	5
5	4	5	3	3	6	-	4	2	5	3
6	4	5	3	5	2	4	-	6	5	7
7	4	3	5	1	6	2	6	-	3	3
8	5	4	4	2	7	5	5	3	-	2
9	5	4	6	4	5	3	7	3	2	-

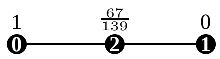
$\geq$ 24; SMAPO# 818

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	4	5	4	5	5	5
1	1	-	2	2	3	6	5	4	4	6
2	1	2	-	4	5	4	3	4	4	6
3	3	2	4	-	5	4	5	2	2	4
4	4	3	5	5	-	5	2	7	7	5
5	5	6	4	4	5	-	5	2	6	4
6	4	5	3	5	2	5	-	5	5	7
7	5	4	4	2	7	2	5	-	4	6
8	5	4	4	2	7	6	5	4	-	2
9	5	6	6	4	5	4	7	6	2	-

$\geq$ 26; SMAPO# 1075

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	6	8	8	8	9	10	10
1	2	-	4	4	6	10	10	7	8	10
2	2	4	-	8	10	6	6	9	8	12
3	6	4	8	-	10	6	10	3	4	8
4	8	6	10	10	-	10	4	13	14	10
5	8	10	6	6	10	-	8	3	10	6
6	8	10	6	10	4	8	-	11	10	14
7	9	7	9	3	13	3	11	-	7	9
8	10	8	8	4	14	10	10	7	-	4
9	10	10	12	8	10	6	14	9	4	-

$\geq 48; (\frac{67}{139}, \frac{72}{139})$



TT-disjoint at 4

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	4	4	4	4	5	5
1	1	-	2	4	3	3	5	5	4	6
2	1	2	-	4	5	5	3	5	6	4
3	3	4	4	-	3	5	3	1	2	4
4	4	3	5	3	-	4	6	2	5	3
5	4	3	5	5	4	-	2	6	5	7
6	4	5	3	3	6	2	-	4	5	5
7	4	5	5	1	2	6	4	-	3	3
8	5	4	6	2	5	5	5	3	-	2
9	5	6	4	4	3	7	5	3	2	-

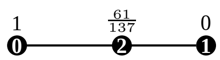
$\geq$ 24; SMAPO# 786

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	5	4	4	5	5	5
1	1	-	2	4	4	3	5	4	4	6
2	1	2	-	4	4	5	3	6	6	4
3	3	4	4	-	4	5	3	2	2	4
4	5	4	4	4	-	5	7	2	6	4
5	4	3	5	5	5	-	2	5	5	7
6	4	5	3	3	7	2	-	5	5	7
7	5	4	6	2	2	5	5	-	4	6
8	5	4	6	2	6	5	5	4	-	2
9	5	6	4	4	4	7	7	6	2	-

$\geq$ 26; SMAPO# 1093

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	6	8	8	8	9	10	10
1	2	-	4	8	6	6	10	9	8	12
2	2	4	-	8	8	10	6	11	12	8
3	6	8	8	-	6	10	6	3	4	8
4	8	6	8	6	-	8	12	3	10	6
5	8	6	10	10	8	-	4	11	10	14
6	8	10	6	6	12	4	-	9	10	12
7	9	9	11	3	3	11	9	-	7	9
8	10	8	12	4	10	10	10	7	-	4
9	10	12	8	8	6	14	12	9	4	-

$\geq 48; (\frac{67}{139}, \frac{72}{139})$



TT-disjoint at 1

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	4	4	4	4	3	4
1	1	-	2	4	3	3	5	5	4	3
2	1	2	-	4	5	5	3	3	4	5
3	3	4	4	-	1	5	3	5	2	3
4	4	3	5	1	-	4	2	6	3	2
5	4	3	5	5	4	-	6	2	5	4
6	4	5	3	3	2	6	-	4	3	2
7	4	5	3	5	6	2	4	-	5	6
8	3	4	4	2	3	5	3	5	-	1
9	4	3	5	3	2	4	2	6	1	-

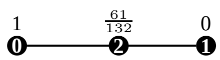
$\geq$ 22; SMAPO# 559

	0	1	2	3	4	5	6	7	8	9
0	-	2	1	5	4	4	4	4	6	6
1	2	-	3	3	4	4	6	6	4	6
2	1	3	-	4	5	5	3	3	5	7
3	5	3	4	-	1	7	3	5	3	5
4	4	4	5	1	-	6	2	6	4	4
5	4	4	5	7	6	-	6	2	8	6
6	4	6	3	3	2	6	-	4	6	4
7	4	6	3	5	6	2	4	-	6	8
8	6	4	5	3	4	8	6	6	-	2
9	6	6	7	5	4	6	4	8	2	-

$\geq$ 28; SMAPO# 1389

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	8	8	8	8	8	9	10
1	2	-	4	6	6	6	10	10	7	8
2	2	4	-	8	10	10	6	6	9	12
3	8	6	8	-	2	12	6	10	5	8
4	8	6	10	2	-	10	4	12	7	6
5	8	6	10	12	10	-	12	4	13	10
6	8	10	6	6	4	12	-	8	9	6
7	8	10	6	10	12	4	8	-	11	14
8	9	7	9	5	7	13	9	11	-	3
9	10	8	12	8	6	10	6	14	3	-

$\geq 48; (\frac{61}{137}, \frac{76}{137})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	4	4	4	4	4	3	3
1	1	-	2	3	3	3	5	5	4	4
2	1	2	-	5	5	5	3	3	4	4
3	4	3	5	-	2	4	2	6	1	3
4	4	3	5	2	-	4	2	6	3	1
5	4	3	5	4	4	-	6	2	5	5
6	4	5	3	2	2	6	-	4	3	3
7	4	5	3	6	6	2	4	-	5	5
8	3	4	4	1	3	5	3	5	-	2
9	3	4	4	3	1	5	3	5	2	-

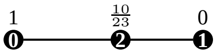
$\geq$ 22; SMAPO# 559

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	4	4	4	6	6
1	1	-	3	3	3	3	5	5	5	5
2	2	3	-	6	6	6	4	4	4	4
3	4	3	6	-	4	4	2	6	2	6
4	4	3	6	4	-	4	2	6	6	2
5	4	3	6	4	4	-	6	2	6	6
6	4	5	4	2	2	6	-	4	4	4
7	4	5	4	6	6	2	4	-	8	8
8	6	5	4	2	6	6	4	8	-	4
9	6	5	4	6	2	6	4	8	4	-

$\geq$ 28; SMAPO# 1404

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	8	8	8	8	8	9	9
1	2	-	4	6	6	6	10	10	9	9
2	2	4	-	10	10	10	6	6	7	7
3	8	6	10	-	6	8	4	12	3	9
4	8	6	10	6	-	8	4	12	9	3
5	8	6	10	8	8	-	12	4	11	11
6	8	10	6	4	4	12	-	8	7	7
7	8	10	6	12	12	4	8	-	13	13
8	9	9	7	3	9	11	7	13	-	6
9	9	9	7	9	3	11	7	13	6	-

$\geq 48; (\frac{61}{132}, \frac{71}{132})$



TT-disjoint at 6

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	5	4	5	4
1	2	-	5	3	5	3	3	4	3	4
2	3	5	-	4	4	4	2	3	4	5
3	3	3	4	-	4	2	4	3	4	1
4	3	5	4	4	-	4	4	5	2	3
5	3	3	4	2	4	-	4	1	4	3
6	5	3	2	4	4	4	-	5	4	3
7	4	4	3	3	5	1	5	-	3	2
8	5	3	4	4	2	4	4	3	-	5
9	4	4	5	1	3	3	3	2	5	-

$\geq$ 24; SMAPO# 662

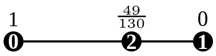
	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	4	4	4	4	5
1	1	-	3	4	4	5	3	5	3	6
2	2	3	-	5	5	6	2	4	4	5
3	3	4	5	-	6	3	5	5	5	2
4	3	4	5	6	-	5	3	5	1	4
5	4	5	6	3	5	-	4	2	6	5
6	4	3	2	5	3	4	-	6	4	5
7	4	5	4	5	5	2	6	-	4	3
8	4	3	4	5	1	6	4	4	-	5
9	5	6	5	2	4	5	5	3	5	-

$\geq$ 26; SMAPO# 909

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	6	6	7	8	8	9	9
1	3	-	8	7	9	8	5	9	6	10
2	5	8	-	9	9	10	3	7	8	10
3	6	7	9	-	10	5	8	8	9	3
4	6	9	9	10	-	9	6	10	3	7
5	7	8	10	5	9	-	7	3	10	8
6	8	5	3	8	6	7	-	10	7	7
7	8	9	7	8	10	3	10	-	7	5
8	9	6	8	9	3	10	7	7	-	10
9	9	10	10	3	7	8	7	5	10	-

$\geq 48; (\frac{10}{23}, \frac{13}{23})$

RHS = 50



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	4	2	2	4	3	3
1	1	-	2	2	3	3	3	5	4	4
2	1	2	-	4	5	3	3	3	2	4
3	3	2	4	-	3	3	3	5	2	2
4	4	3	5	3	-	4	4	2	5	5
5	2	3	3	3	4	-	2	4	1	3
6	2	3	3	3	4	2	-	4	3	1
7	4	5	3	5	2	4	4	-	3	5
8	3	4	2	2	5	1	3	3	-	2
9	3	4	4	2	5	3	1	5	2	-

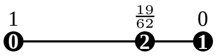
$\geq$ 20; SMAPO# 365

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	6	6	4	6	6
1	1	-	3	3	3	5	5	5	7	7
2	2	3	-	6	6	4	4	4	6	6
3	4	3	6	-	4	6	6	6	4	4
4	4	3	6	4	-	6	6	2	8	8
5	6	5	4	6	6	-	4	8	2	6
6	6	5	4	6	6	4	-	8	6	2
7	4	5	4	6	2	8	8	-	6	6
8	6	7	6	4	8	2	6	6	-	4
9	6	7	6	4	8	6	2	6	4	-

$\geq$ 32; SMAPO# 1859

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	7	8	8	8	8	9	9
1	2	-	4	5	6	8	8	10	11	11
2	2	4	-	9	10	6	6	6	7	9
3	7	5	9	-	7	9	9	11	6	6
4	8	6	10	7	-	10	10	4	13	13
5	8	8	6	9	10	-	6	12	3	9
6	8	8	6	9	10	6	-	12	9	3
7	8	10	6	11	4	12	12	-	9	11
8	9	11	7	6	13	3	9	9	-	6
9	9	11	9	6	13	9	3	11	6	-

$\geq 50; (\frac{49}{130}, \frac{81}{130})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	2	3	3	3	4	4
1	1	-	3	1	3	2	2	2	3	3
2	2	3	-	2	2	3	3	3	2	2
3	2	1	2	-	2	1	3	3	2	2
4	2	3	2	2	-	1	3	3	4	4
5	3	2	3	1	1	-	2	2	3	3
6	3	2	3	3	3	2	-	2	1	3
7	3	2	3	3	3	2	2	-	3	1
8	4	3	2	2	4	3	1	3	-	2
9	4	3	2	2	4	3	3	1	2	-

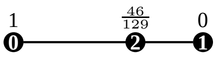
$\geq$ 16; SMAPO# 200

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	5	7	7	6	6
1	1	-	3	2	4	4	6	6	7	7
2	2	3	-	5	3	5	5	5	6	6
3	3	2	5	-	6	4	8	8	5	5
4	3	4	3	6	-	2	8	8	7	7
5	5	4	5	4	2	-	6	6	9	9
6	7	6	5	8	8	6	-	6	3	9
7	7	6	5	8	8	6	6	-	9	3
8	6	7	6	5	7	9	3	9	-	6
9	6	7	6	5	7	9	9	3	6	-

$\geq$ 36; SMAPO# 2888

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	5	5	8	10	10	10	10
1	2	-	5	3	7	6	8	8	10	10
2	3	5	-	6	4	7	7	7	7	7
3	5	3	6	-	8	5	11	11	7	7
4	5	7	4	8	-	3	11	11	11	11
5	8	6	7	5	3	-	8	8	12	12
6	10	8	7	11	11	8	-	8	4	12
7	10	8	7	11	11	8	8	-	12	4
8	10	10	7	7	11	12	4	12	-	8
9	10	10	7	7	11	12	12	4	8	-

$\geq 50; (\frac{19}{62}, \frac{43}{62})$



TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	2	3	3	3	4	4
1	1	-	3	2	3	2	2	2	3	3
2	2	3	-	3	2	1	3	3	4	4
3	3	2	3	-	1	4	4	4	3	3
4	2	3	2	1	-	3	3	3	2	4
5	3	2	1	4	3	-	4	4	3	5
6	3	2	3	4	3	4	-	2	1	3
7	3	2	3	4	3	4	2	-	3	1
8	4	3	4	3	2	3	1	3	-	2
9	4	3	4	3	4	5	3	1	2	-

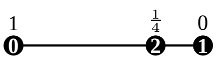
$\geq$ 18; SMAPO# 276

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	4	4	5	5	8	8
1	2	-	3	3	4	2	7	7	6	6
2	3	3	-	4	3	1	6	6	7	7
3	3	3	4	-	1	3	8	8	5	5
4	4	4	3	1	-	4	7	7	6	6
5	4	2	1	3	4	-	7	7	6	6
6	5	7	6	8	7	7	-	6	3	9
7	5	7	6	8	7	7	6	-	9	3
8	8	6	7	5	6	6	3	9	-	6
9	8	6	7	5	6	6	9	3	6	-

$\geq$ 34; SMAPO# 2402

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	5	5	6	7	7	11	11
1	2	-	6	5	7	4	9	9	9	9
2	4	6	-	7	5	2	9	9	11	11
3	5	5	7	-	2	7	12	12	8	8
4	5	7	5	2	-	7	10	10	8	10
5	6	4	2	7	7	-	11	11	9	11
6	7	9	9	12	10	11	-	8	4	12
7	7	9	9	12	10	11	8	-	12	4
8	11	9	11	8	8	9	4	12	-	8
9	11	9	11	8	10	11	12	4	8	-

$\geq 50; (\frac{46}{129}, \frac{83}{129})$



TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	1	2	2	1	1	3	3
1	1	-	0	0	3	3	0	0	2	2
2	1	0	-	0	3	3	0	0	2	2
3	1	0	0	-	3	3	0	0	2	2
4	2	3	3	3	-	2	3	3	1	3
5	2	3	3	3	2	-	3	3	3	1
6	1	0	0	0	3	3	-	0	2	2
7	1	0	0	0	3	3	0	-	2	2
8	3	2	2	2	1	3	2	2	-	2
9	3	2	2	2	3	1	2	2	2	-

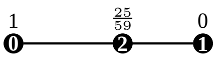
$\geq$ 10; SMAPO# 51

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	6	5	5	8	9	7	7
1	3	-	8	9	6	6	5	6	6	8
2	5	8	-	7	8	8	3	8	6	8
3	6	9	7	-	7	7	6	3	9	5
4	5	6	8	7	-	4	7	8	2	6
5	5	6	8	7	4	-	7	8	6	2
6	8	5	3	6	7	7	-	9	9	5
7	9	6	8	3	8	8	9	-	6	8
8	7	6	6	9	2	6	9	6	-	4
9	7	8	8	5	6	2	5	8	4	-

$\geq$ 42; SMAPO# 4540

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	6	6	6	8	9	9	9
1	3	-	8	9	9	9	5	6	8	10
2	5	8	-	7	11	11	3	8	8	10
3	6	9	7	-	10	10	6	3	11	7
4	6	9	11	10	-	6	10	11	3	9
5	6	9	11	10	6	-	10	11	9	3
6	8	5	3	6	10	10	-	9	11	7
7	9	6	8	3	11	11	9	-	8	10
8	9	8	8	11	3	9	11	8	-	6
9	9	10	10	7	9	3	7	10	6	-

$\geq 50; (\frac{1}{4}, \frac{3}{4})$



TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	3	3	4	4	4	5	5
1	2	-	4	5	5	4	4	4	3	3
2	4	4	-	3	5	2	4	4	5	3
3	3	5	3	-	4	5	3	3	2	4
4	3	5	5	4	-	3	1	3	4	2
5	4	4	2	5	3	-	4	4	3	5
6	4	4	4	3	1	4	-	2	3	3
7	4	4	4	3	3	4	2	-	3	1
8	5	3	5	2	4	3	3	3	-	4
9	5	3	3	4	2	5	3	1	4	-

$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	4	4	4	4	4	5	6
1	2	-	5	4	4	4	6	6	3	4
2	3	5	-	3	5	1	5	5	4	5
3	4	4	3	-	6	4	4	4	1	6
4	4	4	5	6	-	4	2	6	5	4
5	4	4	1	4	4	-	6	6	3	6
6	4	6	5	4	2	6	-	4	5	6
7	4	6	5	4	6	6	4	-	5	2
8	5	3	4	1	5	3	5	5	-	5
9	6	4	5	6	4	6	6	2	5	-

$\geq$ 28; SMAPO# 1246

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	6	6	7	7	7	9	10
1	3	-	9	9	9	8	10	10	6	7
2	6	9	-	6	10	3	9	9	9	8
3	6	9	6	-	10	9	7	7	3	10
4	6	9	10	10	-	7	3	9	9	6
5	7	8	3	9	7	-	10	10	6	11
6	7	10	9	7	3	10	-	6	8	9
7	7	10	9	7	9	10	6	-	8	3
8	9	6	9	3	9	6	8	8	-	9
9	10	7	8	10	6	11	9	3	9	-

$\geq 50; (\frac{25}{59}, \frac{34}{59})$

TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	2	2	2	3	3	2
1	1	-	3	3	1	1	1	2	2	1
2	2	3	-	2	2	2	2	1	3	2
3	2	3	2	-	2	2	2	3	1	2
4	2	1	2	2	-	0	0	1	1	0
5	2	1	2	2	0	-	0	1	1	0
6	2	1	2	2	0	0	-	1	1	0
7	3	2	1	3	1	1	1	-	2	1
8	3	2	3	1	1	1	1	2	-	1
9	2	1	2	2	0	0	0	1	1	-

$\geq$ 10; SMAPO# 7

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	5	5	6	8	7	7	9
1	3	-	6	6	8	9	5	6	8	6
2	5	6	-	4	8	7	7	2	6	8
3	5	6	4	-	8	7	7	6	2	8
4	5	8	8	8	-	7	3	6	8	8
5	6	9	7	7	7	-	6	9	5	3
6	8	5	7	7	3	6	-	9	5	9
7	7	6	2	6	6	9	9	-	4	6
8	7	8	6	2	8	5	5	4	-	8
9	9	6	8	8	8	3	9	6	8	-

$\geq$ 42; SMAPO# 4540

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	6	6	7	9	9	9	10
1	3	-	9	9	9	10	6	8	10	7
2	6	9	-	6	10	9	9	3	9	10
3	6	9	6	-	10	9	9	9	3	10
4	6	9	10	10	-	7	3	7	9	8
5	7	10	9	9	7	-	6	10	6	3
6	9	6	9	9	3	6	-	10	6	9
7	9	8	3	9	7	10	10	-	6	7
8	9	10	9	3	9	6	6	6	-	9
9	10	7	10	10	8	3	9	7	9	-

$\geq 50; (\frac{1}{5}, \frac{4}{5})$

RHS $=$ 52

TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	0	2	1	2	2	2	3	3
1	1	-	1	1	2	3	3	3	2	2
2	0	1	-	2	1	2	2	2	3	3
3	2	1	2	-	3	2	2	4	3	3
4	1	2	1	3	-	1	3	3	2	2
5	2	3	2	2	1	-	2	4	3	3
6	2	3	2	2	3	2	-	2	1	3
7	2	3	2	4	3	4	2	-	3	1
8	3	2	3	3	2	3	1	3	-	2
9	3	2	3	3	2	3	3	1	2	-

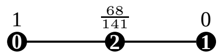
$\geq$ 14; SMAPO# 131

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	4	5	8	8	7	7
1	1	-	3	2	5	4	7	7	8	8
2	2	3	-	5	4	7	6	6	9	9
3	3	2	5	-	7	4	7	9	6	6
4	4	5	4	7	-	3	10	10	7	7
5	5	4	7	4	3	-	7	9	10	10
6	8	7	6	7	10	7	-	6	3	9
7	8	7	6	9	10	9	6	-	9	3
8	7	8	9	6	7	10	3	9	-	6
9	7	8	9	6	7	10	9	3	6	-

$\geq$ 40; SMAPO# 4264

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	4	5	7	10	10	10	10
1	2	-	4	2	7	7	10	10	10	10
2	2	4	-	6	5	9	8	8	12	12
3	4	2	6	-	9	5	8	12	8	8
4	5	7	5	9	-	4	13	13	9	9
5	7	7	9	5	4	-	9	13	13	13
6	10	10	8	8	13	9	-	8	4	12
7	10	10	8	12	13	13	8	-	12	4
8	10	10	12	8	9	13	4	12	-	8
9	10	10	12	8	9	13	12	4	8	-

$\geq 52; (\frac{37}{140}, \frac{103}{140})$



TT-disjoint at 4

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	3	3	5	5	5	5
1	1	-	2	2	4	2	4	4	6	6
2	1	2	-	4	2	2	6	6	4	4
3	3	2	4	-	4	2	6	6	4	4
4	3	4	2	4	-	2	4	4	6	6
5	3	2	2	2	2	-	6	6	6	6
6	5	4	6	6	4	6	-	4	2	6
7	5	4	6	6	4	6	4	-	6	2
8	5	6	4	4	6	6	2	6	-	4
9	5	6	4	4	6	6	6	2	4	-

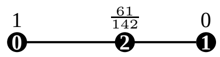
$\geq$ 26; SMAPO# 949

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	4	4	5	5	5	5
1	1	-	2	2	5	3	4	4	6	6
2	1	2	-	4	3	5	6	6	4	6
3	3	2	4	-	5	3	6	6	4	4
4	4	5	3	5	-	2	5	5	5	7
5	4	3	5	3	2	-	5	5	7	7
6	5	4	6	6	5	5	-	4	2	6
7	5	4	6	6	5	5	4	-	6	2
8	5	6	4	4	5	7	2	6	-	4
9	5	6	6	4	7	7	6	2	4	-

$\geq$ 28; SMAPO# 1340

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	6	6	7	10	10	10	10
1	2	-	4	4	8	5	8	8	12	12
2	2	4	-	8	4	7	12	12	8	10
3	6	4	8	-	8	5	12	12	8	8
4	6	8	4	8	-	3	8	8	10	12
5	7	5	7	5	3	-	11	11	13	13
6	10	8	12	12	8	11	-	8	4	12
7	10	8	12	12	8	11	8	-	12	4
8	10	12	8	8	10	13	4	12	-	8
9	10	12	10	8	12	13	12	4	8	-

$\geq 52; (\frac{68}{141}, \frac{73}{141})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	4	4	4	3	3	4	4
1	1	-	2	3	3	5	4	4	5	5
2	1	2	-	5	5	3	4	4	3	3
3	4	3	5	-	4	2	5	5	6	6
4	4	3	5	4	-	6	3	3	2	2
5	4	5	3	2	6	-	5	5	4	4
6	3	4	4	5	3	5	-	2	1	3
7	3	4	4	5	3	5	2	-	3	1
8	4	5	3	6	2	4	1	3	-	2
9	4	5	3	6	2	4	3	1	2	-

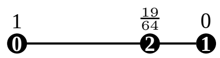
$\geq$ 22; SMAPO# 559

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	4	6	6	6	6
1	1	-	3	3	3	5	5	5	7	7
2	2	3	-	6	6	4	4	4	6	6
3	4	3	6	-	4	2	6	6	8	8
4	4	3	6	4	-	6	6	6	4	4
5	4	5	4	2	6	-	8	8	6	6
6	6	5	4	6	6	8	-	4	2	6
7	6	5	4	6	6	8	4	-	6	2
8	6	7	6	8	4	6	2	6	-	4
9	6	7	6	8	4	6	6	2	4	-

$\geq$ 32; SMAPO# 1859

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	8	8	8	9	9	10	10
1	2	-	4	6	6	10	9	9	12	12
2	2	4	-	10	10	6	7	7	8	8
3	8	6	10	-	8	4	11	11	14	14
4	8	6	10	8	-	12	9	9	6	6
5	8	10	6	4	12	-	13	13	10	10
6	9	9	7	11	9	13	-	6	3	9
7	9	9	7	11	9	13	6	-	9	3
8	10	12	8	14	6	10	3	9	-	6
9	10	12	8	14	6	10	9	3	6	-

$\geq 52; (\frac{61}{142}, \frac{81}{142})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	0	1	1	2	3	2	3	2	3
1	0	-	1	1	2	3	2	3	2	3
2	1	1	-	2	3	2	3	2	3	4
3	1	1	2	-	3	2	1	2	3	2
4	2	2	3	3	-	3	2	1	2	3
5	3	3	2	2	3	-	1	2	3	4
6	2	2	3	1	2	1	-	3	2	3
7	3	3	2	2	1	2	3	-	3	2
8	2	2	3	3	2	3	2	3	-	1
9	3	3	4	2	3	4	3	2	1	-

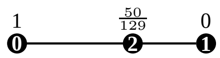
$\geq$ 14; SMAPO# 136

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	6	5	5	6	6	8	7
1	2	-	5	4	7	7	8	8	6	9
2	3	5	-	3	6	4	7	5	7	8
3	6	4	3	-	7	7	4	6	8	5
4	5	7	6	7	-	8	5	3	5	8
5	5	7	4	7	8	-	3	5	7	8
6	6	8	7	4	5	3	-	8	8	7
7	6	8	5	6	3	5	8	-	8	5
8	8	6	7	8	5	7	8	8	-	3
9	7	9	8	5	8	8	7	5	3	-

$\geq$ 40; SMAPO# 3917

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	7	7	8	8	9	10	10
1	2	-	5	5	9	10	10	11	8	12
2	3	5	-	4	8	5	9	6	9	11
3	7	5	4	-	10	9	5	8	11	7
4	7	9	8	10	-	11	7	4	7	11
5	8	10	5	9	11	-	4	7	10	12
6	8	10	9	5	7	4	-	11	10	10
7	9	11	6	8	4	7	11	-	11	7
8	10	8	9	11	7	10	10	11	-	4
9	10	12	11	7	11	12	10	7	4	-

$\geq 52; (\frac{19}{64}, \frac{45}{64})$



TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	3	4	3	4	4	5	5
1	2	-	4	5	4	5	4	4	3	3
2	4	4	-	5	2	3	4	4	3	5
3	3	5	5	-	3	4	3	3	2	4
4	4	4	2	3	-	5	4	4	5	3
5	3	5	3	4	5	-	1	3	4	2
6	4	4	4	3	4	1	-	2	3	3
7	4	4	4	3	4	3	2	-	3	1
8	5	3	3	2	5	4	3	3	-	4
9	5	3	5	4	3	2	3	1	4	-

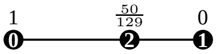
$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	4	4	5	5	5	5	7
1	2	-	5	4	4	5	7	7	3	5
2	3	5	-	5	1	4	6	6	4	6
3	4	4	5	-	4	7	5	5	1	7
4	4	4	1	4	-	5	7	7	5	5
5	5	5	4	7	5	-	2	6	6	4
6	5	7	6	5	7	2	-	4	4	6
7	5	7	6	5	7	6	4	-	4	2
8	5	3	4	1	5	6	4	4	-	6
9	7	5	6	7	5	4	6	2	6	-

$\geq$ 30; SMAPO# 1525

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	6	7	7	8	8	9	11
1	3	-	9	9	8	10	11	11	6	8
2	6	9	-	10	3	7	10	10	7	11
3	6	9	10	-	7	11	8	8	3	11
4	7	8	3	7	-	10	11	11	10	8
5	7	10	7	11	10	-	3	9	10	6
6	8	11	10	8	11	3	-	6	7	9
7	8	11	10	8	11	9	6	-	7	3
8	9	6	7	3	10	10	7	7	-	10
9	11	8	11	11	8	6	9	3	10	-

$\geq 52; (\frac{50}{129}, \frac{79}{129})$



TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	3	4	3	4	4	5	5
1	2	-	4	5	4	5	4	4	3	3
2	4	4	-	5	2	3	4	4	3	5
3	3	5	5	-	3	4	3	3	2	4
4	4	4	2	3	-	5	4	4	5	3
5	3	5	3	4	5	-	1	3	4	2
6	4	4	4	3	4	1	-	2	3	3
7	4	4	4	3	4	3	2	-	3	1
8	5	3	3	2	5	4	3	3	-	4
9	5	3	5	4	3	2	3	1	4	-

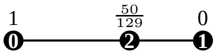
$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	4	4	5	5	5	5	7
1	2	-	5	4	4	5	5	7	3	5
2	3	5	-	5	1	4	6	6	4	6
3	4	4	5	-	4	7	5	5	1	7
4	4	4	1	4	-	5	7	7	5	5
5	5	5	4	7	5	-	2	6	6	4
6	5	5	6	5	7	2	-	4	4	6
7	5	7	6	5	7	6	4	-	6	2
8	5	3	4	1	5	6	4	6	-	6
9	7	5	6	7	5	4	6	2	6	-

$\geq$ 30; SMAPO# 1524

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	6	7	7	8	8	9	11
1	3	-	9	9	8	10	9	11	6	8
2	6	9	-	10	3	7	10	10	7	11
3	6	9	10	-	7	11	8	8	3	11
4	7	8	3	7	-	10	11	11	10	8
5	7	10	7	11	10	-	3	9	10	6
6	8	9	10	8	11	3	-	6	7	9
7	8	11	10	8	11	9	6	-	9	3
8	9	6	7	3	10	10	7	9	-	10
9	11	8	11	11	8	6	9	3	10	-

$\geq 52; (\frac{50}{129}, \frac{79}{129})$



TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	3	4	3	4	4	5	5
1	2	-	4	5	4	5	4	4	3	3
2	4	4	-	5	2	3	4	4	3	5
3	3	5	5	-	3	4	3	3	2	4
4	4	4	2	3	-	5	4	4	5	3
5	3	5	3	4	5	-	3	1	4	2
6	4	4	4	3	4	3	-	2	3	1
7	4	4	4	3	4	1	2	-	3	3
8	5	3	3	2	5	4	3	3	-	4
9	5	3	5	4	3	2	1	3	4	-

$\geq$ 24; SMAPO# 662

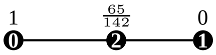
	0	1	2	3	4	5	6	7	8	9
0	-	2	3	4	4	5	5	5	5	7
1	2	-	5	4	4	5	5	7	3	5
2	3	5	-	5	1	4	6	6	4	6
3	4	4	5	-	4	7	5	5	1	7
4	4	4	1	4	-	5	7	7	5	5
5	5	5	4	7	5	-	6	2	6	4
6	5	5	6	5	7	6	-	4	4	2
7	5	7	6	5	7	2	4	-	6	6
8	5	3	4	1	5	6	4	6	-	6
9	7	5	6	7	5	4	2	6	6	-

$\geq$ 30; SMAPO# 1526

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	6	7	7	8	8	9	11
1	3	-	9	9	8	10	9	11	6	8
2	6	9	-	10	3	7	10	10	7	11
3	6	9	10	-	7	11	8	8	3	11
4	7	8	3	7	-	10	11	11	10	8
5	7	10	7	11	10	-	9	3	10	6
6	8	9	10	8	11	9	-	6	7	3
7	8	11	10	8	11	3	6	-	9	9
8	9	6	7	3	10	10	7	9	-	10
9	11	8	11	11	8	6	3	9	10	-

$\geq 52; (\frac{50}{129}, \frac{79}{129})$

RHS = 54



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	3	5	3	5	5	5
1	1	-	3	3	4	4	4	6	6	6
2	2	3	-	2	5	5	3	5	5	5
3	4	3	2	-	5	5	5	5	3	3
4	3	4	5	5	-	4	6	2	4	6
5	5	4	5	5	4	-	6	6	6	2
6	3	4	3	5	6	6	-	4	2	4
7	5	6	5	5	2	6	4	-	6	4
8	5	6	5	3	4	6	2	6	-	4
9	5	6	5	3	6	2	4	4	4	-

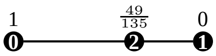
$\geq$ 28; SMAPO# 1236

	0	1	2	3	4	5	6	7	8	9
0	-	0	2	3	4	4	6	6	6	6
1	0	-	2	3	4	4	6	6	6	6
2	2	2	-	3	6	6	4	4	6	6
3	3	3	3	-	5	5	5	5	3	3
4	4	4	6	5	-	4	6	2	4	6
5	4	4	6	5	4	-	6	6	6	2
6	6	6	4	5	6	6	-	4	2	6
7	6	6	4	5	2	6	4	-	6	4
8	6	6	6	3	4	6	2	6	-	4
9	6	6	6	3	6	2	6	4	4	-

$\geq$ 28; SMAPO# 1264

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	7	7	9	9	11	11	11
1	1	-	4	6	8	8	10	12	12	12
2	3	4	-	4	10	10	6	8	10	10
3	7	6	4	-	10	10	10	10	6	6
4	7	8	10	10	-	8	12	4	8	12
5	9	8	10	10	8	-	12	12	12	4
6	9	10	6	10	12	12	-	8	4	10
7	11	12	8	10	4	12	8	-	12	8
8	11	12	10	6	8	12	4	12	-	8
9	11	12	10	6	12	4	10	8	8	-

$\geq 54; (\frac{65}{142}, \frac{77}{142})$



TT-disjoint at 1

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	4	4	2	2	3	3
1	1	-	2	2	3	5	3	3	4	4
2	1	2	-	4	5	3	3	3	2	4
3	3	2	4	-	3	5	3	3	2	2
4	4	3	5	3	-	2	4	4	5	5
5	4	5	3	5	2	-	4	4	3	5
6	2	3	3	3	4	4	-	2	1	3
7	2	3	3	3	4	4	2	-	3	1
8	3	4	2	2	5	3	1	3	-	2
9	3	4	4	2	5	5	3	1	2	-

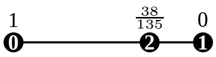
$\geq$ 20; SMAPO# 365

	0	1	2	3	4	5	6	7	8	9
0	-	2	1	3	4	4	7	7	6	6
1	2	-	3	3	4	6	5	5	8	8
2	1	3	-	4	5	3	6	6	5	7
3	3	3	4	-	3	5	8	8	5	5
4	4	4	5	3	-	2	7	7	8	8
5	4	6	3	5	2	-	7	7	6	8
6	7	5	6	8	7	7	-	6	3	9
7	7	5	6	8	7	7	6	-	9	3
8	6	8	5	5	8	6	3	9	-	6
9	6	8	7	5	8	8	9	3	6	-

$\geq$ 36; SMAPO# 2862

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	6	8	8	9	9	9	9
1	2	-	4	4	6	10	7	7	11	11
2	2	4	-	8	10	6	9	9	7	11
3	6	4	8	-	6	10	11	11	7	7
4	8	6	10	6	-	4	11	11	13	13
5	8	10	6	10	4	-	11	11	9	13
6	9	7	9	11	11	11	-	8	4	12
7	9	7	9	11	11	11	8	-	12	4
8	9	11	7	7	13	9	4	12	-	8
9	9	11	11	7	13	13	12	4	8	-

$\geq 54; (\frac{49}{135}, \frac{86}{135})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	0	1	1	2	3	2	2	3	3
1	0	-	1	1	2	3	2	2	3	3
2	1	1	-	2	3	2	3	3	2	4
3	1	1	2	-	3	2	1	3	2	2
4	2	2	3	3	-	3	2	2	1	3
5	3	3	2	2	3	-	1	3	2	4
6	2	2	3	1	2	1	-	2	3	3
7	2	2	3	3	2	3	2	-	3	1
8	3	3	2	2	1	2	3	3	-	2
9	3	3	4	2	3	4	3	1	2	-

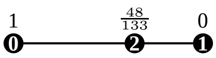
$\geq$ 14; SMAPO# 136

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	6	6	5	6	8	7	7
1	2	-	5	4	6	7	8	6	9	9
2	3	5	-	3	7	4	7	7	6	8
3	6	4	3	-	8	7	4	8	7	5
4	6	6	7	8	-	9	6	6	3	9
5	5	7	4	7	9	-	3	9	6	8
6	6	8	7	4	6	3	-	8	9	7
7	8	6	7	8	6	9	8	-	9	3
8	7	9	6	7	3	6	9	9	-	6
9	7	9	8	5	9	8	7	3	6	-

$\geq$ 42; SMAPO# 4547

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	7	8	8	8	10	10	10
1	2	-	5	5	8	10	10	8	12	12
2	3	5	-	4	9	5	9	9	7	11
3	7	5	4	-	11	9	5	11	9	7
4	8	8	9	11	-	12	8	8	4	12
5	8	10	5	9	12	-	4	12	8	12
6	8	10	9	5	8	4	-	10	12	10
7	10	8	9	11	8	12	10	-	12	4
8	10	12	7	9	4	8	12	12	-	8
9	10	12	11	7	12	12	10	4	8	-

$\geq 54; (\frac{38}{135}, \frac{97}{135})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	2	3	5	4	3	3
1	1	-	3	4	3	2	4	3	4	4
2	2	3	-	5	4	1	3	2	3	3
3	3	4	5	-	3	4	2	5	4	4
4	2	3	4	3	-	3	5	2	1	3
5	3	2	1	4	3	-	4	3	2	2
6	5	4	3	2	5	4	-	3	4	4
7	4	3	2	5	2	3	3	-	3	1
8	3	4	3	4	1	2	4	3	-	2
9	3	4	3	4	3	2	4	1	2	-

$\geq$ 20; SMAPO# 363

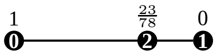
	0	1	2	3	4	5	6	7	8	9
0	-	1	3	4	5	5	4	7	8	8
1	1	-	4	3	6	4	5	6	7	7
2	3	4	-	5	6	2	3	6	7	7
3	4	3	5	-	5	5	2	7	6	6
4	5	6	6	5	-	8	5	6	3	9
5	5	4	2	5	8	-	5	8	5	5
6	4	5	3	2	5	5	-	5	8	8
7	7	6	6	7	6	8	5	-	9	3
8	8	7	7	6	3	5	8	9	-	6
9	8	7	7	6	9	5	8	3	6	-

$\geq$ 36; SMAPO# 2834

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	6	7	8	9	11	11	11
1	2	-	7	6	9	6	9	9	11	11
2	5	7	-	9	10	3	6	8	10	10
3	6	6	9	-	7	8	3	11	9	9
4	7	9	10	7	-	11	10	8	4	12
5	8	6	3	8	11	-	9	11	7	7
6	9	9	6	3	10	9	-	8	12	12
7	11	9	8	11	8	11	8	-	12	4
8	11	11	10	9	4	7	12	12	-	8
9	11	11	10	9	12	7	12	4	8	-

$\geq 54; (\frac{48}{133}, \frac{85}{133})$

RHS = 56



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	2	3	3	3	4	4
1	1	-	3	2	3	4	4	4	5	3
2	2	3	-	1	2	3	3	3	4	4
3	3	2	1	-	3	2	2	2	3	3
4	2	3	2	3	-	1	3	3	4	2
5	3	4	3	2	1	-	4	4	3	3
6	3	4	3	2	3	4	-	2	1	3
7	3	4	3	2	3	4	2	-	3	1
8	4	5	4	3	4	3	1	3	-	2
9	4	3	4	3	2	3	3	1	2	-

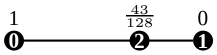
$\geq$ 18; SMAPO# 276

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	5	5	8	8	7	9
1	1	-	3	2	6	6	7	7	8	8
2	2	3	-	3	5	7	6	6	9	9
3	3	2	3	-	6	4	7	7	6	10
4	5	6	5	6	-	2	11	11	8	8
5	5	6	7	4	2	-	9	9	6	10
6	8	7	6	7	11	9	-	6	3	9
7	8	7	6	7	11	9	6	-	9	3
8	7	8	9	6	8	6	3	9	-	6
9	9	8	9	10	8	10	9	3	6	-

$\geq$ 40; SMAPO# 4359

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	6	7	8	11	11	11	13
1	2	-	5	4	9	10	11	11	13	11
2	3	5	-	3	6	9	8	8	12	12
3	6	4	3	-	9	6	9	9	9	13
4	7	9	6	9	-	3	14	14	12	10
5	8	10	9	6	3	-	13	13	9	13
6	11	11	8	9	14	13	-	8	4	12
7	11	11	8	9	14	13	8	-	12	4
8	11	13	12	9	12	9	4	12	-	8
9	13	11	12	13	10	13	12	4	8	-

$\geq 56; (\frac{23}{78}, \frac{55}{78})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	3	2	4	4
1	1	-	3	3	4	2	4	3	3	3
2	2	3	-	4	3	3	5	4	2	4
3	2	3	4	-	5	3	3	4	4	2
4	3	4	3	5	-	4	2	3	5	3
5	3	2	3	3	4	-	4	1	3	3
6	3	4	5	3	2	4	-	3	3	5
7	2	3	4	4	3	1	3	-	2	2
8	4	3	2	4	5	3	3	2	-	4
9	4	3	4	2	3	3	5	2	4	-

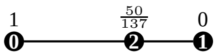
$\geq$ 22; SMAPO# 464

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	5	3	5	5	6	5	6
1	1	-	4	4	4	4	6	7	4	7
2	3	4	-	8	4	8	6	7	2	7
3	5	4	8	-	8	6	6	9	6	3
4	3	4	4	8	-	8	2	5	6	5
5	5	4	8	6	8	-	8	3	6	9
6	5	6	6	6	2	8	-	7	4	7
7	6	7	7	9	5	3	7	-	5	6
8	5	4	2	6	6	6	4	5	-	7
9	6	7	7	3	5	9	7	6	7	-

$\geq$ 36; SMAPO# 2853

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	6	6	8	8	8	9	10
1	2	-	7	6	8	6	10	10	7	10
2	5	7	-	11	7	11	11	11	4	11
3	6	6	11	-	12	8	8	12	9	4
4	6	8	7	12	-	12	4	8	11	8
5	8	6	11	8	12	-	12	4	9	12
6	8	10	11	8	4	12	-	10	7	12
7	8	10	11	12	8	4	10	-	7	8
8	9	7	4	9	11	9	7	7	-	11
9	10	10	11	4	8	12	12	8	11	-

$\geq 56; (\frac{43}{128}, \frac{85}{128})$



TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	5	5	4	4
1	2	-	5	5	3	3	3	3	4	4
2	3	5	-	4	4	4	2	4	3	5
3	3	5	4	-	4	4	4	2	5	3
4	3	3	4	4	-	2	4	4	1	3
5	3	3	4	4	2	-	4	4	3	1
6	5	3	2	4	4	4	-	4	5	3
7	5	3	4	2	4	4	4	-	3	5
8	4	4	3	5	1	3	5	3	-	2
9	4	4	5	3	3	1	3	5	2	-

$\geq$ 24; SMAPO# 662

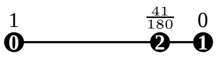
	0	1	2	3	4	5	6	7	8	9
0	-	2	3	4	5	5	4	5	8	8
1	2	-	3	4	7	7	2	3	6	6
2	3	3	-	3	6	6	1	2	5	7
3	4	4	3	-	9	9	4	1	6	6
4	5	7	6	9	-	6	7	8	3	9
5	5	7	6	9	6	-	7	8	9	3
6	4	2	1	4	7	7	-	3	6	6
7	5	3	2	1	8	8	3	-	5	7
8	8	6	5	6	3	9	6	5	-	6
9	8	6	7	6	9	3	6	7	6	-

$\geq$ 34; SMAPO# 2606

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	6	7	7	8	9	11	11
1	3	-	8	9	10	10	5	6	10	10
2	5	8	-	7	10	10	3	6	8	12
3	6	9	7	-	13	13	8	3	11	9
4	7	10	10	13	-	8	11	12	4	12
5	7	10	10	13	8	-	11	12	12	4
6	8	5	3	8	11	11	-	7	11	9
7	9	6	6	3	12	12	7	-	8	12
8	11	10	8	11	4	12	11	8	-	8
9	11	10	12	9	12	4	9	12	8	-

$\geq 56; (\frac{50}{137}, \frac{87}{137})$

RHS = 58



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	0	0	1	2	3	2	2	3	3
1	0	-	0	1	2	3	2	2	3	3
2	0	0	-	1	2	3	2	2	3	3
3	1	1	1	-	3	2	3	3	4	2
4	2	2	2	3	-	1	4	4	3	3
5	3	3	3	2	1	-	3	3	4	2
6	2	2	2	3	4	3	-	2	3	1
7	2	2	2	3	4	3	2	-	1	3
8	3	3	3	4	3	4	3	1	-	2
9	3	3	3	2	3	2	1	3	2	-

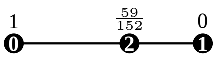
$\geq$ 14; SMAPO# 146

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	5	6	6	8	9	8	11
1	1	-	3	4	5	7	9	8	9	12
2	2	3	-	3	8	6	10	11	8	11
3	5	4	3	-	7	5	7	8	11	10
4	6	5	8	7	-	2	12	11	8	11
5	6	7	6	5	2	-	12	13	10	9
6	8	9	10	7	12	12	-	7	10	3
7	9	8	11	8	11	13	7	-	3	10
8	8	9	8	11	8	10	10	3	-	7
9	11	12	11	10	11	9	3	10	7	-

$\geq$ 46; SMAPO# 6334

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	5	8	9	10	11	11	14
1	1	-	3	4	7	10	11	10	12	15
2	2	3	-	3	10	9	12	13	11	14
3	5	4	3	-	9	6	9	10	14	11
4	8	7	10	9	-	3	16	15	11	14
5	9	10	9	6	3	-	15	16	14	11
6	10	11	12	9	16	15	-	9	13	4
7	11	10	13	10	15	16	9	-	4	13
8	11	12	11	14	11	14	13	4	-	9
9	14	15	14	11	14	11	4	13	9	-

$\geq 58; (\frac{41}{180}, \frac{139}{180})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	7	6	6	8	9	9	10
1	2	-	5	5	8	8	10	7	11	10
2	3	5	-	8	5	3	9	10	6	7
3	7	5	8	-	3	7	11	8	8	7
4	6	8	5	3	-	4	10	11	9	8
5	6	8	3	7	4	-	6	7	9	10
6	8	10	9	11	10	6	-	7	11	4
7	9	7	10	8	11	7	7	-	4	11
8	9	11	6	8	9	9	11	4	-	7
9	10	10	7	7	8	10	4	11	7	-

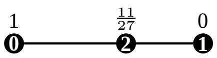
$\geq$ 48; SMAPO# 6626

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	8	10	10	12	13	15	16
1	2	-	7	6	8	12	14	11	17	16
2	5	7	-	11	7	5	15	16	10	11
3	8	6	11	-	4	8	16	13	11	10
4	10	8	7	4	-	4	14	15	15	14
5	10	12	5	8	4	-	10	11	15	16
6	12	14	15	16	14	10	-	11	17	6
7	13	11	16	13	15	11	11	-	6	17
8	15	17	10	11	15	15	17	6	-	11
9	16	16	11	10	14	16	6	17	11	-

$\geq$ 70; SMAPO# 12240

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	7	8	8	10	11	12	13
1	2	-	6	5	8	10	12	9	14	13
2	4	6	-	9	6	4	12	13	8	9
3	7	5	9	-	3	7	13	10	9	8
4	8	8	6	3	-	4	12	13	12	11
5	8	10	4	7	4	-	8	9	12	13
6	10	12	12	13	12	8	-	9	14	5
7	11	9	13	10	13	9	9	-	5	14
8	12	14	8	9	12	12	14	5	-	9
9	13	13	9	8	11	13	5	14	9	-

$\geq 58; (\frac{59}{152}, \frac{93}{152})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	2	3	4	3	5	5
1	1	-	4	4	3	4	3	4	4	4
2	3	4	-	4	5	4	5	4	6	2
3	3	4	4	-	5	4	5	4	2	6
4	2	3	5	5	-	1	2	3	3	3
5	3	4	4	4	1	-	3	2	4	4
6	4	3	5	5	2	3	-	1	5	5
7	3	4	4	4	3	2	1	-	4	4
8	5	4	6	2	3	4	5	4	-	4
9	5	4	2	6	3	4	5	4	4	-

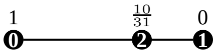
$\geq$ 24; SMAPO# 665

	0	1	2	3	4	5	6	7	8	9
0	-	1	4	3	6	5	5	6	5	5
1	1	-	3	4	5	6	4	7	4	6
2	4	3	-	5	8	5	7	6	7	3
3	3	4	5	-	7	6	6	7	2	6
4	6	5	8	7	-	3	5	8	5	5
5	5	6	5	6	3	-	8	5	6	8
6	5	4	7	6	5	8	-	3	8	8
7	6	7	6	7	8	5	3	-	5	7
8	5	4	7	2	5	6	8	5	-	4
9	5	6	3	6	5	8	8	7	4	-

$\geq$ 36; SMAPO# 2766

	0	1	2	3	4	5	6	7	8	9
0	-	2	6	6	8	8	9	9	10	10
1	2	-	6	8	8	10	7	11	8	10
2	6	6	-	8	12	8	11	9	12	4
3	6	8	8	-	12	10	11	11	4	12
4	8	8	12	12	-	4	7	11	8	8
5	8	10	8	10	4	-	11	7	10	12
6	9	7	11	11	7	11	-	4	13	13
7	9	11	9	11	11	7	4	-	9	11
8	10	8	12	4	8	10	13	9	-	8
9	10	10	4	12	8	12	13	11	8	-

$\geq 58; (\frac{11}{27}, \frac{16}{27})$



TT-disjoint at 8

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	3	4	4	4	4
1	1	-	3	2	4	4	3	3	5	5
2	2	3	-	1	3	3	4	4	4	4
3	3	2	1	-	4	4	5	5	3	3
4	3	4	3	4	-	2	5	3	5	3
5	3	4	3	4	2	-	3	5	3	5
6	4	3	4	5	5	3	-	4	4	2
7	4	3	4	5	3	5	4	-	2	4
8	4	5	4	3	5	3	4	2	-	4
9	4	5	4	3	3	5	2	4	4	-

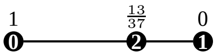
$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	1	4	5	5	7	7	7	8	9
1	1	-	5	4	6	8	6	6	9	8
2	4	5	-	1	7	7	7	9	6	7
3	5	4	1	-	8	8	8	8	7	6
4	5	6	7	8	-	2	6	4	7	4
5	7	8	7	8	2	-	4	6	5	6
6	7	6	7	8	6	4	-	6	5	2
7	7	6	9	8	4	6	6	-	3	8
8	8	9	6	7	7	5	5	3	-	5
9	9	8	7	6	4	6	2	8	5	-

$\geq$ 36; SMAPO# 2820

	0	1	2	3	4	5	6	7	8	9
0	-	2	6	8	8	10	11	11	11	13
1	2	-	8	6	10	12	9	9	13	13
2	6	8	-	2	10	10	11	13	9	11
3	8	6	2	-	12	12	13	13	9	9
4	8	10	10	12	-	4	11	7	11	7
5	10	12	10	12	4	-	7	11	7	11
6	11	9	11	13	11	7	-	10	8	4
7	11	9	13	13	7	11	10	-	4	12
8	11	13	9	9	11	7	8	4	-	8
9	13	13	11	9	7	11	4	12	8	-

$\geq 58; (\frac{10}{31}, \frac{21}{31})$



TT-disjoint at 5

	0	1	2	3	4	5	6	7	8	9
0	-	0	0	1	3	2	0	3	5	5
1	0	-	0	1	3	2	0	3	5	5
2	0	0	-	1	3	2	0	3	5	5
3	1	1	1	-	4	1	1	4	4	4
4	3	3	3	4	-	5	3	4	2	6
5	2	2	2	1	5	-	2	5	3	3
6	0	0	0	1	3	2	-	3	5	5
7	3	3	3	4	4	5	3	-	6	2
8	5	5	5	4	2	3	5	6	-	4
9	5	5	5	4	6	3	5	2	4	-

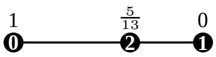
$\geq$ 18; SMAPO# 313

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	5	5	8	9	9	7	7
1	3	-	7	8	8	5	8	6	8	8
2	6	7	-	9	5	6	3	9	7	7
3	5	8	9	-	8	3	6	6	6	6
4	5	8	5	8	-	7	8	4	2	6
5	8	5	6	3	7	-	9	7	5	5
6	9	8	3	6	8	9	-	6	8	8
7	9	6	9	6	4	7	6	-	6	2
8	7	8	7	6	2	5	8	6	-	4
9	7	8	7	6	6	5	8	2	4	-

$\geq$ 42; SMAPO# 4540

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	6	8	9	9	12	12	12
1	3	-	7	9	11	6	8	9	13	13
2	6	7	-	10	8	7	3	12	12	12
3	6	9	10	-	12	3	7	10	10	10
4	8	11	8	12	-	11	11	8	4	12
5	9	6	7	3	11	-	10	11	7	7
6	9	8	3	7	11	10	-	9	13	13
7	12	9	12	10	8	11	9	-	12	4
8	12	13	12	10	4	7	13	12	-	8
9	12	13	12	10	12	7	13	4	8	-

$\geq 58; (\frac{13}{37}, \frac{24}{37})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	2	2	3	3	5	5
1	1	-	4	4	3	3	2	2	4	4
2	3	4	-	4	5	5	4	4	6	2
3	3	4	4	-	5	5	4	4	2	6
4	2	3	5	5	-	2	1	1	3	3
5	2	3	5	5	2	-	1	1	3	3
6	3	2	4	4	1	1	-	2	4	4
7	3	2	4	4	1	1	2	-	4	4
8	5	4	6	2	3	3	4	4	-	4
9	5	4	2	6	3	3	4	4	4	-

$\geq$ 22; SMAPO# 482

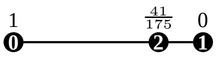
	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	5	6	6	7	6	6
1	2	-	5	6	7	8	6	5	4	8
2	5	5	-	5	8	7	5	6	7	3
3	4	6	5	-	7	6	6	7	2	6
4	5	7	8	7	-	7	3	4	5	5
5	6	8	7	6	7	-	4	3	4	6
6	6	6	5	6	3	4	-	7	6	8
7	7	5	6	7	4	3	7	-	7	7
8	6	4	7	2	5	4	6	7	-	4
9	6	8	3	6	5	6	8	7	4	-

$\geq$ 38; SMAPO# 3287

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	7	8	9	10	11	11
1	3	-	8	10	10	11	8	7	8	12
2	7	8	-	8	12	11	8	9	12	4
3	7	10	8	-	12	11	10	11	4	12
4	7	10	12	12	-	9	4	5	8	8
5	8	11	11	11	9	-	5	4	7	9
6	9	8	8	10	4	5	-	9	10	12
7	10	7	9	11	5	4	9	-	11	11
8	11	8	12	4	8	7	10	11	-	8
9	11	12	4	12	8	9	12	11	8	-

$\geq 58; (\frac{5}{13}, \frac{8}{13})$

RHS = 60



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	0	0	1	2	3	2	2	3	3
1	0	-	0	1	2	3	2	2	3	3
2	0	0	-	1	2	3	2	2	3	3
3	1	1	1	-	3	2	3	3	4	2
4	2	2	2	3	-	1	4	4	3	3
5	3	3	3	2	1	-	3	3	4	2
6	2	2	2	3	4	3	-	2	1	3
7	2	2	2	3	4	3	2	-	3	1
8	3	3	3	4	3	4	1	3	-	2
9	3	3	3	2	3	2	3	1	2	-

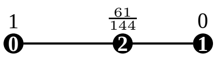
$\geq$ 14; SMAPO# 146

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	5	6	6	8	9	8	11
1	1	-	3	4	5	7	9	8	9	12
2	2	3	-	3	8	6	10	11	8	11
3	5	4	3	-	7	5	7	8	11	10
4	6	5	8	7	-	2	12	11	8	11
5	6	7	6	5	2	-	12	13	10	9
6	8	9	10	7	12	12	-	7	4	11
7	9	8	11	8	11	13	7	-	11	4
8	8	9	8	11	8	10	4	11	-	7
9	11	12	11	10	11	9	11	4	7	-

$\geq$ 48; SMAPO# 6971

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	5	8	9	10	11	11	14
1	1	-	3	4	7	10	11	10	12	15
2	2	3	-	3	10	9	12	13	11	14
3	5	4	3	-	9	6	9	10	14	11
4	8	7	10	9	-	3	16	15	11	14
5	9	10	9	6	3	-	15	16	14	11
6	10	11	12	9	16	15	-	9	5	14
7	11	10	13	10	15	16	9	-	14	5
8	11	12	11	14	11	14	5	14	-	9
9	14	15	14	11	14	11	14	5	9	-

$\geq 60; (\frac{41}{175}, \frac{134}{175})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	3	9	7	13	13	13	13	11	11
1	3	-	8	10	10	10	16	16	8	14
2	9	8	-	10	18	18	12	12	14	6
3	7	10	10	-	16	16	14	14	4	12
4	13	10	18	16	-	12	6	18	12	12
5	13	10	18	16	12	-	18	6	16	16
6	13	16	12	14	6	18	-	12	14	18
7	13	16	12	14	18	6	12	-	10	14
8	11	8	14	4	12	16	14	10	-	8
9	11	14	6	12	12	16	18	14	8	-

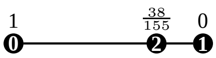
$\geq$ 80; SMAPO# 13644

	0	1	2	3	4	5	6	7	8	9
0	-	3	11	11	14	14	14	14	19	19
1	3	-	12	14	11	11	17	17	16	16
2	11	12	-	16	23	23	17	17	24	8
3	11	14	16	-	23	23	19	19	8	24
4	14	11	23	23	-	12	6	18	15	15
5	14	11	23	23	12	-	18	6	23	23
6	14	17	17	19	6	18	-	12	19	21
7	14	17	17	19	18	6	12	-	17	19
8	19	16	24	8	15	23	19	17	-	16
9	19	16	8	24	15	23	21	19	16	-

$\geq$ 104; SMAPO# 14960

	0	1	2	3	4	5	6	7	8	9
0	-	2	6	6	9	9	9	9	10	10
1	2	-	6	8	7	7	11	11	8	10
2	6	6	-	8	13	13	9	9	12	4
3	6	8	8	-	13	13	11	11	4	12
4	9	7	13	13	-	8	4	12	9	9
5	9	7	13	13	8	-	12	4	13	13
6	9	11	9	11	4	12	-	8	11	13
7	9	11	9	11	12	4	8	-	9	11
8	10	8	12	4	9	13	11	9	-	8
9	10	10	4	12	9	13	13	11	8	-

$\geq 60; (\frac{61}{144}, \frac{83}{144})$



TT-disjoint at 4

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	2	2	2	2	3	3
1	1	-	2	1	3	3	3	3	2	2
2	1	2	-	3	1	3	3	3	2	2
3	2	1	3	-	2	4	2	4	3	3
4	2	3	1	2	-	4	2	4	3	3
5	2	3	3	4	4	-	2	2	3	1
6	2	3	3	2	2	2	-	2	3	3
7	2	3	3	4	4	2	2	-	1	3
8	3	2	2	3	3	3	3	1	-	2
9	3	2	2	3	3	1	3	3	2	-

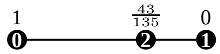
$\geq$ 16; SMAPO# 200

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	4	5	6	7	8	8	10
1	2	-	4	2	7	8	5	10	6	8
2	2	4	-	6	3	8	5	10	10	10
3	4	2	6	-	5	10	3	10	8	10
4	5	7	3	5	-	11	4	11	7	7
5	6	8	8	10	11	-	7	8	12	4
6	7	5	5	3	4	7	-	7	11	11
7	8	10	10	10	11	8	7	-	4	12
8	8	6	10	8	7	12	11	4	-	8
9	10	8	10	10	7	4	11	12	8	-

$\geq$ 46; SMAPO# 6141

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	6	6	8	9	10	11	13
1	3	-	6	3	9	11	8	13	8	10
2	3	6	-	9	3	11	8	13	12	12
3	6	3	9	-	6	14	5	14	11	13
4	6	9	3	6	-	14	5	14	9	9
5	8	11	11	14	14	-	9	10	15	5
6	9	8	8	5	5	9	-	9	14	14
7	10	13	13	14	14	10	9	-	5	15
8	11	8	12	11	9	15	14	5	-	10
9	13	10	12	13	9	5	14	15	10	-

$\geq 60; (\frac{38}{155}, \frac{117}{155})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	2	4	3	4
1	1	-	3	3	4	4	3	3	2	3
2	2	3	-	4	3	5	4	2	3	4
3	2	3	4	-	5	3	4	4	3	2
4	3	4	3	5	-	2	3	5	4	3
5	3	4	5	3	2	-	3	3	4	5
6	2	3	4	4	3	3	-	2	1	2
7	4	3	2	4	5	3	2	-	3	4
8	3	2	3	3	4	4	1	3	-	3
9	4	3	4	2	3	5	2	4	3	-

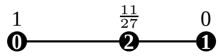
≥ 22; SMAPO# 464

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	6	4	6	7	6	8	7
1	2	-	6	6	6	8	9	4	6	9
2	4	6	-	8	4	6	7	2	8	7
3	6	6	8	-	8	6	9	6	6	3
4	4	6	4	8	-	2	5	6	8	5
5	6	8	6	6	2	-	7	4	8	7
6	7	9	7	9	5	7	-	5	3	6
7	6	4	2	6	6	4	5	-	6	7
8	8	6	8	6	8	8	3	6	-	9
9	7	9	7	3	5	7	6	7	9	-

≥ 40; SMAPO# 3930

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	7	7	9	9	10	11	11
1	3	-	9	8	10	12	12	7	8	12
2	6	9	-	11	7	11	11	4	11	11
3	7	8	11	-	12	8	12	9	8	4
4	7	10	7	12	-	4	8	11	12	8
5	9	12	11	8	4	-	10	7	12	12
6	9	12	11	12	8	10	-	7	4	8
7	10	7	4	9	11	7	7	-	9	11
8	11	8	11	8	12	12	4	9	-	12
9	11	12	11	4	8	12	8	11	12	-

≥ 60; $(\frac{43}{135}, \frac{92}{135})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	2	3	3	4	5	5
1	1	-	4	4	3	4	4	3	4	4
2	3	4	-	4	5	4	4	5	6	2
3	3	4	4	-	5	4	4	5	2	6
4	2	3	5	5	-	1	3	2	3	3
5	3	4	4	4	1	-	2	3	4	4
6	3	4	4	4	3	2	-	1	4	4
7	4	3	5	5	2	3	1	-	5	5
8	5	4	6	2	3	4	4	5	-	4
9	5	4	2	6	3	4	4	5	4	-

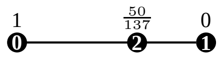
≥ 24; SMAPO# 665

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	5	6	6	7	6	6
1	2	-	5	6	7	6	8	5	4	8
2	5	5	-	5	8	5	7	6	7	3
3	4	6	5	-	7	6	6	7	2	6
4	5	7	8	7	-	3	7	4	5	5
5	6	6	5	6	3	-	4	7	6	8
6	6	8	7	6	7	4	-	3	4	6
7	7	5	6	7	4	7	3	-	7	7
8	6	4	7	2	5	6	4	7	-	4
9	6	8	3	6	5	8	6	7	4	-

≥ 38; SMAPO# 3287

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	7	9	9	11	11	11
1	3	-	8	10	10	10	12	8	8	12
2	7	8	-	8	12	8	10	10	12	4
3	7	10	8	-	12	10	10	12	4	12
4	7	10	12	12	-	4	10	6	8	8
5	9	10	8	10	4	-	6	10	10	12
6	9	12	10	10	10	6	-	4	8	10
7	11	8	10	12	6	10	4	-	12	12
8	11	8	12	4	8	10	8	12	-	8
9	11	12	4	12	8	12	10	12	8	-

≥ 60; $(\frac{11}{27}, \frac{16}{27})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	2	2	3	3	5	5
1	1	-	4	4	3	3	2	2	4	4
2	3	4	-	4	5	5	4	4	6	2
3	3	4	4	-	5	5	4	4	2	6
4	2	3	5	5	-	2	1	1	3	3
5	2	3	5	5	2	-	1	1	3	3
6	3	2	4	4	1	1	-	2	4	4
7	3	2	4	4	1	1	2	-	4	4
8	5	4	6	2	3	3	4	4	-	4
9	5	4	2	6	3	3	4	4	4	-

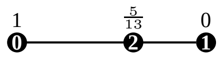
≥ 22; SMAPO# 482

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	6	6	7	7	6	6
1	2	-	5	6	6	8	5	7	4	8
2	5	5	-	5	9	7	8	6	7	3
3	4	6	5	-	8	6	7	7	2	6
4	6	6	9	8	-	8	5	3	6	6
5	6	8	7	6	8	-	3	5	4	6
6	7	5	8	7	5	3	-	8	7	7
7	7	7	6	7	3	5	8	-	7	9
8	6	4	7	2	6	4	7	7	-	4
9	6	8	3	6	6	6	7	9	4	-

≥ 40; SMAPO# 3893

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	8	8	10	10	11	11
1	3	-	8	10	9	11	7	9	8	12
2	7	8	-	8	13	11	11	9	12	4
3	7	10	8	-	13	11	11	11	4	12
4	8	9	13	13	-	10	6	4	9	9
5	8	11	11	11	10	-	4	6	7	9
6	10	7	11	11	6	4	-	10	11	11
7	10	9	9	11	4	6	10	-	11	13
8	11	8	12	4	9	7	11	11	-	8
9	11	12	4	12	9	9	11	13	8	-

≥ 60; $(\frac{50}{137}, \frac{87}{137})$



TT-disjoint at 6

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	3	4	4	4	4
1	1	-	3	4	4	2	3	5	5	3
2	2	3	-	3	3	1	4	4	4	4
3	3	4	3	-	2	4	5	5	3	3
4	3	4	3	2	-	4	3	3	5	5
5	3	2	1	4	4	-	5	3	3	5
6	4	3	4	5	3	5	-	4	2	4
7	4	5	4	5	3	3	4	-	4	2
8	4	5	4	3	5	3	2	4	-	4
9	4	3	4	3	5	5	4	2	4	-

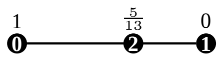
≥ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	3	4	4	6	7	7	6	6	8
1	3	-	7	7	7	4	6	7	7	5
2	4	7	-	6	6	3	5	6	8	6
3	4	7	6	-	2	5	7	6	4	4
4	6	7	6	2	-	7	5	4	6	6
5	7	4	3	5	7	-	8	5	5	7
6	7	6	5	7	5	8	-	5	3	5
7	6	7	6	6	4	5	5	-	6	2
8	6	7	8	4	6	5	3	6	-	8
9	8	5	6	4	6	7	5	2	8	-

≥ 38; SMAPO# 3287

	0	1	2	3	4	5	6	7	8	9
0	-	4	6	7	9	10	10	10	10	12
1	4	-	10	11	11	6	8	12	12	8
2	6	10	-	9	9	4	8	10	12	10
3	7	11	9	-	4	9	11	11	7	7
4	9	11	9	4	-	11	7	7	11	11
5	10	6	4	9	11	-	12	8	8	12
6	10	8	8	11	7	12	-	8	4	8
7	10	12	10	11	7	8	8	-	10	4
8	10	12	12	7	11	8	4	10	-	12
9	12	8	10	7	11	12	8	4	12	-

≥ 60; $(\frac{5}{13}, \frac{8}{13})$



TT-disjoint at 6

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	3	4	4	4	4
1	1	-	3	4	4	2	3	5	5	3
2	2	3	-	3	3	1	4	4	4	4
3	3	4	3	-	2	4	5	5	3	3
4	3	4	3	2	-	4	3	3	5	5
5	3	2	1	4	4	-	5	3	3	5
6	4	3	4	5	3	5	-	4	2	4
7	4	5	4	5	3	3	4	-	4	2
8	4	5	4	3	5	3	2	4	-	4
9	4	3	4	3	5	5	4	2	4	-

≥ 24; SMAPO# 662

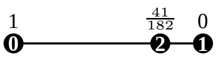
	0	1	2	3	4	5	6	7	8	9
0	-	3	4	4	6	7	7	6	6	8
1	3	-	7	7	7	4	6	7	7	5
2	4	7	-	6	6	3	5	6	8	6
3	4	7	6	-	2	5	7	6	4	4
4	6	7	6	2	-	5	5	4	6	6
5	7	4	3	5	5	-	8	5	5	7
6	7	6	5	7	5	8	-	7	3	5
7	6	7	6	6	4	5	7	-	6	2
8	6	7	8	4	6	5	3	6	-	8
9	8	5	6	4	6	7	5	2	8	-

≥ 38; SMAPO# 3286

	0	1	2	3	4	5	6	7	8	9
0	-	4	6	7	9	10	10	10	10	12
1	4	-	10	11	11	6	8	12	12	8
2	6	10	-	9	9	4	8	10	12	10
3	7	11	9	-	4	9	11	11	7	7
4	9	11	9	4	-	9	7	7	11	11
5	10	6	4	9	9	-	12	8	8	12
6	10	8	8	11	7	12	-	10	4	8
7	10	12	10	11	7	8	10	-	10	4
8	10	12	12	7	11	8	4	10	-	12
9	12	8	10	7	11	12	8	4	12	-

≥ 60; $(\frac{5}{13}, \frac{8}{13})$

RHS = 62



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	0	0	1	2	3	2	2	3	3
1	0	-	0	1	2	3	2	2	3	3
2	0	0	-	1	2	3	2	2	3	3
3	1	1	1	-	3	2	3	3	4	2
4	2	2	2	3	-	1	4	4	3	3
5	3	3	3	2	1	-	3	3	4	2
6	2	2	2	3	4	3	-	2	1	3
7	2	2	2	3	4	3	2	-	3	1
8	3	3	3	4	3	4	1	3	-	2
9	3	3	3	2	3	2	3	1	2	-

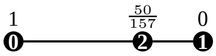
$\geq$ 14; SMAPO# 146

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	5	6	6	9	9	9	11
1	1	-	3	4	5	7	8	8	10	12
2	2	3	-	3	8	6	11	11	9	11
3	5	4	3	-	7	5	8	8	12	10
4	6	5	8	7	-	2	13	13	9	11
5	6	7	6	5	2	-	13	13	11	9
6	9	8	11	8	13	13	-	8	4	12
7	9	8	11	8	13	13	8	-	12	4
8	9	10	9	12	9	11	4	12	-	8
9	11	12	11	10	11	9	12	4	8	-

$\geq$ 50; SMAPO# 7693

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	5	8	9	11	11	12	14
1	1	-	3	4	7	10	10	10	13	15
2	2	3	-	3	10	9	13	13	12	14
3	5	4	3	-	9	6	10	10	15	11
4	8	7	10	9	-	3	17	17	12	14
5	9	10	9	6	3	-	16	16	15	11
6	11	10	13	10	17	16	-	10	5	15
7	11	10	13	10	17	16	10	-	15	5
8	12	13	12	15	12	15	5	15	-	10
9	14	15	14	11	14	11	15	5	10	-

$\geq 62; (\frac{41}{182}, \frac{141}{182})$



TT-disjoint at 8

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	3	4	4	5	5
1	1	-	3	4	4	2	3	3	4	4
2	2	3	-	5	5	1	4	4	3	3
3	3	4	5	-	4	4	5	3	4	2
4	3	4	5	4	-	4	3	5	2	4
5	3	2	1	4	4	-	3	3	4	4
6	4	3	4	5	3	3	-	2	5	3
7	4	3	4	3	5	3	2	-	3	5
8	5	4	3	4	2	4	5	3	-	4
9	5	4	3	2	4	4	3	5	4	-

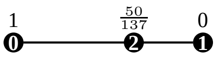
$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	6	6	7	6	8	9	8
1	2	-	7	8	8	5	6	8	7	10
2	5	7	-	9	9	2	9	9	8	7
3	6	8	9	-	6	7	6	4	5	2
4	6	8	9	6	-	9	4	6	3	8
5	7	5	2	7	9	-	7	7	6	9
6	6	6	9	6	4	7	-	2	7	4
7	8	8	9	4	6	7	2	-	5	6
8	9	7	8	5	3	6	7	5	-	5
9	8	10	7	2	8	9	4	6	5	-

$\geq$ 40; SMAPO# 4109

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	9	9	10	10	12	13	13
1	3	-	10	12	12	7	9	11	10	14
2	7	10	-	14	14	3	13	13	10	10
3	9	12	14	-	10	11	11	7	8	4
4	9	12	14	10	-	13	7	11	4	12
5	10	7	3	11	13	-	10	10	9	13
6	10	9	13	11	7	10	-	4	11	7
7	12	11	13	7	11	10	4	-	7	11
8	13	10	10	8	4	9	11	7	-	8
9	13	14	10	4	12	13	7	11	8	-

$\geq 62; (\frac{50}{157}, \frac{107}{157})$



TT-disjoint at 8

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	4	5	5	4
1	2	-	3	5	5	3	4	3	3	4
2	3	3	-	4	4	2	3	4	4	1
3	3	5	4	-	4	4	5	2	4	3
4	3	5	4	4	-	4	3	4	2	5
5	3	3	2	4	4	-	1	4	4	3
6	4	4	3	5	3	1	-	3	5	2
7	5	3	4	2	4	4	3	-	4	5
8	5	3	4	4	2	4	5	4	-	3
9	4	4	1	3	5	3	2	5	3	-

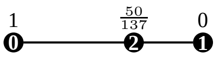
$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	4	4	5	6	6	7	7
1	2	-	6	6	6	7	6	4	5	7
2	4	6	-	8	8	5	8	6	7	3
3	4	6	8	-	8	7	8	2	5	5
4	4	6	8	8	-	9	6	6	3	7
5	5	7	5	7	9	-	3	7	6	8
6	6	6	8	8	6	3	-	6	9	5
7	6	4	6	2	6	7	6	-	7	7
8	7	5	7	5	3	6	9	7	-	6
9	7	7	3	5	7	8	5	7	6	-

$\geq$ 40; SMAPO# 3899

	0	1	2	3	4	5	6	7	8	9
0	-	4	7	7	7	8	10	11	11	11
1	4	-	9	11	11	10	10	7	7	11
2	7	9	-	12	12	7	11	10	10	4
3	7	11	12	-	12	11	13	4	8	8
4	7	11	12	12	-	13	9	10	4	12
5	8	10	7	11	13	-	4	11	9	11
6	10	10	11	13	9	4	-	9	13	7
7	11	7	10	4	10	11	9	-	10	12
8	11	7	10	8	4	9	13	10	-	8
9	11	11	4	8	12	11	7	12	8	-

$\geq 62; (\frac{50}{137}, \frac{87}{137})$



TT-disjoint at 8

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	4	5	5	4
1	2	-	3	5	5	3	4	3	3	4
2	3	3	-	4	4	2	3	4	4	1
3	3	5	4	-	4	4	5	2	4	3
4	3	5	4	4	-	4	3	4	2	5
5	3	3	2	4	4	-	1	4	4	3
6	4	4	3	5	3	1	-	3	5	2
7	5	3	4	2	4	4	3	-	4	5
8	5	3	4	4	2	4	5	4	-	3
9	4	4	1	3	5	3	2	5	3	-

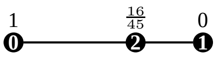
$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	4	4	5	6	6	7	7
1	2	-	6	6	6	7	8	4	5	7
2	4	6	-	8	8	5	8	6	7	3
3	4	6	8	-	8	7	8	2	5	5
4	4	6	8	8	-	9	6	6	3	7
5	5	7	5	7	9	-	3	7	6	8
6	6	8	8	8	6	3	-	6	9	5
7	6	4	6	2	6	7	6	-	5	7
8	7	5	7	5	3	6	9	5	-	6
9	7	7	3	5	7	8	5	7	6	-

$\geq$ 40; SMAPO# 3906

	0	1	2	3	4	5	6	7	8	9
0	-	4	7	7	7	8	10	11	11	11
1	4	-	9	11	11	10	12	7	7	11
2	7	9	-	12	12	7	11	10	10	4
3	7	11	12	-	12	11	13	4	8	8
4	7	11	12	12	-	13	9	10	4	12
5	8	10	7	11	13	-	4	11	9	11
6	10	12	11	13	9	4	-	9	13	7
7	11	7	10	4	10	11	9	-	8	12
8	11	7	10	8	4	9	13	8	-	8
9	11	11	4	8	12	11	7	12	8	-

$\geq 62; (\frac{50}{137}, \frac{87}{137})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	2	2	3	3	3	5	4
1	1	-	4	3	3	4	2	2	4	3
2	3	4	-	3	5	4	4	4	2	5
3	2	3	3	-	4	3	3	1	5	2
4	2	3	5	4	-	3	1	3	3	2
5	3	4	4	3	3	-	4	2	4	1
6	3	2	4	3	1	4	-	2	4	3
7	3	2	4	1	3	2	2	-	4	3
8	5	4	2	5	3	4	4	4	-	3
9	4	3	5	2	2	1	3	3	3	-

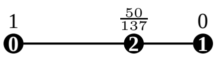
$\geq$ 20; SMAPO# 363

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	5	5	6	8	8	6	7
1	3	-	6	8	8	7	5	5	7	8
2	5	6	-	6	8	5	5	7	3	8
3	5	8	6	-	8	9	7	3	7	6
4	5	8	8	8	-	7	3	7	5	6
5	6	7	5	9	7	-	8	6	8	3
6	8	5	5	7	3	8	-	6	8	7
7	8	5	7	3	7	6	6	-	8	9
8	6	7	3	7	5	8	8	8	-	5
9	7	8	8	6	6	3	7	9	5	-

$\geq$ 44; SMAPO# 5074

	0	1	2	3	4	5	6	7	8	9
0	-	4	7	7	7	9	11	11	11	11
1	4	-	9	11	11	11	7	7	11	11
2	7	9	-	8	12	8	8	10	4	12
3	7	11	8	-	12	12	10	4	12	8
4	7	11	12	12	-	10	4	10	8	8
5	9	11	8	12	10	-	12	8	12	4
6	11	7	8	10	4	12	-	8	12	10
7	11	7	10	4	10	8	8	-	12	12
8	11	11	4	12	8	12	12	12	-	8
9	11	11	12	8	8	4	10	12	8	-

$\geq 62; (\frac{16}{45}, \frac{29}{45})$



TT-disjoint at 7

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	4	5	5	4
1	2	-	5	5	3	3	4	3	3	4
2	3	5	-	4	4	4	3	2	4	5
3	3	5	4	-	4	4	5	4	2	3
4	3	3	4	4	-	2	3	4	4	1
5	3	3	4	4	2	-	1	4	4	3
6	4	4	3	5	3	1	-	5	3	2
7	5	3	2	4	4	4	5	-	4	3
8	5	3	4	2	4	4	3	4	-	5
9	4	4	5	3	1	3	2	3	5	-

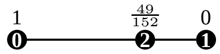
$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	4	5	6	5	7	6	8
1	2	-	6	6	7	6	7	5	4	8
2	4	6	-	8	7	8	5	3	6	8
3	4	6	8	-	9	8	7	5	2	6
4	5	7	7	9	-	5	8	6	7	3
5	6	6	8	8	5	-	3	5	6	8
6	5	7	5	7	8	3	-	8	5	5
7	7	5	3	5	6	5	8	-	5	7
8	6	4	6	2	7	6	5	5	-	8
9	8	8	8	6	3	8	5	7	8	-

$\geq$ 40; SMAPO# 3896

	0	1	2	3	4	5	6	7	8	9
0	-	4	7	7	8	9	9	11	11	12
1	4	-	11	11	10	9	11	7	7	12
2	7	11	-	12	11	12	8	4	10	13
3	7	11	12	-	13	12	12	8	4	9
4	8	10	11	13	-	7	11	9	11	4
5	9	9	12	12	7	-	4	8	10	11
6	9	11	8	12	11	4	-	12	8	7
7	11	7	4	8	9	8	12	-	8	9
8	11	7	10	4	11	10	8	8	-	13
9	12	12	13	9	4	11	7	9	13	-

$\geq 62; (\frac{50}{137}, \frac{87}{137})$



TT-disjoint at 1

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	4	4	2	2	3	3
1	1	-	2	2	3	5	3	3	4	4
2	1	2	-	4	5	3	3	3	2	4
3	3	2	4	-	3	5	3	3	2	2
4	4	3	5	3	-	2	4	4	5	5
5	4	5	3	5	2	-	4	4	3	5
6	2	3	3	3	4	4	-	2	1	3
7	2	3	3	3	4	4	2	-	3	1
8	3	4	2	2	5	3	1	3	-	2
9	3	4	4	2	5	5	3	1	2	-

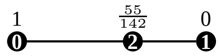
$\geq$ 20; SMAPO# 365

	0	1	2	3	4	5	6	7	8	9
0	-	2	1	4	4	4	8	8	8	8
1	2	-	3	4	4	6	6	6	10	10
2	1	3	-	5	5	3	7	7	7	7
3	4	4	5	-	4	6	10	10	6	6
4	4	4	5	4	-	2	10	10	10	10
5	4	6	3	6	2	-	8	8	8	8
6	8	6	7	10	10	8	-	8	4	12
7	8	6	7	10	10	8	8	-	12	4
8	8	10	7	6	10	8	4	12	-	8
9	8	10	7	6	10	8	12	4	8	-

$\geq$ 44; SMAPO# 5476

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	7	8	8	10	10	11	11
1	2	-	4	5	6	10	8	8	13	13
2	2	4	-	9	10	6	10	10	9	11
3	7	5	9	-	7	11	13	13	8	8
4	8	6	10	7	-	4	14	14	15	15
5	8	10	6	11	4	-	12	12	11	13
6	10	8	10	13	14	12	-	10	5	15
7	10	8	10	13	14	12	10	-	15	5
8	11	13	9	8	15	11	5	15	-	10
9	11	13	11	8	15	13	15	5	10	-

≥ 62; $(\frac{49}{152}, \frac{103}{152})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	2	3	3	5	5	4
1	1	-	4	4	3	4	4	4	4	3
2	3	4	-	4	5	4	4	6	2	5
3	3	4	4	-	5	4	4	2	6	5
4	2	3	5	5	-	1	3	3	3	2
5	3	4	4	4	1	-	2	4	4	3
6	3	4	4	4	3	2	-	4	4	1
7	5	4	6	2	3	4	4	-	4	5
8	5	4	2	6	3	4	4	4	-	5
9	4	3	5	5	2	3	1	5	5	-

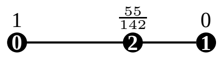
$\geq$ 24; SMAPO# 665

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	5	6	7	6	6	8
1	2	-	5	6	7	8	9	4	8	6
2	5	5	-	5	8	5	6	7	3	7
3	4	6	5	-	7	6	7	2	6	8
4	5	7	8	7	-	3	8	5	5	5
5	6	8	5	6	3	-	5	6	8	8
6	7	9	6	7	8	5	-	5	7	3
7	6	4	7	2	5	6	5	-	4	8
8	6	8	3	6	5	8	7	4	-	8
9	8	6	7	8	5	8	3	8	8	-

$\geq$ 40; SMAPO# 3896

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	7	9	10	11	11	12
1	3	-	8	10	10	12	13	8	12	9
2	7	8	-	8	12	8	9	12	4	11
3	7	10	8	-	12	10	11	4	12	13
4	7	10	12	12	-	4	11	8	8	7
5	9	12	8	10	4	-	7	10	12	11
6	10	13	9	11	11	7	-	9	11	4
7	11	8	12	4	8	10	9	-	8	13
8	11	12	4	12	8	12	11	8	-	13
9	12	9	11	13	7	11	4	13	13	-

≥ 62; $(\frac{55}{142}, \frac{87}{142})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	2	3	3	4	5	5
1	1	-	4	4	3	4	4	3	4	4
2	3	4	-	4	5	4	4	5	6	2
3	3	4	4	-	5	4	4	5	2	6
4	2	3	5	5	-	3	1	2	3	3
5	3	4	4	4	3	-	2	1	4	4
6	3	4	4	4	1	2	-	3	4	4
7	4	3	5	5	2	1	3	-	5	5
8	5	4	6	2	3	4	4	5	-	4
9	5	4	2	6	3	4	4	5	4	-

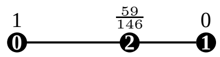
$\geq$ 24; SMAPO# 665

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	6	6	7	7	6	6
1	2	-	5	6	8	8	7	5	4	8
2	5	5	-	5	9	7	6	6	7	3
3	4	6	5	-	8	6	7	7	2	6
4	6	8	9	8	-	8	3	5	6	6
5	6	8	7	6	8	-	5	3	4	8
6	7	7	6	7	3	5	-	8	5	9
7	7	5	6	7	5	3	8	-	7	7
8	6	4	7	2	6	4	5	7	-	4
9	6	8	3	6	6	8	9	7	4	-

$\geq$ 40; SMAPO# 3906

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	8	9	10	11	11	11
1	3	-	8	10	11	12	11	8	8	12
2	7	8	-	8	13	10	9	10	12	4
3	7	10	8	-	13	10	11	12	4	12
4	8	11	13	13	-	11	4	7	9	9
5	9	12	10	10	11	-	7	4	8	12
6	10	11	9	11	4	7	-	11	9	13
7	11	8	10	12	7	4	11	-	12	12
8	11	8	12	4	9	8	9	12	-	8
9	11	12	4	12	9	12	13	12	8	-

≥ 62; $(\frac{55}{142}, \frac{87}{142})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	2	3	3	5	4	5
1	1	-	4	4	3	2	2	4	3	4
2	3	4	-	4	5	6	6	6	5	2
3	3	4	4	-	5	6	6	2	5	6
4	2	3	5	5	-	3	1	3	2	3
5	3	2	6	6	3	-	2	4	1	4
6	3	2	6	6	1	2	-	4	3	4
7	5	4	6	2	3	4	4	-	5	4
8	4	3	5	5	2	1	3	5	-	5
9	5	4	2	6	3	4	4	4	5	-

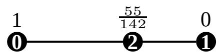
$\geq$ 24; SMAPO# 724

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	6	6	7	6	7	6
1	2	-	5	6	8	6	5	4	7	8
2	5	5	-	5	7	9	8	7	6	3
3	4	6	5	-	6	8	7	2	7	6
4	6	8	7	6	-	8	3	4	5	6
5	6	6	9	8	8	-	5	6	3	6
6	7	5	8	7	3	5	-	7	8	7
7	6	4	7	2	4	6	7	-	7	4
8	7	7	6	7	5	3	8	7	-	9
9	6	8	3	6	6	6	7	4	9	-

$\geq$ 40; SMAPO# 3893

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	8	9	10	11	11	11
1	3	-	8	10	11	8	7	8	10	12
2	7	8	-	8	11	14	13	12	10	4
3	7	10	8	-	11	14	13	4	12	12
4	8	11	11	11	-	11	4	7	7	9
5	9	8	14	14	11	-	7	10	4	10
6	10	7	13	13	4	7	-	11	11	11
7	11	8	12	4	7	10	11	-	12	8
8	11	10	10	12	7	4	11	12	-	14
9	11	12	4	12	9	10	11	8	14	-

≥ 62; $(\frac{59}{146}, \frac{87}{146})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	2	3	3	5	4	5
1	1	-	4	4	3	4	4	4	3	4
2	3	4	-	4	5	4	4	6	5	2
3	3	4	4	-	5	4	4	2	5	6
4	2	3	5	5	-	3	1	3	2	3
5	3	4	4	4	3	-	2	4	1	4
6	3	4	4	4	1	2	-	4	3	4
7	5	4	6	2	3	4	4	-	5	4
8	4	3	5	5	2	1	3	5	-	5
9	5	4	2	6	3	4	4	4	5	-

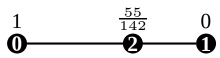
$\geq$ 24; SMAPO# 665

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	6	6	7	6	7	6
1	2	-	5	6	6	8	7	4	5	8
2	5	5	-	5	9	7	6	7	8	3
3	4	6	5	-	8	6	7	2	7	6
4	6	6	9	8	-	8	3	6	5	6
5	6	8	7	6	8	-	5	4	3	6
6	7	7	6	7	3	5	-	7	8	9
7	6	4	7	2	6	4	7	-	7	4
8	7	5	8	7	5	3	8	7	-	7
9	6	8	3	6	6	6	9	4	7	-

$\geq$ 40; SMAPO# 3893

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	8	9	10	11	11	11
1	3	-	8	10	9	12	11	8	8	12
2	7	8	-	8	13	10	9	12	12	4
3	7	10	8	-	13	10	11	4	12	12
4	8	9	13	13	-	11	4	9	7	9
5	9	12	10	10	11	-	7	8	4	10
6	10	11	9	11	4	7	-	11	11	13
7	11	8	12	4	9	8	11	-	12	8
8	11	8	12	12	7	4	11	12	-	12
9	11	12	4	12	9	10	13	8	12	-

≥ 62; $(\frac{55}{142}, \frac{87}{142})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	2	3	3	5	4	5
1	1	-	4	4	3	4	4	4	3	4
2	3	4	-	4	5	4	4	6	5	2
3	3	4	4	-	5	4	4	2	5	6
4	2	3	5	5	-	3	1	3	2	3
5	3	4	4	4	3	-	2	4	1	4
6	3	4	4	4	1	2	-	4	3	4
7	5	4	6	2	3	4	4	-	5	4
8	4	3	5	5	2	1	3	5	-	5
9	5	4	2	6	3	4	4	4	5	-

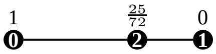
$\geq$ 24; SMAPO# 665

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	6	6	7	6	7	6
1	2	-	5	6	6	8	7	4	5	8
2	5	5	-	5	9	7	6	7	8	3
3	4	6	5	-	8	6	7	2	7	6
4	6	6	9	8	-	8	3	6	5	6
5	6	8	7	6	8	-	5	4	3	8
6	7	7	6	7	3	5	-	5	8	9
7	6	4	7	2	6	4	5	-	7	4
8	7	5	8	7	5	3	8	7	-	7
9	6	8	3	6	6	8	9	4	7	-

$\geq$ 40; SMAPO# 3898

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	8	9	10	11	11	11
1	3	-	8	10	9	12	11	8	8	12
2	7	8	-	8	13	10	9	12	12	4
3	7	10	8	-	13	10	11	4	12	12
4	8	9	13	13	-	11	4	9	7	9
5	9	12	10	10	11	-	7	8	4	12
6	10	11	9	11	4	7	-	9	11	13
7	11	8	12	4	9	8	9	-	12	8
8	11	8	12	12	7	4	11	12	-	12
9	11	12	4	12	9	12	13	8	12	-

≥ 62; $(\frac{55}{142}, \frac{87}{142})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	4	3	4	4	3	4
1	1	-	3	4	5	2	3	3	4	5
2	2	3	-	3	4	1	4	4	3	4
3	3	4	3	-	5	4	5	3	2	3
4	4	5	4	5	-	3	4	2	3	4
5	3	2	1	4	3	-	5	5	4	3
6	4	3	4	5	4	5	-	4	3	2
7	4	3	4	3	2	5	4	-	5	4
8	3	4	3	2	3	4	3	5	-	5
9	4	5	4	3	4	3	2	4	5	-

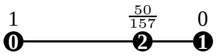
≥ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	5	5	8	7	7	8	7
1	2	-	7	7	7	6	5	5	8	7
2	5	7	-	6	8	3	6	8	9	8
3	5	7	6	-	8	9	8	6	3	6
4	5	7	8	8	-	5	4	2	5	4
5	8	6	3	9	5	-	7	7	8	5
6	7	5	6	8	4	7	-	6	5	2
7	7	5	8	6	2	7	6	-	7	6
8	8	8	9	3	5	8	5	7	-	7
9	7	7	8	6	4	5	2	6	7	-

≥ 40; SMAPO# 3915

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	9	11	11	11	11	11
1	3	-	10	10	12	8	8	8	12	12
2	7	10	-	8	12	4	10	12	12	12
3	7	10	8	-	12	12	12	8	4	8
4	9	12	12	12	-	8	8	4	8	8
5	11	8	4	12	8	-	12	12	12	8
6	11	8	10	12	8	12	-	10	8	4
7	11	8	12	8	4	12	10	-	12	10
8	11	12	12	4	8	12	8	12	-	12
9	11	12	12	8	8	8	4	10	12	-

$\geq 62; (\frac{25}{72}, \frac{47}{72})$



TT-disjoint at 7

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	3	4	4	4	4
1	1	-	3	4	2	4	3	5	3	5
2	2	3	-	3	1	3	4	4	4	4
3	3	4	3	-	4	2	5	5	3	3
4	3	2	1	4	-	4	5	3	5	3
5	3	4	3	2	4	-	3	3	5	5
6	4	3	4	5	5	3	-	4	4	2
7	4	5	4	5	3	3	4	-	2	4
8	4	3	4	3	5	5	4	2	-	4
9	4	5	4	3	3	5	2	4	4	-

≥ 24; SMAPO# 662

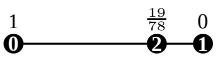
	0	1	2	3	4	5	6	7	8	9
0	-	2	5	5	7	7	7	8	9	9
1	2	-	7	7	5	9	7	10	7	9
2	5	7	-	6	2	8	8	7	10	8
3	5	7	6	-	8	2	6	7	4	4
4	7	5	2	8	-	8	8	7	8	6
5	7	9	8	2	8	-	4	5	6	6
6	7	7	8	6	8	4	-	5	6	2
7	8	10	7	7	7	5	5	-	3	5
8	9	7	10	4	8	6	6	3	-	8
9	9	9	8	4	6	6	2	5	8	-

≥ 40; SMAPO# 4130

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	8	10	10	11	11	13	13
1	3	-	10	11	7	13	10	14	10	14
2	7	10	-	9	3	11	12	10	14	12
3	8	11	9	-	12	4	11	11	7	7
4	10	7	3	12	-	12	13	9	13	9
5	10	13	11	4	12	-	7	7	11	11
6	11	10	12	11	13	7	-	8	10	4
7	11	14	10	11	9	7	8	-	4	8
8	13	10	14	7	13	11	10	4	-	12
9	13	14	12	7	9	11	4	8	12	-

$\geq 62; (\frac{50}{157}, \frac{107}{157})$

RHS = 64



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	0	1	1	2	3	2	3	2	3
1	0	-	1	1	2	3	2	3	2	3
2	1	1	-	2	3	2	3	2	3	4
3	1	1	2	-	3	2	3	2	1	2
4	2	2	3	3	-	3	2	1	2	3
5	3	3	2	2	3	-	3	2	1	4
6	2	2	3	3	2	3	-	3	2	1
7	3	3	2	2	1	2	3	-	3	2
8	2	2	3	1	2	1	2	3	-	3
9	3	3	4	2	3	4	1	2	3	-

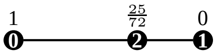
≥ 14; SMAPO# 136

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	7	6	7	9	8	9	9
1	2	-	5	5	8	9	7	10	11	11
2	3	5	-	4	9	6	8	7	10	10
3	7	5	4	-	9	10	10	9	6	6
4	6	8	9	9	-	11	7	4	7	11
5	7	9	6	10	11	-	10	7	4	10
6	9	7	8	10	7	10	-	11	10	4
7	8	10	7	9	4	7	11	-	11	7
8	9	11	10	6	7	4	10	11	-	8
9	9	11	10	6	11	10	4	7	8	-

≥ 52; SMAPO# 7787

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	8	8	10	11	11	11	12
1	2	-	5	6	10	12	9	13	13	14
2	3	5	-	5	11	7	10	8	12	13
3	8	6	5	-	12	12	13	11	7	8
4	8	10	11	12	-	14	9	5	9	14
5	10	12	7	12	14	-	13	9	5	14
6	11	9	10	13	9	13	-	14	12	5
7	11	13	8	11	5	9	14	-	14	9
8	11	13	12	7	9	5	12	14	-	11
9	12	14	13	8	14	14	5	9	11	-

$\geq 64; (\frac{19}{78}, \frac{59}{78})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	3	4	5	4	5
1	1	-	3	4	4	2	3	4	3	4
2	2	3	-	5	5	1	4	3	4	3
3	3	4	5	-	4	4	5	2	3	4
4	3	4	5	4	-	4	3	4	5	2
5	3	2	1	4	4	-	3	4	3	4
6	4	3	4	5	3	3	-	3	2	5
7	5	4	3	2	4	4	3	-	5	4
8	4	3	4	3	5	3	2	5	-	3
9	5	4	3	4	2	4	5	4	3	-

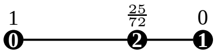
≥ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	6	5	8	8	7	8	7
1	3	-	8	7	8	5	5	6	7	8
2	5	8	-	9	8	3	7	6	7	6
3	6	7	9	-	5	6	8	3	6	7
4	5	8	8	5	-	7	5	8	7	2
5	8	5	3	6	7	-	8	9	6	7
6	8	5	7	8	5	8	-	5	2	7
7	7	6	6	3	8	9	5	-	7	6
8	8	7	7	6	7	6	2	7	-	5
9	7	8	6	7	2	7	7	6	5	-

≥ 42; SMAPO# 4504

	0	1	2	3	4	5	6	7	8	9
0	-	4	7	8	8	11	12	12	12	12
1	4	-	11	10	12	7	8	10	10	12
2	7	11	-	13	13	4	11	9	11	9
3	8	10	13	-	8	9	12	4	8	10
4	8	12	13	8	-	11	8	12	12	4
5	11	7	4	9	11	-	11	13	9	11
6	12	8	11	12	8	11	-	8	4	12
7	12	10	9	4	12	13	8	-	12	10
8	12	10	11	8	12	9	4	12	-	8
9	12	12	9	10	4	11	12	10	8	-

$\geq 64; (\frac{25}{72}, \frac{47}{72})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	3	4	4	5	5
1	1	-	3	4	4	2	3	3	4	4
2	2	3	-	5	5	1	4	4	3	3
3	3	4	5	-	4	4	5	3	2	4
4	3	4	5	4	-	4	3	5	4	2
5	3	2	1	4	4	-	3	3	4	4
6	4	3	4	5	3	3	-	2	3	5
7	4	3	4	3	5	3	2	-	5	3
8	5	4	3	2	4	4	3	5	-	4
9	5	4	3	4	2	4	5	3	4	-

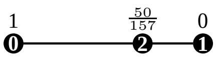
≥ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	6	5	8	8	8	7	7
1	3	-	8	7	8	5	5	7	8	8
2	5	8	-	9	8	3	7	7	6	6
3	6	7	9	-	5	6	8	6	3	7
4	5	8	8	5	-	7	5	7	8	2
5	8	5	3	6	7	-	6	6	9	7
6	8	5	7	8	5	6	-	2	5	7
7	8	7	7	6	7	6	2	-	7	5
8	7	8	6	3	8	9	5	7	-	6
9	7	8	6	7	2	7	7	5	6	-

≥ 42; SMAPO# 4505

	0	1	2	3	4	5	6	7	8	9
0	-	4	7	8	8	11	12	12	12	12
1	4	-	11	10	12	7	8	10	12	12
2	7	11	-	13	13	4	11	11	9	9
3	8	10	13	-	8	9	12	8	4	10
4	8	12	13	8	-	11	8	12	12	4
5	11	7	4	9	11	-	9	9	13	11
6	12	8	11	12	8	9	-	4	8	12
7	12	10	11	8	12	9	4	-	12	8
8	12	12	9	4	12	13	8	12	-	10
9	12	12	9	10	4	11	12	8	10	-

$\geq 64; (\frac{25}{72}, \frac{47}{72})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	3	4	5	3	4	4	5
1	2	-	4	5	4	3	5	4	4	5
2	4	4	-	5	2	3	3	4	4	5
3	3	5	5	-	3	2	4	3	3	4
4	4	4	2	3	-	5	5	4	4	3
5	5	3	3	2	5	-	4	3	3	4
6	3	5	3	4	5	4	-	1	3	2
7	4	4	4	3	4	3	1	-	2	3
8	4	4	4	3	4	3	3	2	-	1
9	5	3	5	4	3	4	2	3	1	-

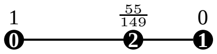
≥ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	3	4	6	8	8	8
1	1	-	3	5	4	3	7	9	9	7
2	2	3	-	6	1	4	6	10	10	8
3	4	5	6	-	5	2	10	6	6	10
4	3	4	1	5	-	5	7	11	11	7
5	4	3	4	2	5	-	8	8	8	8
6	6	7	6	10	7	8	-	4	12	8
7	8	9	10	6	11	8	4	-	8	12
8	8	9	10	6	11	8	12	8	-	4
9	8	7	8	10	7	8	8	12	4	-

≥ 42; SMAPO# 4983

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	6	7	9	9	12	12	13
1	3	-	7	9	8	6	12	13	13	10
2	6	7	-	10	3	7	9	14	14	13
3	6	9	10	-	7	3	13	8	8	13
4	7	8	3	7	-	10	12	15	15	10
5	9	6	7	3	10	-	12	11	11	12
6	9	12	9	13	12	12	-	5	15	10
7	12	13	14	8	15	11	5	-	10	15
8	12	13	14	8	15	11	15	10	-	5
9	13	10	13	13	10	12	10	15	5	-

$\geq 64; (\frac{50}{157}, \frac{107}{157})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	2	3	3	5	5	4
1	1	-	4	4	3	4	4	4	4	3
2	3	4	-	4	5	4	4	6	2	5
3	3	4	4	-	5	4	4	2	6	5
4	2	3	5	5	-	1	3	3	3	2
5	3	4	4	4	1	-	2	4	4	3
6	3	4	4	4	3	2	-	4	4	1
7	5	4	6	2	3	4	4	-	4	5
8	5	4	2	6	3	4	4	4	-	5
9	4	3	5	5	2	3	1	5	5	-

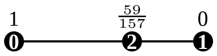
$\geq$ 24; SMAPO# 665

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	6	7	7	6	6	8
1	2	-	5	6	6	9	9	4	8	6
2	5	5	-	5	9	6	6	7	3	9
3	4	6	5	-	8	7	7	2	6	8
4	6	6	9	8	-	3	9	6	6	6
5	7	9	6	7	3	-	6	7	9	9
6	7	9	6	7	9	6	-	5	7	3
7	6	4	7	2	6	7	5	-	4	8
8	6	8	3	6	6	9	7	4	-	8
9	8	6	9	8	6	9	3	8	8	-

$\geq$ 42; SMAPO# 4551

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	8	10	10	11	11	12
1	3	-	8	10	9	13	13	8	12	9
2	7	8	-	8	13	9	9	12	4	13
3	7	10	8	-	13	11	11	4	12	13
4	8	9	13	13	-	4	12	9	9	8
5	10	13	9	11	4	-	8	11	13	12
6	10	13	9	11	12	8	-	9	11	4
7	11	8	12	4	9	11	9	-	8	13
8	11	12	4	12	9	13	11	8	-	13
9	12	9	13	13	8	12	4	13	13	-

$\geq 64; (\frac{55}{149}, \frac{94}{149})$



TT-disjoint at 4

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	4	4	4	3	3	4
1	1	-	3	3	3	5	3	2	4	5
2	2	3	-	2	6	4	6	1	3	4
3	2	3	2	-	6	4	6	3	1	4
4	4	3	6	6	-	6	4	5	5	2
5	4	5	4	4	6	-	2	3	5	4
6	4	3	6	6	4	2	-	5	5	6
7	3	2	1	3	5	3	5	-	2	3
8	3	4	3	1	5	5	5	2	-	5
9	4	5	4	4	2	4	6	3	5	-

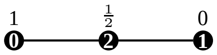
$\geq$ 24; SMAPO# 724

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	5	5	4	6	8	8	8
1	2	-	7	7	5	6	4	6	8	6
2	5	7	-	6	10	7	9	3	9	9
3	5	7	6	-	10	7	9	9	3	7
4	5	5	10	10	-	7	5	7	7	3
5	4	6	7	7	7	-	2	4	8	4
6	6	4	9	9	5	2	-	6	8	6
7	8	6	3	9	7	4	6	-	6	6
8	8	8	9	3	7	8	8	6	-	10
9	8	6	9	7	3	4	6	6	10	-

$\geq$ 42; SMAPO# 4653

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	8	8	10	11	11	12
1	3	-	10	10	7	11	7	8	12	11
2	7	10	-	8	15	11	15	4	12	13
3	7	10	8	-	15	11	15	12	4	11
4	8	7	15	15	-	12	8	11	11	4
5	8	11	11	11	12	-	4	7	13	8
6	10	7	15	15	8	4	-	11	13	12
7	11	8	4	12	11	7	11	-	8	9
8	11	12	12	4	11	13	13	8	-	15
9	12	11	13	11	4	8	12	9	15	-

$\geq 64; (\frac{59}{157}, \frac{98}{157})$



TT-disjoint at 5

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	9	10	11	11	13	13
1	3	-	10	10	10	7	8	14	10	14
2	7	10	-	8	14	3	8	12	14	10
3	7	10	8	-	14	11	4	10	14	14
4	9	10	14	14	-	13	10	12	8	4
5	10	7	3	11	13	-	7	9	13	13
6	11	8	8	4	10	7	-	14	14	12
7	11	14	12	10	12	9	14	-	4	8
8	13	10	14	14	8	13	14	4	-	12
9	13	14	10	14	4	13	12	8	12	-

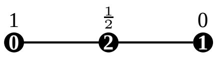
$\geq$ 64; SMAPO# 11099

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	9	11	11	11	13	13
1	3	-	10	10	10	8	8	14	10	14
2	7	10	-	8	14	4	12	12	14	10
3	7	10	8	-	14	12	4	10	14	14
4	9	10	14	14	-	10	10	12	8	4
5	11	8	4	12	10	-	8	10	14	14
6	11	8	12	4	10	8	-	14	14	12
7	11	14	12	10	12	10	14	-	4	8
8	13	10	14	14	8	14	14	4	-	12
9	13	14	10	14	4	14	12	8	12	-

$\geq$ 66; SMAPO# 11468

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	9	10	11	11	13	13
1	3	-	10	10	10	7	8	14	10	14
2	7	10	-	8	14	3	10	12	14	10
3	7	10	8	-	14	11	4	10	14	14
4	9	10	14	14	-	11	10	12	8	4
5	10	7	3	11	11	-	7	9	13	13
6	11	8	10	4	10	7	-	14	14	12
7	11	14	12	10	12	9	14	-	4	8
8	13	10	14	14	8	13	14	4	-	12
9	13	14	10	14	4	13	12	8	12	-

$\geq 64; (\frac{1}{2}, \frac{1}{2})$



TT-disjoint at 5

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	9	10	11	11	13	13
1	3	-	10	10	10	7	8	14	10	14
2	7	10	-	8	14	3	8	12	14	10
3	7	10	8	-	14	11	4	10	14	14
4	9	10	14	14	-	13	10	4	8	12
5	10	7	3	11	13	-	7	9	13	13
6	11	8	8	4	10	7	-	14	14	12
7	11	14	12	10	4	9	14	-	12	8
8	13	10	14	14	8	13	14	12	-	4
9	13	14	10	14	12	13	12	8	4	-

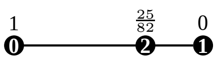
$\geq$ 64; SMAPO# 11100

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	9	11	11	11	13	13
1	3	-	10	10	10	8	8	14	10	14
2	7	10	-	8	14	4	12	12	14	10
3	7	10	8	-	14	12	4	10	14	14
4	9	10	14	14	-	10	10	4	8	12
5	11	8	4	12	10	-	8	10	14	14
6	11	8	12	4	10	8	-	14	14	12
7	11	14	12	10	4	10	14	-	12	8
8	13	10	14	14	8	14	14	12	-	4
9	13	14	10	14	12	14	12	8	4	-

$\geq$ 66; SMAPO# 11469

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	9	10	11	11	13	13
1	3	-	10	10	10	7	8	14	10	14
2	7	10	-	8	14	3	10	12	14	10
3	7	10	8	-	14	11	4	10	14	14
4	9	10	14	14	-	11	10	4	8	12
5	10	7	3	11	11	-	7	9	13	13
6	11	8	10	4	10	7	-	14	14	12
7	11	14	12	10	4	9	14	-	12	8
8	13	10	14	14	8	13	14	12	-	4
9	13	14	10	14	12	13	12	8	4	-

$\geq 64; (\frac{1}{2}, \frac{1}{2})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	3	3	4	5	5
1	1	-	3	4	4	2	3	3	4	4
2	2	3	-	5	5	1	4	4	3	3
3	3	4	5	-	4	4	5	3	4	2
4	3	4	5	4	-	4	3	5	2	4
5	3	2	1	4	4	-	3	3	4	4
6	4	3	4	5	3	3	-	2	5	3
7	4	3	4	3	5	3	2	-	3	5
8	5	4	3	4	2	4	5	3	-	4
9	5	4	3	2	4	4	3	5	4	-

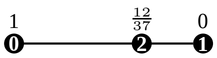
$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	7	6	7	9	9	8	8
1	2	-	7	7	8	5	7	7	8	10
2	5	7	-	10	9	2	8	10	7	7
3	7	7	10	-	5	8	8	6	5	3
4	6	8	9	5	-	9	5	7	2	8
5	7	5	2	8	9	-	6	8	9	9
6	9	7	8	8	5	6	-	2	7	5
7	9	7	10	6	7	8	2	-	5	7
8	8	8	7	5	2	9	7	5	-	6
9	8	10	7	3	8	9	5	7	6	-

$\geq$ 42; SMAPO# 4710

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	9	9	10	13	13	13	13
1	3	-	10	10	12	7	10	10	12	14
2	7	10	-	14	14	3	12	14	10	10
3	9	10	14	-	8	11	12	8	8	4
4	9	12	14	8	-	13	8	12	4	12
5	10	7	3	11	13	-	9	11	13	13
6	13	10	12	12	8	9	-	4	12	8
7	13	10	14	8	12	11	4	-	8	12
8	13	12	10	8	4	13	12	8	-	10
9	13	14	10	4	12	13	8	12	10	-

$\geq 64; (\frac{25}{82}, \frac{57}{82})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	2	2	3	3	5	3	4
1	1	-	4	3	3	2	2	4	2	3
2	3	4	-	5	3	4	4	2	4	5
3	2	3	5	-	4	3	1	3	3	2
4	2	3	3	4	-	3	3	5	1	2
5	3	2	4	3	3	-	4	4	2	1
6	3	2	4	1	3	4	-	4	4	3
7	5	4	2	3	5	4	4	-	4	3
8	3	2	4	3	1	2	4	4	-	3
9	4	3	5	2	2	1	3	3	3	-

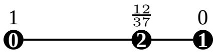
$\geq$ 20; SMAPO# 363

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	5	6	7	8	6	9	8
1	3	-	8	8	9	6	5	7	6	9
2	5	8	-	8	7	6	5	3	8	9
3	5	8	8	-	9	8	3	5	8	7
4	6	9	7	9	-	9	6	8	3	6
5	7	6	6	8	9	-	9	9	6	3
6	8	5	5	3	6	9	-	8	9	6
7	6	7	3	5	8	9	8	-	9	6
8	9	6	8	8	3	6	9	9	-	9
9	8	9	9	7	6	3	6	6	9	-

$\geq$ 46; SMAPO# 5697

	0	1	2	3	4	5	6	7	8	9
0	-	4	7	7	8	10	11	11	12	12
1	4	-	11	11	12	8	7	11	8	12
2	7	11	-	12	9	9	8	4	11	13
3	7	11	12	-	13	11	4	8	11	9
4	8	12	9	13	-	12	9	13	4	8
5	10	8	9	11	12	-	13	13	8	4
6	11	7	8	4	9	13	-	12	13	9
7	11	11	4	8	13	13	12	-	13	9
8	12	8	11	11	4	8	13	13	-	12
9	12	12	13	9	8	4	9	9	12	-

$\geq 64; (\frac{12}{37}, \frac{25}{37})$



TT-disjoint at 8

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	3	4	5	3
1	1	-	3	3	4	4	2	3	4	4
2	2	3	-	4	5	3	1	2	3	3
3	2	3	4	-	3	1	3	2	5	3
4	3	4	5	3	-	4	4	5	2	4
5	3	4	3	1	4	-	2	3	4	2
6	3	2	1	3	4	2	-	3	4	2
7	4	3	2	2	5	3	3	-	3	1
8	5	4	3	5	2	4	4	3	-	4
9	3	4	3	3	4	2	2	1	4	-

$\geq$ 20; SMAPO# 363

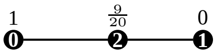
	0	1	2	3	4	5	6	7	8	9
0	-	3	5	6	5	7	8	8	8	9
1	3	-	8	7	8	8	5	5	5	8
2	5	8	-	9	8	6	3	7	7	10
3	6	7	9	-	5	3	8	6	8	9
4	5	8	8	5	-	8	7	7	3	8
5	7	8	6	3	8	-	7	9	5	6
6	8	5	3	8	7	7	-	10	10	7
7	8	5	7	6	7	9	10	-	6	3
8	8	5	7	8	3	5	10	6	-	9
9	9	8	10	9	8	6	7	3	9	-

$\geq$ 46; SMAPO# 5739

	0	1	2	3	4	5	6	7	8	9
0	-	4	7	8	8	10	11	12	12	12
1	4	-	11	10	12	12	7	8	8	12
2	7	11	-	13	13	9	4	9	9	13
3	8	10	13	-	8	4	11	8	12	12
4	8	12	13	8	-	12	11	12	4	12
5	10	12	9	4	12	-	9	12	8	8
6	11	7	4	11	11	9	-	13	13	9
7	12	8	9	8	12	12	13	-	8	4
8	12	8	9	12	4	8	13	8	-	12
9	12	12	13	12	12	8	9	4	12	-

$\geq 64; (\frac{12}{37}, \frac{25}{37})$

RHS = 66



TT-disjoint at 6

	0	1	2	3	4	5	6	7	8	9
0	-	0	2	4	3	4	6	4	6	6
1	0	-	2	4	3	4	6	4	6	6
2	2	2	-	6	5	6	4	6	4	8
3	4	4	6	-	1	6	2	4	6	4
4	3	3	5	1	-	7	3	3	5	5
5	4	4	6	6	7	-	6	8	2	6
6	6	6	4	2	3	6	-	6	4	4
7	4	4	6	4	3	8	6	-	6	2
8	6	6	4	6	5	2	4	6	-	8
9	6	6	8	4	5	6	4	2	8	-

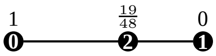
$\geq$ 28; SMAPO# 1414

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	4	6	5	6	8	6	8
1	2	-	4	6	4	5	8	6	8	10
2	2	4	-	4	6	7	4	8	4	8
3	4	6	4	-	2	7	2	8	6	8
4	6	4	6	2	-	9	4	6	8	10
5	5	5	7	7	9	-	5	11	3	7
6	6	8	4	2	4	5	-	10	6	6
7	8	6	8	8	6	11	10	-	8	4
8	6	8	4	6	8	3	6	8	-	10
9	8	10	8	8	10	7	6	4	10	-

$\geq$ 40; SMAPO# 4336

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	8	9	9	11	12	12	14
1	2	-	6	10	7	9	13	10	14	16
2	4	6	-	10	11	13	7	14	8	16
3	8	10	10	-	3	13	3	12	12	12
4	9	7	11	3	-	16	6	9	13	15
5	9	9	13	13	16	-	10	19	5	13
6	11	13	7	3	6	10	-	15	9	9
7	12	10	14	12	9	19	15	-	14	6
8	12	14	8	12	13	5	9	14	-	18
9	14	16	16	12	15	13	9	6	18	-

$\geq 66; (\frac{9}{20}, \frac{11}{20})$



TT-disjoint at 6

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	4	4	5	6	6
1	1	-	3	3	5	5	3	4	5	7
2	2	3	-	6	4	4	6	3	4	6
3	4	3	6	-	2	6	4	5	6	8
4	4	5	4	2	-	6	6	7	8	6
5	4	5	4	6	6	-	2	1	4	4
6	4	3	6	4	6	2	-	3	6	4
7	5	4	3	5	7	1	3	-	3	5
8	6	5	4	6	8	4	6	3	-	2
9	6	7	6	8	6	4	4	5	2	-

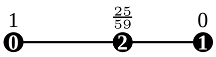
$\geq$ 28; SMAPO# 1389

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	5	5	5	7	7	9	9
1	1	-	3	4	6	6	6	6	8	10
2	2	3	-	7	5	7	9	5	7	11
3	5	4	7	-	2	6	6	8	10	10
4	5	6	5	2	-	8	6	10	12	8
5	5	6	7	6	8	-	2	2	8	8
6	7	6	9	6	6	2	-	4	10	6
7	7	6	5	8	10	2	4	-	6	10
8	9	8	7	10	12	8	10	6	-	4
9	9	10	11	10	8	8	6	10	4	-

$\geq$ 40; SMAPO# 4428

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	9	9	9	10	12	15	15
1	2	-	6	7	11	11	8	10	13	17
2	4	6	-	13	9	11	14	8	11	17
3	9	7	13	-	4	12	9	13	16	18
4	9	11	9	4	-	14	11	17	20	14
5	9	11	11	12	14	-	3	3	12	12
6	10	8	14	9	11	3	-	6	15	9
7	12	10	8	13	17	3	6	-	9	15
8	15	13	11	16	20	12	15	9	-	6
9	15	17	17	18	14	12	9	15	6	-

$\geq 66; (\frac{19}{48}, \frac{29}{48})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	4	4	4	6	6	6
1	1	-	3	1	3	3	5	5	5	7
2	2	3	-	4	6	6	4	4	4	6
3	2	1	4	-	2	2	4	4	4	6
4	4	3	6	2	-	4	2	6	6	8
5	4	3	6	2	4	-	6	6	2	4
6	4	5	4	4	2	6	-	8	8	6
7	6	5	4	4	6	6	8	-	4	2
8	6	5	4	4	6	2	8	4	-	6
9	6	7	6	6	8	4	6	2	6	-

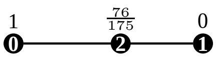
$\geq$ 28; SMAPO# 1405

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	5	6	7	7	6	6	7
1	2	-	4	3	6	5	9	8	8	9
2	2	4	-	7	8	9	5	6	6	5
3	5	3	7	-	3	4	6	5	7	8
4	6	6	8	3	-	7	3	6	8	9
5	7	5	9	4	7	-	10	9	3	6
6	7	9	5	6	3	10	-	9	9	6
7	6	8	6	5	6	9	9	-	6	3
8	6	8	6	7	8	3	9	6	-	9
9	7	9	5	8	9	6	6	3	9	-

$\geq$ 40; SMAPO# 4159

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	7	10	11	11	12	12	13
1	3	-	6	4	9	8	14	13	13	16
2	3	6	-	10	13	14	8	9	9	10
3	7	4	10	-	5	6	10	9	11	14
4	10	9	13	5	-	11	5	12	14	17
5	11	8	14	6	11	-	16	15	5	10
6	11	14	8	10	5	16	-	17	17	12
7	12	13	9	9	12	15	17	-	10	5
8	12	13	9	11	14	5	17	10	-	15
9	13	16	10	14	17	10	12	5	15	-

$\geq 66; (\frac{25}{59}, \frac{34}{59})$



TT-disjoint at 1

	0	1	2	3	4	5	6	7	8	9
0	-	2	1	4	5	4	6	4	4	6
1	2	-	3	4	3	6	4	4	6	6
2	1	3	-	5	4	3	5	5	3	7
3	4	4	5	-	1	2	4	6	6	4
4	5	3	4	1	-	3	3	7	5	5
5	4	6	3	2	3	-	6	6	4	4
6	6	4	5	4	3	6	-	8	6	2
7	4	4	5	6	7	6	8	-	2	6
8	4	6	3	6	5	4	6	2	-	8
9	6	6	7	4	5	4	2	6	8	-

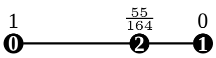
$\geq$ 28; SMAPO# 1389

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	4	4	6	6	8	8	7
1	2	-	4	6	4	8	6	6	10	5
2	2	4	-	6	6	4	8	10	6	9
3	4	6	6	-	2	2	6	8	8	5
4	4	4	6	2	-	4	4	8	8	7
5	6	8	4	2	4	-	8	10	6	5
6	6	6	8	6	4	8	-	10	8	3
7	8	6	10	8	8	10	10	-	4	7
8	8	10	6	8	8	6	8	4	-	11
9	7	5	9	5	7	5	3	7	11	-

$\geq$ 40; SMAPO# 4336

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	8	9	10	12	12	12	13
1	3	-	6	9	6	13	9	9	15	10
2	3	6	-	11	10	7	13	15	9	16
3	8	9	11	-	3	4	10	14	14	9
4	9	6	10	3	-	7	7	15	13	12
5	10	13	7	4	7	-	14	16	10	9
6	12	9	13	10	7	14	-	18	14	5
7	12	9	15	14	15	16	18	-	6	13
8	12	15	9	14	13	10	14	6	-	19
9	13	10	16	9	12	9	5	13	19	-

$\geq 66; (\frac{76}{175}, \frac{99}{175})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	2	5	5	3	3	4
1	1	-	4	4	3	4	4	4	4	3
2	3	4	-	4	5	6	2	4	4	5
3	3	4	4	-	5	2	6	4	4	5
4	2	3	5	5	-	3	3	1	3	2
5	5	4	6	2	3	-	4	4	4	5
6	5	4	2	6	3	4	-	4	4	5
7	3	4	4	4	1	4	4	-	2	3
8	3	4	4	4	3	4	4	2	-	1
9	4	3	5	5	2	5	5	3	1	-

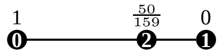
$\geq$ 24; SMAPO# 665

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	7	6	6	8	8	9
1	2	-	5	6	7	4	8	10	10	7
2	5	5	-	5	10	7	3	7	7	10
3	4	6	5	-	9	2	6	8	8	9
4	7	7	10	9	-	7	7	3	9	6
5	6	4	7	2	7	-	4	6	8	9
6	6	8	3	6	7	4	-	8	10	9
7	8	10	7	8	3	6	8	-	6	9
8	8	10	7	8	9	8	10	6	-	3
9	9	7	10	9	6	9	9	9	3	-

$\geq$ 44; SMAPO# 5213

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	9	11	11	11	11	13
1	3	-	8	10	10	8	12	14	14	10
2	7	8	-	8	14	12	4	10	10	14
3	7	10	8	-	14	4	12	12	12	14
4	9	10	14	14	-	10	10	4	12	8
5	11	8	12	4	10	-	8	10	12	14
6	11	12	4	12	10	8	-	12	14	14
7	11	14	10	12	4	10	12	-	8	12
8	11	14	10	12	12	12	14	8	-	4
9	13	10	14	14	8	14	14	12	4	-

$\geq 66; (\frac{55}{164}, \frac{109}{164})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	3	4	4	5	5
1	1	-	3	4	4	2	3	3	4	4
2	2	3	-	5	5	1	4	4	3	3
3	3	4	5	-	4	4	5	3	2	4
4	3	4	5	4	-	4	3	5	4	2
5	3	2	1	4	4	-	3	3	4	4
6	4	3	4	5	3	3	-	2	3	5
7	4	3	4	3	5	3	2	-	5	3
8	5	4	3	2	4	4	3	5	-	4
9	5	4	3	4	2	4	5	3	4	-

$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	7	6	8	9	9	8	8
1	3	-	8	8	9	5	6	8	9	9
2	5	8	-	10	9	3	8	10	7	7
3	7	8	10	-	5	7	8	6	3	5
4	6	9	9	5	-	8	5	7	8	2
5	8	5	3	7	8	-	7	7	10	8
6	9	6	8	8	5	7	-	2	5	7
7	9	8	10	6	7	7	2	-	7	5
8	8	9	7	3	8	10	5	7	-	6
9	8	9	7	5	2	8	7	5	6	-

$\geq$ 44; SMAPO# 5203

	0	1	2	3	4	5	6	7	8	9
0	-	4	7	9	9	11	13	13	13	13
1	4	-	11	11	13	7	9	11	13	13
2	7	11	-	14	14	4	12	14	10	10
3	9	11	14	-	8	10	12	8	4	8
4	9	13	14	8	-	12	8	12	12	4
5	11	7	4	10	12	-	10	10	14	12
6	13	9	12	12	8	10	-	4	8	12
7	13	11	14	8	12	10	4	-	12	8
8	13	13	10	4	12	14	8	12	-	10
9	13	13	10	8	4	12	12	8	10	-

$\geq$ 66; $(\frac{50}{159}, \frac{109}{159})$

RHS = 68

TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	4	2	3	4
1	1	-	3	3	4	2	3	3	4	3
2	2	3	-	4	3	3	2	4	5	4
3	2	3	4	-	5	3	4	4	3	2
4	3	4	3	5	-	4	5	3	2	3
5	3	2	3	3	4	-	3	1	4	3
6	4	3	2	4	5	3	-	2	3	4
7	2	3	4	4	3	1	2	-	3	2
8	3	4	5	3	2	4	3	3	-	5
9	4	3	4	2	3	3	4	2	5	-

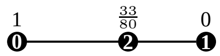
$\geq$ 22; SMAPO# 464

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	6	5	6	6	8	8	8
1	1	-	4	5	6	5	5	9	9	9
2	3	4	-	9	6	9	3	9	9	9
3	6	5	9	-	11	8	8	12	8	4
4	5	6	6	11	-	11	9	7	3	7
5	6	5	9	8	11	-	8	4	12	12
6	6	5	3	8	9	8	-	6	6	10
7	8	9	9	12	7	4	6	-	10	8
8	8	9	9	8	3	12	6	10	-	10
9	8	9	9	4	7	12	10	8	10	-

$\geq$ 48; SMAPO# 6826

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	7	8	9	10	10	11	12
1	2	-	7	7	10	7	8	12	13	12
2	5	7	-	12	9	12	5	13	14	13
3	7	7	12	-	15	10	11	15	10	5
4	8	10	9	15	-	15	14	10	5	10
5	9	7	12	10	15	-	11	5	16	15
6	10	8	5	11	14	11	-	8	9	14
7	10	12	13	15	10	5	8	-	13	10
8	11	13	14	10	5	16	9	13	-	15
9	12	12	13	5	10	15	14	10	15	-

$\geq$ 68; $(\frac{43}{158}, \frac{115}{158})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	4	4	5	5	4	4
1	1	-	4	4	3	3	4	4	5	5
2	3	4	-	4	7	7	6	2	5	5
3	3	4	4	-	7	7	2	6	5	5
4	4	3	7	7	-	4	5	5	2	6
5	4	3	7	7	4	-	5	5	6	2
6	5	4	6	2	5	5	-	4	5	5
7	5	4	2	6	5	5	4	-	5	5
8	4	5	5	5	2	6	5	5	-	4
9	4	5	5	5	6	2	5	5	4	-

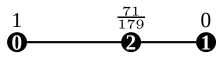
$\geq$ 30; SMAPO# 1506

	0	1	2	3	4	5	6	7	8	9
0	-	1	5	4	6	6	6	6	7	7
1	1	-	4	5	5	5	5	7	8	8
2	5	4	-	5	9	9	7	3	6	6
3	4	5	5	-	8	8	2	6	7	7
4	6	5	9	8	-	6	6	6	3	9
5	6	5	9	8	6	-	8	8	9	3
6	6	5	7	2	6	8	-	4	7	5
7	6	7	3	6	6	8	4	-	9	7
8	7	8	6	7	3	9	7	9	-	6
9	7	8	6	7	9	3	5	7	6	-

$\geq$ 40; SMAPO# 3975

	0	1	2	3	4	5	6	7	8	9
0	-	2	7	7	10	10	11	11	11	11
1	2	-	7	9	8	8	9	11	13	13
2	7	7	-	8	15	15	12	4	10	10
3	7	9	8	-	15	15	4	12	12	12
4	10	8	15	15	-	10	11	11	5	15
5	10	8	15	15	10	-	13	13	15	5
6	11	9	12	4	11	13	-	8	12	10
7	11	11	4	12	11	13	8	-	14	12
8	11	13	10	12	5	15	12	14	-	10
9	11	13	10	12	15	5	10	12	10	-

$\geq$ 68; $(\frac{33}{80}, \frac{47}{80})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	6	4	4	6	4
1	1	-	3	3	3	5	5	3	5	5
2	2	3	-	6	6	4	4	6	4	4
3	4	3	6	-	4	2	2	4	6	6
4	4	3	6	4	-	6	2	4	2	6
5	6	5	4	2	6	-	4	6	4	8
6	4	5	4	2	2	4	-	6	4	4
7	4	3	6	4	4	6	6	-	6	2
8	6	5	4	6	2	4	4	6	-	8
9	4	5	4	6	6	8	4	2	8	-

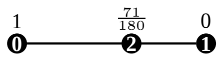
$\geq$ 28; SMAPO# 1404

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	6	7	5	7	8	6	8
1	2	-	4	6	5	7	9	6	8	10
2	2	4	-	8	9	5	5	10	6	6
3	6	6	8	-	7	3	3	8	6	10
4	7	5	9	7	-	6	4	7	3	11
5	5	7	5	3	6	-	6	9	3	9
6	7	9	5	3	4	6	-	11	5	7
7	8	6	10	8	7	9	11	-	10	4
8	6	8	6	6	3	3	5	10	-	10
9	8	10	6	10	11	9	7	4	10	-

$\geq$ 42; SMAPO# 4862

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	10	11	11	11	12	12	12
1	3	-	6	9	8	12	14	9	13	15
2	3	6	-	13	14	8	8	15	9	9
3	10	9	13	-	11	5	5	12	12	16
4	11	8	14	11	-	12	6	11	5	17
5	11	12	8	5	12	-	10	15	7	17
6	11	14	8	5	6	10	-	17	9	11
7	12	9	15	12	11	15	17	-	16	6
8	12	13	9	12	5	7	9	16	-	18
9	12	15	9	16	17	17	11	6	18	-

$\geq$ 68; $(\frac{71}{179}, \frac{108}{179})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	4	4	6	6	4
1	1	-	3	3	3	5	3	5	5	5
2	2	3	-	6	6	4	6	4	4	4
3	4	3	6	-	4	2	4	2	6	6
4	4	3	6	4	-	2	4	6	2	6
5	4	5	4	2	2	-	6	4	4	4
6	4	3	6	4	4	6	-	6	6	2
7	6	5	4	2	6	4	6	-	4	8
8	6	5	4	6	2	4	6	4	-	8
9	4	5	4	6	6	4	2	8	8	-

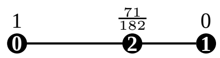
$\geq$ 28; SMAPO# 1404

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	6	7	7	8	6	6	8
1	2	-	4	6	5	9	6	6	8	10
2	2	4	-	8	9	5	10	6	6	6
3	6	6	8	-	5	3	8	2	6	10
4	7	5	9	5	-	4	7	7	3	11
5	7	9	5	3	4	-	11	5	7	7
6	8	6	10	8	7	11	-	8	10	4
7	6	6	6	2	7	5	8	-	4	10
8	6	8	6	6	3	7	10	4	-	10
9	8	10	6	10	11	7	4	10	10	-

$\geq$ 42; SMAPO# 4864

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	10	11	11	12	12	12	12
1	3	-	6	9	8	14	9	11	13	15
2	3	6	-	13	14	8	15	9	9	9
3	10	9	13	-	9	5	12	4	12	16
4	11	8	14	9	-	6	11	13	5	17
5	11	14	8	5	6	-	17	9	11	11
6	12	9	15	12	11	17	-	14	16	6
7	12	11	9	4	13	9	14	-	8	18
8	12	13	9	12	5	11	16	8	-	18
9	12	15	9	16	17	11	6	18	18	-

$\geq$ 68; $(\frac{71}{180}, \frac{109}{180})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	4	4	6	6	4
1	1	-	3	3	3	5	3	5	5	5
2	2	3	-	6	6	4	6	4	4	4
3	4	3	6	-	4	2	4	2	6	6
4	4	3	6	4	-	2	4	6	2	6
5	4	5	4	2	2	-	6	4	4	4
6	4	3	6	4	4	6	-	6	6	2
7	6	5	4	2	6	4	6	-	4	8
8	6	5	4	6	2	4	6	4	-	8
9	4	5	4	6	6	4	2	8	8	-

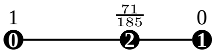
$\geq$ 28; SMAPO# 1404

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	6	7	7	8	6	6	8
1	2	-	4	6	5	9	6	8	8	10
2	2	4	-	8	9	5	10	6	6	6
3	6	6	8	-	5	3	8	2	6	10
4	7	5	9	5	-	4	7	7	3	11
5	7	9	5	3	4	-	11	5	5	7
6	8	6	10	8	7	11	-	10	10	4
7	6	8	6	2	7	5	10	-	4	10
8	6	8	6	6	3	5	10	4	-	10
9	8	10	6	10	11	7	4	10	10	-

$\geq$ 42; SMAPO# 4865

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	10	11	11	12	12	12	12
1	3	-	6	9	8	14	9	13	13	15
2	3	6	-	13	14	8	15	9	9	9
3	10	9	13	-	9	5	12	4	12	16
4	11	8	14	9	-	6	11	13	5	17
5	11	14	8	5	6	-	17	9	9	11
6	12	9	15	12	11	17	-	16	16	6
7	12	13	9	4	13	9	16	-	8	18
8	12	13	9	12	5	9	16	8	-	18
9	12	15	9	16	17	11	6	18	18	-

$\geq$ 68; $(\frac{71}{182}, \frac{111}{182})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	4	4	6	4	6
1	1	-	3	3	3	5	3	5	5	5
2	2	3	-	6	6	4	6	4	4	4
3	4	3	6	-	4	2	4	2	6	6
4	4	3	6	4	-	2	4	6	6	2
5	4	5	4	2	2	-	6	4	4	4
6	4	3	6	4	4	6	-	6	2	6
7	6	5	4	2	6	4	6	-	8	4
8	4	5	4	6	6	4	2	8	-	8
9	6	5	4	6	2	4	6	4	8	-

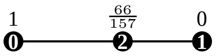
$\geq$ 28; SMAPO# 1404

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	6	7	7	8	6	8	7
1	2	-	4	6	5	9	6	8	10	7
2	2	4	-	8	9	5	10	6	6	7
3	6	6	8	-	5	3	8	2	10	7
4	7	5	9	5	-	4	7	7	11	2
5	7	9	5	3	4	-	11	5	7	6
6	8	6	10	8	7	11	-	10	4	9
7	6	8	6	2	7	5	10	-	10	5
8	8	10	6	10	11	7	4	10	-	11
9	7	7	7	7	2	6	9	5	11	-

$\geq$ 42; SMAPO# 4925

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	10	11	11	12	12	12	13
1	3	-	6	9	8	14	9	13	15	12
2	3	6	-	13	14	8	15	9	9	10
3	10	9	13	-	9	5	12	4	16	13
4	11	8	14	9	-	6	11	13	17	4
5	11	14	8	5	6	-	17	9	11	10
6	12	9	15	12	11	17	-	16	6	15
7	12	13	9	4	13	9	16	-	18	9
8	12	15	9	16	17	11	6	18	-	19
9	13	12	10	13	4	10	15	9	19	-

$\geq 68; (\frac{71}{185}, \frac{114}{185})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	5	5	4	4	4	4
1	1	-	4	4	4	4	5	5	3	3
2	3	4	-	4	6	2	5	5	7	7
3	3	4	4	-	2	6	5	5	7	7
4	5	4	6	2	-	4	5	5	5	5
5	5	4	2	6	4	-	5	5	5	5
6	4	5	5	5	5	5	-	4	6	2
7	4	5	5	5	5	5	4	-	2	6
8	4	3	7	7	5	5	6	2	-	4
9	4	3	7	7	5	5	2	6	4	-

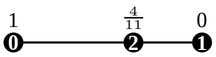
$\geq$ 30; SMAPO# 1506

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	3	5	5	6	6	7	7
1	2	-	4	5	5	7	8	8	5	5
2	4	4	-	5	7	3	6	6	9	9
3	3	5	5	-	2	6	7	7	8	8
4	5	5	7	2	-	4	5	7	6	8
5	5	7	3	6	4	-	7	9	6	8
6	6	8	6	7	5	7	-	6	9	3
7	6	8	6	7	7	9	6	-	3	9
8	7	5	9	8	6	6	9	3	-	6
9	7	5	9	8	8	8	3	9	6	-

$\geq$ 40; SMAPO# 3979

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	6	10	10	10	10	11	11
1	3	-	7	9	9	11	13	13	8	8
2	6	7	-	8	12	4	10	10	15	15
3	6	9	8	-	4	12	12	12	15	15
4	10	9	12	4	-	8	10	12	11	13
5	10	11	4	12	8	-	12	14	11	13
6	10	13	10	12	10	12	-	10	15	5
7	10	13	10	12	12	14	10	-	5	15
8	11	8	15	15	11	11	15	5	-	10
9	11	8	15	15	13	13	5	15	10	-

$\geq 68; (\frac{66}{157}, \frac{91}{157})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	5	5	4	5	4
1	2	-	5	5	5	5	3	4	3	4
2	3	5	-	6	6	4	2	5	4	5
3	3	5	6	-	4	6	4	5	2	3
4	3	5	6	4	-	2	4	3	4	5
5	5	5	4	6	2	-	6	5	4	3
6	5	3	2	4	4	6	-	5	4	3
7	4	4	5	5	3	5	5	-	3	2
8	5	3	4	2	4	4	4	3	-	5
9	4	4	5	3	5	3	3	2	5	-

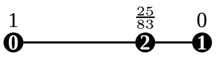
$\geq$ 28; SMAPO# 1227

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	6	6	5	6	7	7	8
1	2	-	6	8	8	7	4	5	5	8
2	4	6	-	8	8	5	2	7	7	8
3	6	8	8	-	6	9	6	9	3	8
4	6	8	8	6	-	3	6	5	9	8
5	5	7	5	9	3	-	7	8	6	5
6	6	4	2	6	6	7	-	9	9	6
7	7	5	7	9	5	8	9	-	6	3
8	7	5	7	3	9	6	9	6	-	9
9	8	8	8	8	8	5	6	3	9	-

$\geq$ 42; SMAPO# 4544

	0	1	2	3	4	5	6	7	8	9
0	-	4	7	8	9	10	11	11	12	12
1	4	-	11	12	13	12	7	9	8	12
2	7	11	-	13	14	9	4	12	11	13
3	8	12	13	-	9	14	9	13	4	10
4	9	13	14	9	-	5	10	8	13	13
5	10	12	9	14	5	-	13	13	10	8
6	11	7	4	9	10	13	-	14	13	9
7	11	9	12	13	8	13	14	-	9	5
8	12	8	11	4	13	10	13	9	-	14
9	12	12	13	10	13	8	9	5	14	-

$\geq 68; (\frac{4}{11}, \frac{7}{11})$



TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	4	3	3	4	4	5	5
1	2	-	4	4	5	5	4	4	3	3
2	4	4	-	2	3	5	4	4	5	3
3	4	4	2	-	5	3	4	4	3	5
4	3	5	3	5	-	4	3	3	2	4
5	3	5	5	3	4	-	1	3	4	2
6	4	4	4	4	3	1	-	2	3	3
7	4	4	4	4	3	3	2	-	3	1
8	5	3	5	3	2	4	3	3	-	4
9	5	3	3	5	4	2	3	1	4	-

$\geq$ 24; SMAPO# 662

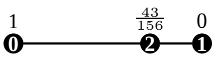
	0	1	2	3	4	5	6	7	8	9
0	-	3	5	5	6	7	8	8	8	11
1	3	-	6	8	7	8	11	11	5	8
2	5	6	-	2	5	8	9	9	7	8
3	5	8	2	-	7	6	9	9	5	10
4	6	7	5	7	-	9	8	8	2	11
5	7	8	8	6	9	-	3	9	9	6
6	8	11	9	9	8	3	-	6	6	9
7	8	11	9	9	8	9	6	-	6	3
8	8	5	7	5	2	9	6	6	-	9
9	11	8	8	10	11	6	9	3	9	-

$\geq$ 46; SMAPO# 6006

	0	1	2	3	4	5	6	7	8	9
0	-	4	8	8	8	9	11	11	12	15
1	4	-	10	12	12	13	15	15	8	11
2	8	10	-	4	8	13	13	13	12	11
3	8	12	4	-	12	9	13	13	8	15
4	8	12	8	12	-	13	11	11	4	15
5	9	13	13	9	13	-	4	12	13	8
6	11	15	13	13	11	4	-	8	9	12
7	11	15	13	13	11	12	8	-	9	4
8	12	8	12	8	4	13	9	9	-	13
9	15	11	11	15	15	8	12	4	13	-

$\geq 68; (\frac{25}{83}, \frac{58}{83})$

RHS = 70



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	2	4	3	4
1	1	-	3	3	4	4	3	3	2	3
2	2	3	-	4	3	5	4	2	3	4
3	2	3	4	-	5	3	4	4	3	2
4	3	4	3	5	-	2	3	5	4	3
5	3	4	5	3	2	-	3	3	4	5
6	2	3	4	4	3	3	-	2	1	2
7	4	3	2	4	5	3	2	-	3	4
8	3	2	3	3	4	4	1	3	-	3
9	4	3	4	2	3	5	2	4	3	-

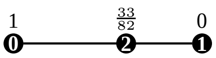
$\geq$ 22; SMAPO# 464

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	6	5	6	8	7	8	8
1	2	-	6	6	7	8	10	5	6	10
2	4	6	-	10	5	8	10	3	10	10
3	6	6	10	-	11	8	12	9	8	4
4	5	7	5	11	-	3	7	8	11	7
5	6	8	8	8	3	-	8	5	10	10
6	8	10	10	12	7	8	-	7	4	8
7	7	5	3	9	8	5	7	-	9	9
8	8	6	10	8	11	10	4	9	-	12
9	8	10	10	4	7	10	8	9	12	-

$\geq$ 50; SMAPO# 7345

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	7	8	9	10	11	11	12
1	3	-	9	8	11	12	13	8	8	13
2	6	9	-	13	8	13	14	5	13	14
3	7	8	13	-	15	10	15	12	10	5
4	8	11	8	15	-	5	10	13	15	10
5	9	12	13	10	5	-	11	8	14	15
6	10	13	14	15	10	11	-	9	5	10
7	11	8	5	12	13	8	9	-	12	13
8	11	8	13	10	15	14	5	12	-	15
9	12	13	14	5	10	15	10	13	15	-

$\geq 70; (\frac{43}{156}, \frac{113}{156})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	4	5	4	4	5	4
1	1	-	4	4	5	4	3	3	4	5
2	3	4	-	4	5	6	7	7	2	5
3	3	4	4	-	5	2	7	7	6	5
4	4	5	5	5	-	5	2	6	5	4
5	5	4	6	2	5	-	5	5	4	5
6	4	3	7	7	2	5	-	4	5	6
7	4	3	7	7	6	5	4	-	5	2
8	5	4	2	6	5	4	5	5	-	5
9	4	5	5	5	4	5	6	2	5	-

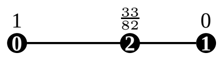
$\geq$ 30; SMAPO# 1506

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	6	6	7	7	6	8
1	2	-	5	6	8	4	5	5	8	8
2	5	5	-	5	7	7	10	10	3	7
3	4	6	5	-	6	2	9	9	6	8
4	6	8	7	6	-	4	3	9	6	6
5	6	4	7	2	4	-	7	7	4	8
6	7	5	10	9	3	7	-	6	9	9
7	7	5	10	9	9	7	6	-	7	3
8	6	8	3	6	6	4	9	7	-	10
9	8	8	7	8	6	8	9	3	10	-

$\geq$ 42; SMAPO# 4653

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	10	11	11	11	11	12
1	3	-	8	10	13	8	8	8	12	13
2	7	8	-	8	11	12	16	16	4	11
3	7	10	8	-	11	4	16	16	12	13
4	10	13	11	11	-	9	5	15	11	10
5	11	8	12	4	9	-	12	12	8	13
6	11	8	16	16	5	12	-	10	14	15
7	11	8	16	16	15	12	10	-	12	5
8	11	12	4	12	11	8	14	12	-	15
9	12	13	11	13	10	13	15	5	15	-

$\geq 70; (\frac{33}{82}, \frac{49}{82})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	4	5	4	4	5	4
1	1	-	4	4	5	4	3	3	4	5
2	3	4	-	4	5	6	7	7	2	5
3	3	4	4	-	5	2	7	7	6	5
4	4	5	5	5	-	5	2	6	5	4
5	5	4	6	2	5	-	5	5	4	5
6	4	3	7	7	2	5	-	4	5	6
7	4	3	7	7	6	5	4	-	5	2
8	5	4	2	6	5	4	5	5	-	5
9	4	5	5	5	4	5	6	2	5	-

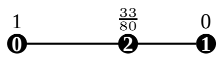
$\geq$ 30; SMAPO# 1506

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	6	6	7	7	6	8
1	2	-	5	6	8	4	5	5	8	8
2	5	5	-	5	7	7	10	10	3	7
3	4	6	5	-	6	2	9	9	6	8
4	6	8	7	6	-	4	3	9	8	6
5	6	4	7	2	4	-	7	7	4	6
6	7	5	10	9	3	7	-	6	9	9
7	7	5	10	9	9	7	6	-	7	3
8	6	8	3	6	8	4	9	7	-	10
9	8	8	7	8	6	6	9	3	10	-

$\geq$ 42; SMAPO# 4652

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	10	11	11	11	11	12
1	3	-	8	10	13	8	8	8	12	13
2	7	8	-	8	11	12	16	16	4	11
3	7	10	8	-	11	4	16	16	12	13
4	10	13	11	11	-	9	5	15	13	10
5	11	8	12	4	9	-	12	12	8	11
6	11	8	16	16	5	12	-	10	14	15
7	11	8	16	16	15	12	10	-	12	5
8	11	12	4	12	13	8	14	12	-	15
9	12	13	11	13	10	11	15	5	15	-

$\geq 70; (\frac{33}{82}, \frac{49}{82})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	4	5	5	4	4	4
1	1	-	4	4	3	4	4	5	5	3
2	3	4	-	4	7	6	2	5	5	7
3	3	4	4	-	7	2	6	5	5	7
4	4	3	7	7	-	5	5	2	6	4
5	5	4	6	2	5	-	4	5	5	5
6	5	4	2	6	5	4	-	5	5	5
7	4	5	5	5	2	5	5	-	4	6
8	4	5	5	5	6	5	5	4	-	2
9	4	3	7	7	4	5	5	6	2	-

$\geq$ 30; SMAPO# 1506

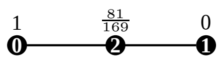
	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	6	6	6	7	7	8
1	2	-	5	6	6	4	8	9	9	6
2	5	5	-	5	9	7	3	6	6	9
3	4	6	5	-	8	2	6	7	7	8
4	6	6	9	8	-	6	6	3	9	6
5	6	4	7	2	6	-	4	7	5	8
6	6	8	3	6	6	4	-	9	7	8
7	7	9	6	7	3	7	9	-	6	9
8	7	9	6	7	9	5	7	6	-	3
9	8	6	9	8	6	8	8	9	3	-

$\geq$ 42; SMAPO# 4551

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	10	11	11	11	11	12
1	3	-	8	10	9	8	12	14	14	9
2	7	8	-	8	15	12	4	10	10	15
3	7	10	8	-	15	4	12	12	12	15
4	10	9	15	15	-	11	11	5	15	10
5	11	8	12	4	11	-	8	12	10	13
6	11	12	4	12	11	8	-	14	12	13
7	11	14	10	12	5	12	14	-	10	15
8	11	14	10	12	15	10	12	10	-	5
9	12	9	15	15	10	13	13	15	5	-

$\geq 70; (\frac{33}{80}, \frac{47}{80})$

RHS = 72



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	5	6	4	5	7	6
1	1	-	3	2	4	5	5	6	6	7
2	2	3	-	5	7	4	4	7	5	6
3	3	2	5	-	4	7	7	4	6	5
4	5	4	7	4	-	7	3	8	6	9
5	6	5	4	7	7	-	8	3	5	8
6	4	5	4	7	3	8	-	7	9	6
7	5	6	7	4	8	3	7	-	8	5
8	7	6	5	6	6	5	9	8	-	3
9	6	7	6	5	9	8	6	5	3	-

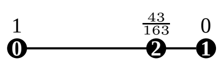
$\geq$ 36; SMAPO# 2790

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	4	6	5	7	6	6	7
1	2	-	4	2	4	5	7	6	6	7
2	2	4	-	6	8	5	5	8	6	5
3	4	2	6	-	4	7	7	4	6	5
4	6	4	8	4	-	7	3	8	6	9
5	5	5	5	7	7	-	8	3	5	8
6	7	7	5	7	3	8	-	9	9	6
7	6	6	8	4	8	3	9	-	8	5
8	6	6	6	6	6	5	9	8	-	3
9	7	7	5	5	9	8	6	5	3	-

$\geq$ 38; SMAPO# 3396

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	7	11	11	11	11	13	13
1	3	-	6	4	8	10	12	12	12	14
2	3	6	-	10	14	8	8	14	10	10
3	7	4	10	-	8	14	14	8	12	10
4	11	8	14	8	-	14	6	16	12	18
5	11	10	8	14	14	-	16	6	10	16
6	11	12	8	14	6	16	-	16	18	12
7	11	12	14	8	16	6	16	-	16	10
8	13	12	10	12	12	10	18	16	-	6
9	13	14	10	10	18	16	12	10	6	-

$\geq 72; (\frac{81}{169}, \frac{88}{169})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	2	3	3	4	4
1	1	-	3	3	4	3	2	4	3	3
2	2	3	-	4	5	4	3	3	4	2
3	2	3	4	-	3	4	3	5	2	4
4	3	4	5	3	-	3	4	2	5	3
5	2	3	4	4	3	-	1	3	2	2
6	3	2	3	3	4	1	-	4	3	3
7	3	4	3	5	2	3	4	-	3	5
8	4	3	4	2	5	2	3	3	-	4
9	4	3	2	4	3	2	3	5	4	-

$\geq$ 22; SMAPO# 464

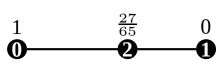
	0	1	2	3	4	5	6	7	8	9
0	-	2	6	5	5	8	8	8	8	8
1	2	-	6	7	7	10	6	8	6	10
2	6	6	-	11	11	12	8	8	8	4
3	5	7	11	-	6	11	11	9	3	11
4	5	7	11	6	-	7	11	3	9	7
5	8	10	12	11	7	-	4	10	8	8
6	8	6	8	11	11	4	-	10	8	12
7	8	8	8	9	3	10	10	-	6	10
8	8	6	8	3	9	8	8	6	-	10
9	8	10	4	11	7	8	12	10	10	-

$\geq$ 52; SMAPO# 7848

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	8	10	11	11	12	12
1	3	-	8	10	11	13	8	12	9	13
2	7	8	-	14	15	15	10	10	11	5
3	7	10	14	-	9	15	14	14	5	15
4	8	11	15	9	-	10	15	5	14	10
5	10	13	15	15	10	-	5	13	10	10
6	11	8	10	14	15	5	-	14	11	15
7	11	12	10	14	5	13	14	-	9	15
8	12	9	11	5	14	10	11	9	-	14
9	12	13	5	15	10	10	15	15	14	-

$\geq 72; (\frac{43}{163}, \frac{120}{163})$

RHS = 74



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	6	4	4	6	6	6
1	1	-	3	3	5	3	5	5	7	7
2	2	3	-	6	4	6	4	4	6	6
3	4	3	6	-	6	4	6	6	4	4
4	6	5	4	6	-	6	8	4	2	6
5	4	3	6	4	6	-	2	6	8	8
6	4	5	4	6	8	2	-	8	6	6
7	6	5	4	6	4	6	8	-	6	2
8	6	7	6	4	2	8	6	6	-	4
9	6	7	6	4	6	8	6	2	4	-

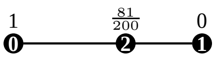
$\geq$ 32; SMAPO# 1859

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	7	5	8	8	7	7	8
1	2	-	4	5	7	6	10	7	9	10
2	2	4	-	9	5	10	6	7	7	6
3	7	5	9	-	6	7	11	8	4	5
4	5	7	5	6	-	9	9	4	2	7
5	8	6	10	7	9	-	4	9	11	12
6	8	10	6	11	9	4	-	11	9	8
7	7	7	7	8	4	9	11	-	6	3
8	7	9	7	4	2	11	9	6	-	5
9	8	10	6	5	7	12	8	3	5	-

$\geq$ 44; SMAPO# 5532

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	11	11	12	12	13	13	14
1	3	-	6	8	12	9	15	12	16	17
2	3	6	-	14	8	15	9	10	12	11
3	11	8	14	-	12	11	17	14	8	9
4	11	12	8	12	-	15	17	8	4	13
5	12	9	15	11	15	-	6	15	19	20
6	12	15	9	17	17	6	-	19	15	14
7	13	12	10	14	8	15	19	-	12	5
8	13	16	12	8	4	19	15	12	-	9
9	14	17	11	9	13	20	14	5	9	-

$\geq 74; (\frac{27}{65}, \frac{38}{65})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	6	4	6	6	6
1	1	-	3	3	3	5	5	5	7	7
2	2	3	-	6	6	4	4	4	6	6
3	4	3	6	-	4	6	6	6	4	4
4	4	3	6	4	-	6	2	6	8	8
5	6	5	4	6	6	-	8	4	6	2
6	4	5	4	6	2	8	-	8	6	6
7	6	5	4	6	6	4	8	-	2	6
8	6	7	6	4	8	6	6	2	-	4
9	6	7	6	4	8	2	6	6	4	-

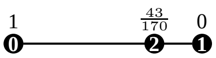
$\geq$ 32; SMAPO# 1859

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	7	8	6	8	7	7	8
1	2	-	4	5	6	8	10	7	9	10
2	2	4	-	9	10	6	6	7	7	6
3	7	5	9	-	7	7	11	6	4	5
4	8	6	10	7	-	10	4	9	11	12
5	6	8	6	7	10	-	10	5	7	2
6	8	10	6	11	4	10	-	11	9	8
7	7	7	7	6	9	5	11	-	2	7
8	7	9	7	4	11	7	9	2	-	5
9	8	10	6	5	12	2	8	7	5	-

$\geq$ 44; SMAPO# 5558

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	11	12	12	12	13	13	14
1	3	-	6	8	9	13	15	12	16	17
2	3	6	-	14	15	9	9	10	12	11
3	11	8	14	-	11	13	17	12	8	9
4	12	9	15	11	-	16	6	15	19	20
5	12	13	9	13	16	-	18	9	13	4
6	12	15	9	17	6	18	-	19	15	14
7	13	12	10	12	15	9	19	-	4	13
8	13	16	12	8	19	13	15	4	-	9
9	14	17	11	9	20	4	14	13	9	-

$\geq 74; (\frac{81}{200}, \frac{119}{200})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	2	4	3	4
1	1	-	3	3	4	4	3	3	2	3
2	2	3	-	4	3	5	4	2	3	4
3	2	3	4	-	5	3	4	4	3	2
4	3	4	3	5	-	2	3	5	4	3
5	3	4	5	3	2	-	3	3	4	5
6	2	3	4	4	3	3	-	2	1	2
7	4	3	2	4	5	3	2	-	3	4
8	3	2	3	3	4	4	1	3	-	3
9	4	3	4	2	3	5	2	4	3	-

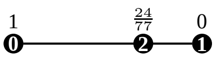
$\geq$ 22; SMAPO# 464

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	7	5	8	9	8	9	9
1	2	-	7	7	7	10	11	6	7	11
2	5	7	-	12	6	9	10	3	12	10
3	7	7	12	-	12	9	12	9	8	4
4	5	7	6	12	-	3	8	9	12	8
5	8	10	9	9	3	-	9	6	11	11
6	9	11	10	12	8	9	-	7	4	8
7	8	6	3	9	9	6	7	-	9	9
8	9	7	12	8	12	11	4	9	-	12
9	9	11	10	4	8	11	8	9	12	-

$\geq$ 54; SMAPO# 8451

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	8	8	11	11	12	12	13
1	3	-	10	9	11	14	14	9	9	14
2	7	10	-	15	9	14	14	5	15	14
3	8	9	15	-	16	11	15	12	10	5
4	8	11	9	16	-	5	11	14	16	11
5	11	14	14	11	5	-	12	9	15	16
6	11	14	14	15	11	12	-	9	5	10
7	12	9	5	12	14	9	9	-	12	13
8	12	9	15	10	16	15	5	12	-	15
9	13	14	14	5	11	16	10	13	15	-

$\geq 74; (\frac{43}{170}, \frac{127}{170})$



TT-disjoint at 4

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	3	3	4	5
1	1	-	3	3	4	2	2	2	3	4
2	2	3	-	4	3	3	1	3	2	5
3	2	3	4	-	5	3	3	1	2	3
4	3	4	3	5	-	4	4	4	5	2
5	3	2	3	3	4	-	2	4	1	4
6	3	2	1	3	4	2	-	4	3	4
7	3	2	3	1	4	4	4	-	3	4
8	4	3	2	2	5	1	3	3	-	3
9	5	4	5	3	2	4	4	4	3	-

$\geq$ 20; SMAPO# 363

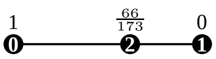
	0	1	2	3	4	5	6	7	8	9
0	-	4	6	6	7	8	10	10	10	9
1	4	-	10	10	9	8	6	6	8	9
2	6	10	-	10	7	10	4	8	6	9
3	6	10	10	-	11	8	8	4	8	7
4	7	9	7	11	-	7	9	7	11	4
5	8	8	10	8	7	-	6	10	4	11
6	10	6	4	8	9	6	-	8	10	9
7	10	6	8	4	7	10	8	-	8	11
8	10	8	6	8	11	4	10	8	-	7
9	9	9	9	7	4	11	9	11	7	-

$\geq$ 56; SMAPO# 8797

	0	1	2	3	4	5	6	7	8	9
0	-	5	8	8	9	11	13	13	14	14
1	5	-	13	13	12	10	8	8	11	13
2	8	13	-	14	9	13	5	11	8	14
3	8	13	14	-	15	11	11	5	10	10
4	9	12	9	15	-	10	12	10	15	5
5	11	10	13	11	10	-	8	14	5	15
6	13	8	5	11	12	8	-	12	13	13
7	13	8	11	5	10	14	12	-	11	15
8	14	11	8	10	15	5	13	11	-	10
9	14	13	14	10	5	15	13	15	10	-

$\geq 74; (\frac{24}{77}, \frac{53}{77})$

RHS = 76



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	5	5	4	4	4	4
1	1	-	4	4	4	4	3	3	5	5
2	3	4	-	4	6	2	7	7	5	5
3	3	4	4	-	2	6	7	7	5	5
4	5	4	6	2	-	4	5	5	5	5
5	5	4	2	6	4	-	5	5	5	5
6	4	3	7	7	5	5	-	4	6	2
7	4	3	7	7	5	5	4	-	2	6
8	4	5	5	5	5	5	6	2	-	4
9	4	5	5	5	5	5	2	6	4	-

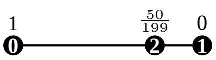
$\geq$ 30; SMAPO# 1506

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	6	6	8	8	8	8
1	2	-	5	6	4	8	6	6	10	10
2	5	5	-	5	7	3	11	11	7	7
3	4	6	5	-	2	6	10	10	8	8
4	6	4	7	2	-	4	8	8	6	8
5	6	8	3	6	4	-	8	10	8	10
6	8	6	11	10	8	8	-	8	12	4
7	8	6	11	10	8	10	8	-	4	12
8	8	10	7	8	6	8	12	4	-	8
9	8	10	7	8	8	10	4	12	8	-

$\geq$ 48; SMAPO# 6698

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	11	11	12	12	12	12
1	3	-	8	10	8	12	9	9	15	15
2	7	8	-	8	12	4	17	17	11	11
3	7	10	8	-	4	12	17	17	13	13
4	11	8	12	4	-	8	13	13	11	13
5	11	12	4	12	8	-	13	15	13	15
6	12	9	17	17	13	13	-	12	18	6
7	12	9	17	17	13	15	12	-	6	18
8	12	15	11	13	11	13	18	6	-	12
9	12	15	11	13	13	15	6	18	12	-

$\geq 76; (\frac{66}{173}, \frac{107}{173})$



TT-disjoint at 4

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	3	4	5	4	5
1	1	-	3	2	4	4	3	4	3	4
2	2	3	-	1	5	5	4	3	4	3
3	3	2	1	-	4	4	3	4	3	4
4	3	4	5	4	-	4	3	4	5	2
5	3	4	5	4	4	-	5	2	3	4
6	4	3	4	3	3	5	-	3	2	5
7	5	4	3	4	4	2	3	-	5	4
8	4	3	4	3	5	3	2	5	-	3
9	5	4	3	4	2	4	5	4	3	-

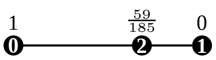
$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	2	6	8	9	8	11	11	12	11
1	2	-	8	6	9	10	9	9	10	13
2	6	8	-	2	13	12	11	9	10	9
3	8	6	2	-	11	12	11	11	8	11
4	9	9	13	11	-	7	8	8	11	4
5	8	10	12	12	7	-	9	3	6	11
6	11	9	11	11	8	9	-	6	3	10
7	11	9	9	11	8	3	6	-	9	8
8	12	10	10	8	11	6	3	9	-	7
9	11	13	9	11	4	11	10	8	7	-

$\geq$ 54; SMAPO# 8502

	0	1	2	3	4	5	6	7	8	9
0	-	3	8	11	11	11	15	16	16	16
1	3	-	11	8	12	14	12	13	13	17
2	8	11	-	3	17	17	15	12	14	12
3	11	8	3	-	14	16	14	15	11	15
4	11	12	17	14	-	10	10	11	15	5
5	11	14	17	16	10	-	14	5	9	15
6	15	12	15	14	10	14	-	9	5	15
7	16	13	12	15	11	5	9	-	14	12
8	16	13	14	11	15	9	5	14	-	10
9	16	17	12	15	5	15	15	12	10	-

$\geq 76; (\frac{50}{199}, \frac{149}{199})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	2	3	3	5	4	5
1	1	-	4	4	3	2	2	4	3	4
2	3	4	-	4	5	6	6	6	5	2
3	3	4	4	-	5	6	6	2	5	6
4	2	3	5	5	-	1	3	3	2	3
5	3	2	6	6	1	-	2	4	3	4
6	3	2	6	6	3	2	-	4	1	4
7	5	4	6	2	3	4	4	-	5	4
8	4	3	5	5	2	3	1	5	-	5
9	5	4	2	6	3	4	4	4	5	-

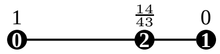
$\geq$ 24; SMAPO# 724

	0	1	2	3	4	5	6	7	8	9
0	-	2	6	5	8	8	9	8	9	8
1	2	-	6	7	10	6	7	6	9	10
2	6	6	-	7	10	12	13	10	9	4
3	5	7	7	-	9	11	12	3	10	9
4	8	10	10	9	-	4	11	6	7	8
5	8	6	12	11	4	-	7	10	11	10
6	9	7	13	12	11	7	-	9	4	9
7	8	6	10	3	6	10	9	-	11	6
8	9	9	9	10	7	11	4	11	-	13
9	8	10	4	9	8	10	9	6	13	-

$\geq$ 54; SMAPO# 8498

	0	1	2	3	4	5	6	7	8	9
0	-	3	8	8	10	11	12	13	13	13
1	3	-	9	11	13	8	9	10	12	14
2	8	9	-	10	14	17	18	15	13	5
3	8	11	10	-	14	17	18	5	15	15
4	10	13	14	14	-	5	14	9	9	11
5	11	8	17	17	5	-	9	14	14	14
6	12	9	18	18	14	9	-	13	5	13
7	13	10	15	5	9	14	13	-	16	10
8	13	12	13	15	9	14	5	16	-	18
9	13	14	5	15	11	14	13	10	18	-

$\geq 76; (\frac{59}{185}, \frac{126}{185})$



TT-disjoint at 1

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	4	4	5	4	5	4
1	2	-	5	5	6	6	3	4	3	4
2	3	5	-	4	5	5	2	3	4	5
3	3	5	4	-	5	5	4	5	2	3
4	4	6	5	5	-	4	3	2	5	6
5	4	6	5	5	4	-	5	6	3	2
6	5	3	2	4	3	5	-	5	4	3
7	4	4	3	5	2	6	5	-	3	4
8	5	3	4	2	5	3	4	3	-	5
9	4	4	5	3	6	2	3	4	5	-

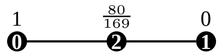
≥ 28; SMAPO# 1227

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	6	7	7	9	10	9	10
1	3	-	9	9	10	10	8	7	8	7
2	6	9	-	6	9	9	3	6	9	10
3	6	9	6	-	9	9	9	10	3	6
4	7	10	9	9	-	6	6	3	8	9
5	7	10	9	9	6	-	8	9	6	3
6	9	8	3	9	6	8	-	9	10	7
7	10	7	6	10	3	9	9	-	7	8
8	9	8	9	3	8	6	10	7	-	9
9	10	7	10	6	9	3	7	8	9	-

≥ 50; SMAPO# 7133

	0	1	2	3	4	5	6	7	8	9
0	-	4	9	9	11	11	14	14	14	14
1	4	-	13	13	15	15	10	10	10	10
2	9	13	-	10	14	14	5	9	13	15
3	9	13	10	-	14	14	13	15	5	9
4	11	15	14	14	-	10	9	5	13	15
5	11	15	14	14	10	-	13	15	9	5
6	14	10	5	13	9	13	-	14	14	10
7	14	10	9	15	5	15	14	-	10	12
8	14	10	13	5	13	9	14	10	-	14
9	14	10	15	9	15	5	10	12	14	-

$\geq 76; (\frac{14}{43}, \frac{29}{43})$



TT-disjoint at 8

	0	1	2	3	4	5	6	7	8	9
0	-	3	4	4	7	6	6	6	7	8
1	3	-	7	7	4	7	7	7	6	5
2	4	7	-	6	3	6	6	8	5	6
3	4	7	6	-	5	2	6	4	7	4
4	7	4	3	5	-	5	5	5	8	7
5	6	7	6	2	5	-	4	6	5	6
6	6	7	6	6	5	4	-	6	7	2
7	6	7	8	4	5	6	6	-	3	8
8	7	6	5	7	8	5	7	3	-	5
9	8	5	6	4	7	6	2	8	5	-

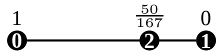
≥ 38; SMAPO# 3286

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	5	5	6	6	6	7	7
1	2	-	5	7	3	8	8	8	5	5
2	3	5	-	6	2	7	7	7	6	6
3	5	7	6	-	6	3	9	5	8	6
4	5	3	2	6	-	5	5	5	8	8
5	6	8	7	3	5	-	6	8	5	9
6	6	8	7	9	5	6	-	8	9	3
7	6	8	7	5	5	8	8	-	3	7
8	7	5	6	8	8	5	9	3	-	6
9	7	5	6	6	8	9	3	7	6	-

≥ 40; SMAPO# 3946

	0	1	2	3	4	5	6	7	8	9
0	-	5	7	9	12	12	12	12	13	15
1	5	-	12	14	7	15	15	15	10	10
2	7	12	-	12	5	13	13	15	10	12
3	9	14	12	-	11	5	15	9	14	10
4	12	7	5	11	-	10	10	10	15	15
5	12	15	13	5	10	-	10	14	9	15
6	12	15	13	15	10	10	-	14	15	5
7	12	15	15	9	10	14	14	-	5	15
8	13	10	10	14	15	9	15	5	-	10
9	15	10	12	10	15	15	5	15	10	-

$\geq 76; (\frac{80}{169}, \frac{89}{169})$



TT-disjoint at 8

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	4	5	5	4
1	2	-	5	3	5	3	4	3	3	4
2	3	5	-	4	4	4	5	2	4	3
3	3	3	4	-	4	2	3	4	4	1
4	3	5	4	4	-	4	3	4	2	5
5	3	3	4	2	4	-	1	4	4	3
6	4	4	5	3	3	1	-	3	5	2
7	5	3	2	4	4	4	3	-	4	5
8	5	3	4	4	2	4	5	4	-	3
9	4	4	3	1	5	3	2	5	3	-

≥ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	6	6	8	8	8	10	10
1	3	-	8	9	9	9	9	5	7	9
2	5	8	-	11	11	9	11	3	7	7
3	6	9	11	-	10	6	10	8	10	4
4	6	9	11	10	-	12	8	8	4	10
5	8	9	9	6	12	-	4	8	8	10
6	8	9	11	10	8	4	-	8	12	6
7	8	5	3	8	8	8	8	-	8	10
8	10	7	7	10	4	8	12	8	-	8
9	10	9	7	4	10	10	6	10	8	-

≥ 54; SMAPO# 8330

	0	1	2	3	4	5	6	7	8	9
0	-	5	8	9	9	11	12	13	14	14
1	5	-	13	12	14	12	13	8	9	13
2	8	13	-	15	15	13	16	5	10	10
3	9	12	15	-	14	8	13	12	13	5
4	9	14	15	14	-	16	11	12	5	15
5	11	12	13	8	16	-	5	12	11	13
6	12	13	16	13	11	5	-	11	16	8
7	13	8	5	12	12	12	11	-	11	15
8	14	9	10	13	5	11	16	11	-	10
9	14	13	10	5	15	13	8	15	10	-

$\geq 76; (\frac{50}{167}, \frac{117}{167})$

TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	2	3	3	3	5	4
1	1	-	3	4	3	2	2	2	4	3
2	2	3	-	5	4	3	1	3	3	2
3	3	4	5	-	3	4	4	4	2	5
4	2	3	4	3	-	3	3	1	5	2
5	3	2	3	4	3	-	4	2	4	1
6	3	2	1	4	3	4	-	4	4	3
7	3	2	3	4	1	2	4	-	4	3
8	5	4	3	2	5	4	4	4	-	3
9	4	3	2	5	2	1	3	3	3	-

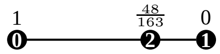
≥ 20; SMAPO# 363

	0	1	2	3	4	5	6	7	8	9
0	-	4	6	7	7	8	10	11	9	10
1	4	-	10	11	11	8	6	7	9	10
2	6	10	-	11	11	10	4	9	7	8
3	7	11	11	-	8	7	7	10	4	11
4	7	11	11	8	-	11	7	4	10	7
5	8	8	10	7	11	-	10	7	11	4
6	10	6	4	7	7	10	-	9	11	8
7	11	7	9	10	4	7	9	-	10	11
8	9	9	7	4	10	11	11	10	-	7
9	10	10	8	11	7	4	8	11	7	-

≥ 58; SMAPO# 9365

	0	1	2	3	4	5	6	7	8	9
0	-	5	8	9	9	11	13	14	14	14
1	5	-	13	14	14	10	8	9	13	13
2	8	13	-	15	15	13	5	12	10	10
3	9	14	15	-	10	10	10	13	5	15
4	9	14	15	10	-	14	10	5	15	9
5	11	10	13	10	14	-	14	9	15	5
6	13	8	5	10	10	14	-	13	15	11
7	14	9	12	13	5	9	13	-	14	14
8	14	13	10	5	15	15	15	14	-	10
9	14	13	10	15	9	5	11	14	10	-

$\geq 76; (\frac{16}{55}, \frac{39}{55})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	2	3	3	3	4	5
1	1	-	3	4	3	2	2	2	3	4
2	2	3	-	5	4	1	3	3	2	3
3	3	4	5	-	3	4	4	4	5	2
4	2	3	4	3	-	3	3	1	2	5
5	3	2	1	4	3	-	4	4	3	4
6	3	2	3	4	3	4	-	2	1	4
7	3	2	3	4	1	4	2	-	3	4
8	4	3	2	5	2	3	1	3	-	3
9	5	4	3	2	5	4	4	4	3	-

≥ 20; SMAPO# 363

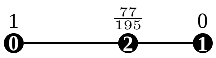
	0	1	2	3	4	5	6	7	8	9
0	-	4	6	7	7	10	10	11	10	9
1	4	-	10	9	11	6	8	7	8	9
2	6	10	-	11	11	4	10	9	8	7
3	7	9	11	-	8	7	7	8	11	4
4	7	11	11	8	-	9	11	4	7	10
5	10	6	4	7	9	-	10	9	8	11
6	10	8	10	7	11	10	-	7	4	11
7	11	7	9	8	4	9	7	-	11	10
8	10	8	8	11	7	8	4	11	-	7
9	9	9	7	4	10	11	11	10	7	-

≥ 58; SMAPO# 9366

	0	1	2	3	4	5	6	7	8	9
0	-	5	8	9	9	13	13	14	14	14
1	5	-	13	12	14	8	10	9	11	13
2	8	13	-	15	15	5	13	12	10	10
3	9	12	15	-	10	10	10	11	15	5
4	9	14	15	10	-	12	14	5	9	15
5	13	8	5	10	12	-	14	13	11	15
6	13	10	13	10	14	14	-	9	5	15
7	14	9	12	11	5	13	9	-	14	14
8	14	11	10	15	9	11	5	14	-	10
9	14	13	10	5	15	15	15	14	10	-

$\geq 76; (\frac{48}{163}, \frac{115}{163})$

RHS = 78



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	0	2	3	4	4	6	6	6	6
1	0	-	2	3	4	4	6	6	6	6
2	2	2	-	3	6	6	4	4	6	6
3	3	3	3	-	5	5	5	5	3	3
4	4	4	6	5	-	4	6	2	4	6
5	4	4	6	5	4	-	6	6	6	2
6	6	6	4	5	6	6	-	4	2	6
7	6	6	4	5	2	6	4	-	6	4
8	6	6	6	3	4	6	2	6	-	4
9	6	6	6	3	6	2	6	4	4	-

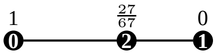
$\geq$ 28; SMAPO# 1264

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	7	6	9	7	8	9	9
1	2	-	5	5	8	7	9	10	11	11
2	3	5	-	4	9	8	6	7	10	10
3	7	5	4	-	9	10	10	9	6	6
4	6	8	9	9	-	7	11	4	7	11
5	9	7	8	10	7	-	10	11	10	4
6	7	9	6	10	11	10	-	7	4	10
7	8	10	7	9	4	11	7	-	11	7
8	9	11	10	6	7	10	4	11	-	8
9	9	11	10	6	11	4	10	7	8	-

$\geq$ 52; SMAPO# 7787

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	10	10	13	13	14	15	15
1	2	-	6	8	12	11	15	16	17	17
2	4	6	-	6	14	13	9	10	15	15
3	10	8	6	-	14	15	15	14	9	9
4	10	12	14	14	-	11	17	6	11	17
5	13	11	13	15	11	-	16	17	16	6
6	13	15	9	15	17	16	-	11	6	16
7	14	16	10	14	6	17	11	-	17	11
8	15	17	15	9	11	16	6	17	-	12
9	15	17	15	9	17	6	16	11	12	-

$\geq 78; (\frac{77}{195}, \frac{118}{195})$



TT-disjoint at 1

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	5	5	5	5	5	5
1	1	-	4	4	4	6	4	6	6	6
2	3	4	-	4	6	8	2	6	6	6
3	3	4	4	-	6	8	6	6	2	6
4	5	4	6	6	-	4	6	6	6	2
5	5	6	8	8	4	-	6	2	6	6
6	5	4	2	6	6	6	-	4	4	4
7	5	6	6	6	6	2	4	-	8	4
8	5	6	6	2	6	6	4	8	-	8
9	5	6	6	6	2	6	4	4	8	-

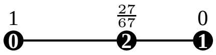
$\geq$ 32; SMAPO# 1863

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	5	7	7	8	8	8	8
1	2	-	5	7	7	9	8	6	6	10
2	5	5	-	6	8	10	3	7	9	11
3	5	7	6	-	10	10	9	9	3	11
4	7	7	8	10	-	6	11	9	7	3
5	7	9	10	10	6	-	11	3	7	9
6	8	8	3	9	11	11	-	8	6	8
7	8	6	7	9	9	3	8	-	10	6
8	8	6	9	3	7	7	6	10	-	10
9	8	10	11	11	3	9	8	6	10	-

$\geq$ 48; SMAPO# 6631

	0	1	2	3	4	5	6	7	8	9
0	-	2	8	8	12	12	13	13	13	13
1	2	-	8	10	10	14	11	11	11	15
2	8	8	-	10	14	18	5	13	15	17
3	8	10	10	-	16	18	15	15	5	17
4	12	10	14	16	-	10	17	15	13	5
5	12	14	18	18	10	-	17	5	13	15
6	13	11	5	15	17	17	-	12	10	12
7	13	11	13	15	15	5	12	-	18	10
8	13	11	15	5	13	13	10	18	-	18
9	13	15	17	17	5	15	12	10	18	-

$\geq 78; (\frac{27}{67}, \frac{40}{67})$



TT-disjoint at 1

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	5	5	5	5	5	5
1	1	-	4	4	4	6	4	6	6	6
2	3	4	-	4	6	8	2	6	6	6
3	3	4	4	-	6	8	6	6	2	6
4	5	4	6	6	-	4	6	2	6	6
5	5	6	8	8	4	-	6	6	6	2
6	5	4	2	6	6	6	-	4	4	4
7	5	6	6	6	2	6	4	-	8	4
8	5	6	6	2	6	6	4	8	-	8
9	5	6	6	6	6	2	4	4	8	-

$\geq$ 32; SMAPO# 1863

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	5	7	7	8	8	8	8
1	2	-	5	7	7	9	8	6	6	10
2	5	5	-	6	8	10	3	7	9	11
3	5	7	6	-	10	10	9	9	3	11
4	7	7	8	10	-	6	11	3	7	9
5	7	9	10	10	6	-	11	9	7	3
6	8	8	3	9	11	11	-	8	6	8
7	8	6	7	9	3	9	8	-	10	6
8	8	6	9	3	7	7	6	10	-	10
9	8	10	11	11	9	3	8	6	10	-

$\geq$ 48; SMAPO# 6630

	0	1	2	3	4	5	6	7	8	9
0	-	2	8	8	12	12	13	13	13	13
1	2	-	8	10	10	14	11	11	11	15
2	8	8	-	10	14	18	5	13	15	17
3	8	10	10	-	16	18	15	15	5	17
4	12	10	14	16	-	10	17	5	13	15
5	12	14	18	18	10	-	17	15	13	5
6	13	11	5	15	17	17	-	12	10	12
7	13	11	13	15	5	15	12	-	18	10
8	13	11	15	5	13	13	10	18	-	18
9	13	15	17	17	15	5	12	10	18	-

$\geq 78; (\frac{27}{67}, \frac{40}{67})$

TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	0	1	1	2	3	2	3	2	3
1	0	-	1	1	2	3	2	3	2	3
2	1	1	-	2	3	2	3	2	3	4
3	1	1	2	-	3	2	1	2	3	2
4	2	2	3	3	-	3	2	1	2	3
5	3	3	2	2	3	-	1	2	3	4
6	2	2	3	1	2	1	-	3	2	3
7	3	3	2	2	1	2	3	-	3	2
8	2	2	3	3	2	3	2	3	-	1
9	3	3	4	2	3	4	3	2	1	-

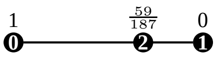
$\geq$ 14; SMAPO# 136

	0	1	2	3	4	5	6	7	8	9
0	-	3	4	9	9	9	10	10	12	11
1	3	-	7	6	10	12	13	13	9	14
2	4	7	-	5	11	7	12	8	10	13
3	9	6	5	-	12	12	7	11	13	8
4	9	10	11	12	-	14	9	5	9	14
5	9	12	7	12	14	-	5	9	13	14
6	10	13	12	7	9	5	-	14	12	11
7	10	13	8	11	5	9	14	-	14	9
8	12	9	10	13	9	13	12	14	-	5
9	11	14	13	8	14	14	11	9	5	-

$\geq$ 66; SMAPO# 11465

	0	1	2	3	4	5	6	7	8	9
0	-	3	4	10	11	12	12	13	14	14
1	3	-	7	7	12	15	15	16	11	17
2	4	7	-	6	13	8	14	9	12	16
3	10	7	6	-	15	14	8	13	16	10
4	11	12	13	15	-	17	11	6	11	17
5	12	15	8	14	17	-	6	11	16	18
6	12	15	14	8	11	6	-	17	14	14
7	13	16	9	13	6	11	17	-	17	11
8	14	11	12	16	11	16	14	17	-	6
9	14	17	16	10	17	18	14	11	6	-

$\geq 78; (\frac{19}{94}, \frac{75}{94})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	2	4	3	3	4	4
1	1	-	3	3	3	5	4	2	3	5
2	2	3	-	6	2	4	3	1	6	4
3	4	3	6	-	6	6	5	5	4	2
4	2	3	2	6	-	4	1	3	6	4
5	4	5	4	6	4	-	5	3	2	4
6	3	4	3	5	1	5	-	2	5	5
7	3	2	1	5	3	3	2	-	5	3
8	4	3	6	4	6	2	5	5	-	6
9	4	5	4	2	4	4	5	3	6	-

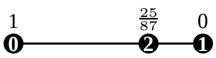
$\geq$ 24; SMAPO# 724

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	6	7	6	9	10	9	10
1	3	-	9	7	10	9	10	7	6	9
2	6	9	-	12	7	10	11	4	11	10
3	6	7	12	-	13	10	9	10	7	4
4	7	10	7	13	-	9	4	11	12	9
5	6	9	10	10	9	-	11	6	3	6
6	9	10	11	9	4	11	-	7	12	13
7	10	7	4	10	11	6	7	-	9	8
8	9	6	11	7	12	3	12	9	-	9
9	10	9	10	4	9	6	13	8	9	-

$\geq$ 56; SMAPO# 8969

	0	1	2	3	4	5	6	7	8	9
0	-	4	8	9	9	10	12	13	13	14
1	4	-	12	9	13	14	14	9	9	14
2	8	12	-	17	9	14	14	5	17	14
3	9	9	17	-	18	15	13	14	10	5
4	9	13	9	18	-	13	5	14	18	13
5	10	14	14	15	13	-	16	9	5	10
6	12	14	14	13	5	16	-	9	17	18
7	13	9	5	14	14	9	9	-	14	11
8	13	9	17	10	18	5	17	14	-	15
9	14	14	14	5	13	10	18	11	15	-

$\geq 78; (\frac{59}{187}, \frac{128}{187})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	3	4	4	5	5
1	1	-	3	4	4	2	3	3	4	4
2	2	3	-	5	5	1	4	4	3	3
3	3	4	5	-	4	4	3	5	2	4
4	3	4	5	4	-	4	5	3	4	2
5	3	2	1	4	4	-	3	3	4	4
6	4	3	4	3	5	3	-	2	5	3
7	4	3	4	5	3	3	2	-	3	5
8	5	4	3	2	4	4	5	3	-	4
9	5	4	3	4	2	4	3	5	4	-

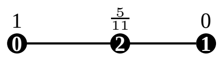
$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	4	6	8	7	10	10	11	10	10
1	4	-	10	10	11	6	10	7	10	10
2	6	10	-	12	11	4	10	9	8	8
3	8	10	12	-	7	8	8	11	4	8
4	7	11	11	7	-	9	9	6	11	3
5	10	6	4	8	9	-	8	9	12	8
6	10	10	10	8	9	8	-	3	10	6
7	11	7	9	11	6	9	3	-	7	9
8	10	10	8	4	11	12	10	7	-	8
9	10	10	8	8	3	8	6	9	8	-

$\geq$ 56; SMAPO# 8806

	0	1	2	3	4	5	6	7	8	9
0	-	5	8	10	10	13	14	15	15	15
1	5	-	13	13	15	8	13	10	14	14
2	8	13	-	16	16	5	14	13	11	11
3	10	13	16	-	10	11	10	15	5	11
4	10	15	16	10	-	13	14	9	15	5
5	13	8	5	11	13	-	11	12	16	12
6	14	13	14	10	14	11	-	5	15	9
7	15	10	13	15	9	12	5	-	10	14
8	15	14	11	5	15	16	15	10	-	12
9	15	14	11	11	5	12	9	14	12	-

$\geq 78; (\frac{25}{87}, \frac{62}{87})$



TT-disjoint at 7

	0	1	2	3	4	5	6	7	8	9
0	-	3	4	4	6	6	7	7	6	8
1	3	-	7	7	7	7	4	6	7	5
2	4	7	-	6	6	8	3	5	6	6
3	4	7	6	-	2	4	5	7	6	4
4	6	7	6	2	-	6	5	5	4	6
5	6	7	8	4	6	-	5	3	6	8
6	7	4	3	5	5	5	-	8	5	7
7	7	6	5	7	5	3	8	-	7	5
8	6	7	6	6	4	6	5	7	-	2
9	8	5	6	4	6	8	7	5	2	-

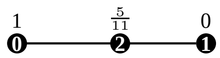
≥ 38; SMAPO# 3286

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	5	6	6	6	7	8	7
1	2	-	6	7	8	8	4	5	8	5
2	4	6	-	7	6	8	2	7	8	7
3	5	7	7	-	3	5	7	8	9	6
4	6	8	6	3	-	8	6	5	6	9
5	6	8	8	5	8	-	6	3	8	7
6	6	4	2	7	6	6	-	9	6	9
7	7	5	7	8	5	3	9	-	9	6
8	8	8	8	9	6	8	6	9	-	3
9	7	5	7	6	9	7	9	6	3	-

≥ 42; SMAPO# 4542

	0	1	2	3	4	5	6	7	8	9
0	-	5	8	9	12	12	13	13	14	15
1	5	-	13	14	15	15	8	10	15	10
2	8	13	-	13	12	16	5	11	14	13
3	9	14	13	-	5	9	12	14	15	10
4	12	15	12	5	-	14	11	9	10	15
5	12	15	16	9	14	-	11	5	14	15
6	13	8	5	12	11	11	-	16	11	16
7	13	10	11	14	9	5	16	-	15	10
8	14	15	14	15	10	14	11	15	-	5
9	15	10	13	10	15	15	16	10	5	-

$\geq 78; (\frac{5}{11}, \frac{6}{11})$



TT-disjoint at 6

	0	1	2	3	4	5	6	7	8	9
0	-	3	4	4	6	7	7	6	6	8
1	3	-	7	7	7	4	6	7	7	5
2	4	7	-	6	6	3	5	6	8	6
3	4	7	6	-	2	5	7	6	4	4
4	6	7	6	2	-	5	5	4	6	6
5	7	4	3	5	5	-	8	5	5	7
6	7	6	5	7	5	8	-	7	3	5
7	6	7	6	6	4	5	7	-	6	2
8	6	7	8	4	6	5	3	6	-	8
9	8	5	6	4	6	7	5	2	8	-

≥ 38; SMAPO# 3286

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	5	6	6	7	8	8	7
1	2	-	6	7	8	4	5	8	8	5
2	4	6	-	5	6	2	7	8	8	7
3	5	7	5	-	3	7	8	9	5	6
4	6	8	6	3	-	6	5	6	8	9
5	6	4	2	7	6	-	9	6	6	9
6	7	5	7	8	5	9	-	9	3	6
7	8	8	8	9	6	6	9	-	8	3
8	8	8	8	5	8	6	3	8	-	7
9	7	5	7	6	9	9	6	3	7	-

≥ 42; SMAPO# 4540

	0	1	2	3	4	5	6	7	8	9
0	-	5	8	9	12	13	13	14	14	15
1	5	-	13	14	15	8	10	15	15	10
2	8	13	-	11	12	5	11	14	16	13
3	9	14	11	-	5	12	14	15	9	10
4	12	15	12	5	-	11	9	10	14	15
5	13	8	5	12	11	-	16	11	11	16
6	13	10	11	14	9	16	-	15	5	10
7	14	15	14	15	10	11	15	-	14	5
8	14	15	16	9	14	11	5	14	-	15
9	15	10	13	10	15	16	10	5	15	-

$\geq 78; (\frac{5}{11}, \frac{6}{11})$

TT-disjoint at 8

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	5	4	5	4
1	2	-	5	3	5	3	3	4	3	4
2	3	5	-	4	4	4	2	5	4	3
3	3	3	4	-	4	2	4	3	4	1
4	3	5	4	4	-	4	4	3	2	5
5	3	3	4	2	4	-	4	1	4	3
6	5	3	2	4	4	4	-	3	4	5
7	4	4	5	3	3	1	3	-	5	2
8	5	3	4	4	2	4	4	5	-	3
9	4	4	3	1	5	3	5	2	3	-

≥ 24; SMAPO# 662

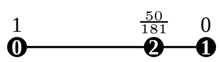
	0	1	2	3	4	5	6	7	8	9
0	-	3	5	6	6	7	8	9	10	10
1	3	-	8	9	9	10	5	10	7	9
2	5	8	-	11	11	10	3	12	7	7
3	6	9	11	-	12	7	8	11	10	4
4	6	9	11	12	-	13	8	9	4	10
5	7	10	10	7	13	-	9	4	9	11
6	8	5	3	8	8	9	-	9	8	10
7	9	10	12	11	9	4	9	-	13	7
8	10	7	7	10	4	9	8	13	-	8
9	10	9	7	4	10	11	10	7	8	-

≥ 56; SMAPO# 8959

	0	1	2	3	4	5	6	7	8	9
0	-	5	8	9	9	10	13	13	14	14
1	5	-	13	12	14	13	8	14	9	13
2	8	13	-	15	15	14	5	17	10	10
3	9	12	15	-	16	9	12	14	13	5
4	9	14	15	16	-	17	12	12	5	15
5	10	13	14	9	17	-	13	5	12	14
6	13	8	5	12	12	13	-	12	11	15
7	13	14	17	14	12	5	12	-	17	9
8	14	9	10	13	5	12	11	17	-	10
9	14	13	10	5	15	14	15	9	10	-

$\geq 78; (\frac{25}{87}, \frac{62}{87})$

RHS = 80



TT-disjoint at 8

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	5	4	5	4
1	2	-	5	3	5	3	3	4	3	4
2	3	5	-	4	4	4	2	5	4	3
3	3	3	4	-	4	2	4	3	4	1
4	3	5	4	4	-	4	4	3	2	5
5	3	3	4	2	4	-	4	1	4	3
6	5	3	2	4	4	4	-	3	4	5
7	4	4	5	3	3	1	3	-	5	2
8	5	3	4	4	2	4	4	5	-	3
9	4	4	3	1	5	3	5	2	3	-

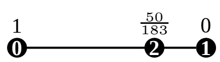
≥ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	7	7	8	9	10	11	11
1	3	-	9	10	10	9	6	11	8	10
2	6	9	-	11	11	10	3	12	7	7
3	7	10	11	-	12	7	10	11	10	4
4	7	10	11	12	-	13	8	9	4	10
5	8	9	10	7	13	-	9	4	9	11
6	9	6	3	10	8	9	-	9	8	10
7	10	11	12	11	9	4	9	-	13	7
8	11	8	7	10	4	9	8	13	-	8
9	11	10	7	4	10	11	10	7	8	-

≥ 58; SMAPO# 9446

	0	1	2	3	4	5	6	7	8	9
0	-	5	9	10	10	11	14	14	15	15
1	5	-	14	13	15	12	9	15	10	14
2	9	14	-	15	15	14	5	17	10	10
3	10	13	15	-	16	9	14	14	13	5
4	10	15	15	16	-	17	12	12	5	15
5	11	12	14	9	17	-	13	5	12	14
6	14	9	5	14	12	13	-	12	11	15
7	14	15	17	14	12	5	12	-	17	9
8	15	10	10	13	5	12	11	17	-	10
9	15	14	10	5	15	14	15	9	10	-

$\geq 80; (\frac{50}{181}, \frac{131}{181})$



TT-disjoint at 6

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	5	4	5	4
1	2	-	5	3	5	3	3	4	3	4
2	3	5	-	4	4	4	2	3	4	5
3	3	3	4	-	4	2	4	3	4	1
4	3	5	4	4	-	4	4	5	2	3
5	3	3	4	2	4	-	4	1	4	3
6	5	3	2	4	4	4	-	5	4	3
7	4	4	3	3	5	1	5	-	3	2
8	5	3	4	4	2	4	4	3	-	5
9	4	4	5	1	3	3	3	2	5	-

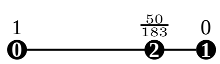
≥ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	7	7	8	10	10	10	11
1	3	-	9	10	10	9	7	11	7	12
2	6	9	-	11	11	12	4	8	8	11
3	7	10	11	-	12	7	9	11	11	4
4	7	10	11	12	-	11	7	11	3	8
5	8	9	12	7	11	-	8	4	10	11
6	10	7	4	9	7	8	-	12	8	9
7	10	11	8	11	11	4	12	-	8	7
8	10	7	8	11	3	10	8	8	-	11
9	11	12	11	4	8	11	9	7	11	-

≥ 58; SMAPO# 9385

	0	1	2	3	4	5	6	7	8	9
0	-	5	9	10	10	11	14	14	15	15
1	5	-	14	13	15	12	9	15	10	16
2	9	14	-	15	15	16	5	11	12	16
3	10	13	15	-	16	9	12	14	15	5
4	10	15	15	16	-	15	10	16	5	11
5	11	12	16	9	15	-	11	5	14	14
6	14	9	5	12	10	11	-	16	11	11
7	14	15	11	14	16	5	16	-	11	9
8	15	10	12	15	5	14	11	11	-	16
9	15	16	16	5	11	14	11	9	16	-

$\geq 80; (\frac{50}{183}, \frac{133}{183})$



TT-disjoint at 6

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	5	4	5	4
1	2	-	5	3	5	3	3	4	3	4
2	3	5	-	4	4	4	2	3	4	5
3	3	3	4	-	4	2	4	3	4	1
4	3	5	4	4	-	4	4	5	2	3
5	3	3	4	2	4	-	4	1	4	3
6	5	3	2	4	4	4	-	5	4	3
7	4	4	3	3	5	1	5	-	3	2
8	5	3	4	4	2	4	4	3	-	5
9	4	4	5	1	3	3	3	2	5	-

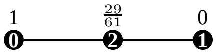
≥ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	7	7	10	10	10	10	11
1	3	-	9	10	10	9	7	11	7	12
2	6	9	-	9	11	12	4	8	8	11
3	7	10	9	-	12	7	9	11	11	4
4	7	10	11	12	-	11	7	11	3	8
5	10	9	12	7	11	-	8	4	10	11
6	10	7	4	9	7	8	-	12	8	9
7	10	11	8	11	11	4	12	-	8	7
8	10	7	8	11	3	10	8	8	-	11
9	11	12	11	4	8	11	9	7	11	-

≥ 58; SMAPO# 9386

	0	1	2	3	4	5	6	7	8	9
0	-	5	9	10	10	13	14	14	15	15
1	5	-	14	13	15	12	9	15	10	16
2	9	14	-	13	15	16	5	11	12	16
3	10	13	13	-	16	9	12	14	15	5
4	10	15	15	16	-	15	10	16	5	11
5	13	12	16	9	15	-	11	5	14	14
6	14	9	5	12	10	11	-	16	11	11
7	14	15	11	14	16	5	16	-	11	9
8	15	10	12	15	5	14	11	11	-	16
9	15	16	16	5	11	14	11	9	16	-

$\geq 80; (\frac{50}{183}, \frac{133}{183})$



TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	5	5	5	6	7	7	8
1	3	-	8	6	6	8	7	8	4	5
2	5	8	-	6	2	8	9	6	4	7
3	5	6	6	-	4	6	7	8	2	7
4	5	6	2	4	-	6	7	8	6	5
5	5	8	8	6	6	-	5	8	6	3
6	6	7	9	7	7	5	-	3	5	8
7	7	8	6	8	8	8	3	-	8	5
8	7	4	4	2	6	6	5	8	-	5
9	8	5	7	7	5	3	8	5	5	-

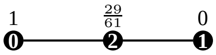
$\geq$ 40; SMAPO# 3896

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	5	6	6	5	7	8	8
1	3	-	6	8	7	7	8	8	5	5
2	5	6	-	8	3	9	8	6	5	7
3	5	8	8	-	5	7	8	8	3	7
4	6	7	3	5	-	6	7	9	8	6
5	6	7	9	7	6	-	5	7	8	2
6	5	8	8	8	7	5	-	2	7	7
7	7	8	6	8	9	7	2	-	9	5
8	8	5	5	3	8	8	7	9	-	6
9	8	5	7	7	6	2	7	5	6	-

$\geq$ 42; SMAPO# 4524

	0	1	2	3	4	5	6	7	8	9
0	-	5	9	9	10	10	10	13	14	15
1	5	-	14	14	13	15	15	16	9	10
2	9	14	-	14	5	17	17	12	9	14
3	9	14	14	-	9	13	15	16	5	14
4	10	13	5	9	-	12	14	17	14	11
5	10	15	17	13	12	-	10	15	14	5
6	10	15	17	15	14	10	-	5	12	15
7	13	16	12	16	17	15	5	-	17	10
8	14	9	9	5	14	14	12	17	-	11
9	15	10	14	14	11	5	15	10	11	-

$\geq 80; (\frac{29}{61}, \frac{32}{61})$



TT-disjoint at 7

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	4	5	4	6	7	7	6
1	2	-	6	6	7	6	6	5	7	4
2	4	6	-	8	5	8	8	7	3	6
3	4	6	8	-	9	8	6	3	7	6
4	5	7	5	9	-	7	3	6	8	7
5	4	6	8	8	7	-	8	5	5	2
6	6	6	8	6	3	8	-	9	5	6
7	7	5	7	3	6	5	9	-	6	7
8	7	7	3	7	8	5	5	6	-	7
9	6	4	6	6	7	2	6	7	7	-

$\geq$ 40; SMAPO# 3899

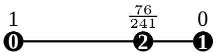
	0	1	2	3	4	5	6	7	8	9
0	-	3	5	5	5	6	7	8	7	9
1	3	-	6	8	6	9	6	5	8	6
2	5	6	-	8	4	7	6	7	2	8
3	5	8	8	-	8	7	6	3	8	8
4	5	6	4	8	-	7	2	7	6	8
5	6	9	7	7	7	-	9	6	5	3
6	7	6	6	6	2	9	-	9	4	6
7	8	5	7	3	7	6	9	-	5	9
8	7	8	2	8	6	5	4	5	-	8
9	9	6	8	8	8	3	6	9	8	-

$\geq$ 42; SMAPO# 4540

	0	1	2	3	4	5	6	7	8	9
0	-	5	9	9	10	10	13	14	14	15
1	5	-	12	14	13	15	12	9	15	10
2	9	12	-	16	9	15	14	13	5	14
3	9	14	16	-	17	15	12	5	15	14
4	10	13	9	17	-	14	5	12	14	15
5	10	15	15	15	14	-	17	10	10	5
6	13	12	14	12	5	17	-	17	9	12
7	14	9	13	5	12	10	17	-	10	15
8	14	15	5	15	14	10	9	10	-	15
9	15	10	14	14	15	5	12	15	15	-

$\geq 80; (\frac{29}{61}, \frac{32}{61})$

RHS = 82



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	4	4	4	4	5	6	6
1	1	-	2	3	3	5	5	4	7	5
2	1	2	-	5	5	5	3	6	5	7
3	4	3	5	-	4	2	6	3	4	6
4	4	3	5	4	-	6	2	5	8	6
5	4	5	5	2	6	-	4	1	4	4
6	4	5	3	6	2	4	-	3	6	4
7	5	4	6	3	5	1	3	-	5	3
8	6	7	5	4	8	4	6	5	-	2
9	6	5	7	6	6	4	4	3	2	-

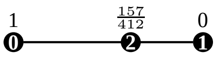
$\geq$ 26; SMAPO# 1162

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	9	8	8	8	11	12	14
1	2	-	4	7	6	8	10	9	14	12
2	2	4	-	7	10	10	6	13	10	16
3	9	7	7	-	9	3	13	6	9	15
4	8	6	10	9	-	10	4	11	16	14
5	8	8	10	3	10	-	10	3	12	12
6	8	10	6	13	4	10	-	7	16	10
7	11	9	13	6	11	3	7	-	15	9
8	12	14	10	9	16	12	16	15	-	6
9	14	12	16	15	14	12	10	9	6	-

$\geq$ 58; SMAPO# 9854

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	12	12	12	12	16	18	20
1	3	-	6	9	9	13	15	13	21	17
2	3	6	-	11	15	15	9	19	15	23
3	12	9	11	-	12	4	18	8	12	20
4	12	9	15	12	-	16	6	16	24	20
5	12	13	15	4	16	-	14	4	16	16
6	12	15	9	18	6	14	-	10	22	14
7	16	13	19	8	16	4	10	-	20	12
8	18	21	15	12	24	16	22	20	-	8
9	20	17	23	20	20	16	14	12	8	-

$\geq 82; (\frac{76}{241}, \frac{165}{241})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	5	8	8	9	10	11	13
1	2	-	4	3	10	10	11	12	9	11
2	2	4	-	7	6	6	11	10	11	13
3	5	3	7	-	13	13	8	9	8	10
4	8	10	6	13	-	10	5	14	15	9
5	8	10	6	13	10	-	13	12	5	17
6	9	11	11	8	5	13	-	9	16	14
7	10	12	10	9	14	12	9	-	17	5
8	11	9	11	8	15	5	16	17	-	12
9	13	11	13	10	9	17	14	5	12	-

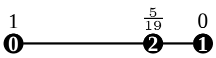
$\geq$ 62; SMAPO# 10880

	0	1	2	3	4	5	6	7	8	9
0	-	4	4	8	14	14	15	18	21	21
1	4	-	8	4	18	18	15	18	17	17
2	4	8	-	12	10	10	19	16	19	19
3	8	4	12	-	22	22	13	16	13	13
4	14	18	10	22	-	18	9	24	27	15
5	14	18	10	22	18	-	21	18	9	27
6	15	15	19	13	9	21	-	15	24	24
7	18	18	16	16	24	18	15	-	27	9
8	21	17	19	13	27	9	24	27	-	18
9	21	17	19	13	15	27	24	9	18	-

$\geq$ 104; SMAPO# 14973

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	6	11	11	12	14	16	17
1	3	-	6	3	14	14	13	15	13	14
2	3	6	-	9	8	8	15	13	15	16
3	6	3	9	-	17	17	10	12	10	11
4	11	14	8	17	-	14	7	19	21	12
5	11	14	8	17	14	-	17	15	7	22
6	12	13	15	10	7	17	-	12	20	19
7	14	15	13	12	19	15	12	-	22	7
8	16	13	15	10	21	7	20	22	-	15
9	17	14	16	11	12	22	19	7	15	-

$\geq 82; (\frac{157}{412}, \frac{255}{412})$



TT-disjoint at 7

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	5	5	4	4
1	2	-	5	3	3	5	3	3	4	4
2	3	5	-	4	4	4	2	4	3	5
3	3	3	4	-	2	4	4	4	3	1
4	3	3	4	2	-	4	4	4	1	3
5	3	5	4	4	4	-	4	2	5	3
6	5	3	2	4	4	4	-	4	5	3
7	5	3	4	4	4	2	4	-	3	5
8	4	4	3	3	1	5	5	3	-	2
9	4	4	5	1	3	3	3	5	2	-

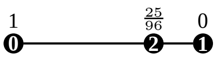
$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	7	7	7	9	11	11	11
1	3	-	9	10	10	10	6	8	10	12
2	6	9	-	11	11	11	3	7	7	13
3	7	10	11	-	8	14	10	10	12	4
4	7	10	11	8	-	14	10	12	4	12
5	7	10	11	14	14	-	8	4	10	10
6	9	6	3	10	10	8	-	8	10	10
7	11	8	7	10	12	4	8	-	8	14
8	11	10	7	12	4	10	10	8	-	8
9	11	12	13	4	12	10	10	14	8	-

$\geq$ 60; SMAPO# 10108

	0	1	2	3	4	5	6	7	8	9
0	-	5	9	10	10	10	14	15	15	15
1	5	-	14	13	13	15	9	10	14	16
2	9	14	-	15	15	15	5	10	10	18
3	10	13	15	-	10	18	14	13	15	5
4	10	13	15	10	-	18	14	15	5	15
5	10	15	15	18	18	-	12	5	15	13
6	14	9	5	14	14	12	-	11	15	13
7	15	10	10	13	15	5	11	-	10	18
8	15	14	10	15	5	15	15	10	-	10
9	15	16	18	5	15	13	13	18	10	-

$\geq 82; (\frac{5}{19}, \frac{14}{19})$



TT-disjoint at 6

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	5	5	4	4
1	2	-	5	3	3	5	3	3	4	4
2	3	5	-	4	4	4	2	4	3	5
3	3	3	4	-	2	4	4	4	1	3
4	3	3	4	2	-	4	4	4	3	1
5	3	5	4	4	4	-	4	2	5	3
6	5	3	2	4	4	4	-	4	5	3
7	5	3	4	4	4	2	4	-	3	5
8	4	4	3	1	3	5	5	3	-	2
9	4	4	5	3	1	3	3	5	2	-

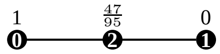
$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	7	7	7	10	10	11	11
1	3	-	9	10	10	10	7	7	12	12
2	6	9	-	13	13	11	4	8	9	11
3	7	10	13	-	8	12	9	11	4	12
4	7	10	13	8	-	12	11	11	12	4
5	7	10	11	12	12	-	7	3	12	8
6	10	7	4	9	11	7	-	8	13	9
7	10	7	8	11	11	3	8	-	9	11
8	11	12	9	4	12	12	13	9	-	8
9	11	12	11	12	4	8	9	11	8	-

$\geq$ 60; SMAPO# 9988

	0	1	2	3	4	5	6	7	8	9
0	-	5	9	10	10	10	14	15	15	15
1	5	-	14	13	13	15	9	10	16	16
2	9	14	-	17	17	15	5	12	12	16
3	10	13	17	-	10	16	12	15	5	15
4	10	13	17	10	-	16	14	15	15	5
5	10	15	15	16	16	-	10	5	17	11
6	14	9	5	12	14	10	-	11	17	11
7	15	10	12	15	15	5	11	-	12	16
8	15	16	12	5	15	17	17	12	-	10
9	15	16	16	15	5	11	11	16	10	-

$\geq 82; (\frac{25}{96}, \frac{71}{96})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	6	5	8	8	7	7	8
1	3	-	8	7	8	5	5	6	8	7
2	5	8	-	9	8	3	7	6	6	7
3	6	7	9	-	5	6	8	3	7	6
4	5	8	8	5	-	7	5	8	2	7
5	8	5	3	6	7	-	8	9	7	6
6	8	5	7	8	5	8	-	5	7	2
7	7	6	6	3	8	9	5	-	6	7
8	7	8	6	7	2	7	7	6	-	5
9	8	7	7	6	7	6	2	7	5	-

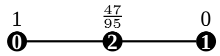
≥ 42; SMAPO# 4504

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	5	5	6	7	8	8	8
1	2	-	6	7	7	4	5	6	8	6
2	4	6	-	9	9	2	7	6	6	6
3	5	7	9	-	6	7	8	3	9	5
4	5	7	9	6	-	7	6	7	3	9
5	6	4	2	7	7	-	7	8	8	6
6	7	5	7	8	6	7	-	5	9	3
7	8	6	6	3	7	8	5	-	8	8
8	8	8	6	9	3	8	9	8	-	6
9	8	6	6	5	9	6	3	8	6	-

≥ 42; SMAPO# 4542

	0	1	2	3	4	5	6	7	8	9
0	-	5	9	10	10	14	15	15	15	16
1	5	-	14	13	15	9	10	12	16	13
2	9	14	-	17	17	5	14	12	12	13
3	10	13	17	-	10	12	15	5	15	10
4	10	15	17	10	-	14	11	15	5	16
5	14	9	5	12	14	-	15	17	15	12
6	15	10	14	15	11	15	-	10	16	5
7	15	12	12	5	15	17	10	-	14	15
8	15	16	12	15	5	15	16	14	-	11
9	16	13	13	10	16	12	5	15	11	-

$\geq 82; (\frac{47}{95}, \frac{48}{95})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	6	5	8	8	7	7	8
1	3	-	8	7	8	5	5	8	8	7
2	5	8	-	9	8	3	7	6	6	7
3	6	7	9	-	5	6	8	3	7	6
4	5	8	8	5	-	7	5	8	2	7
5	8	5	3	6	7	-	6	9	7	6
6	8	5	7	8	5	6	-	5	7	2
7	7	8	6	3	8	9	5	-	6	7
8	7	8	6	7	2	7	7	6	-	5
9	8	7	7	6	7	6	2	7	5	-

≥ 42; SMAPO# 4505

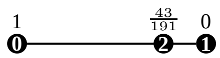
	0	1	2	3	4	5	6	7	8	9
0	-	2	4	5	5	6	7	8	8	8
1	2	-	6	7	7	4	5	8	8	6
2	4	6	-	9	9	2	7	6	6	6
3	5	7	9	-	6	7	8	3	9	5
4	5	7	9	6	-	7	6	7	3	9
5	6	4	2	7	7	-	5	8	8	6
6	7	5	7	8	6	5	-	5	9	3
7	8	8	6	3	7	8	5	-	8	8
8	8	8	6	9	3	8	9	8	-	6
9	8	6	6	5	9	6	3	8	6	-

≥ 42; SMAPO# 4540

	0	1	2	3	4	5	6	7	8	9
0	-	5	9	10	10	14	15	15	15	16
1	5	-	14	13	15	9	10	16	16	13
2	9	14	-	17	17	5	14	12	12	13
3	10	13	17	-	10	12	15	5	15	10
4	10	15	17	10	-	14	11	15	5	16
5	14	9	5	12	14	-	11	17	15	12
6	15	10	14	15	11	11	-	10	16	5
7	15	16	12	5	15	17	10	-	14	15
8	15	16	12	15	5	15	16	14	-	11
9	16	13	13	10	16	12	5	15	11	-

$\geq 82; (\frac{47}{95}, \frac{48}{95})$

RHS = 84



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	3	2	4	4
1	1	-	3	3	4	2	4	3	3	3
2	2	3	-	4	5	3	3	4	4	2
3	2	3	4	-	3	3	5	4	2	4
4	3	4	5	3	-	4	2	3	5	3
5	3	2	3	3	4	-	4	1	3	3
6	3	4	3	5	2	4	-	3	3	5
7	2	3	4	4	3	1	3	-	2	2
8	4	3	4	2	5	3	3	2	-	4
9	4	3	2	4	3	3	5	2	4	-

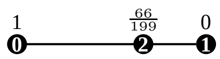
≥ 22; SMAPO# 464

	0	1	2	3	4	5	6	7	8	9
0	-	2	7	6	7	9	9	10	10	10
1	2	-	7	8	9	7	9	12	8	12
2	7	7	-	13	14	10	10	15	11	5
3	6	8	13	-	7	13	11	14	4	14
4	7	9	14	7	-	14	4	9	11	9
5	9	7	10	13	14	-	12	5	11	15
6	9	9	10	11	4	12	-	11	7	13
7	10	12	15	14	9	5	11	-	10	10
8	10	8	11	4	11	11	7	10	-	12
9	10	12	5	14	9	15	13	10	12	-

≥ 64; SMAPO# 11117

	0	1	2	3	4	5	6	7	8	9
0	-	3	8	8	10	12	12	12	14	14
1	3	-	9	11	13	9	13	15	11	15
2	8	9	-	16	18	12	12	18	14	6
3	8	11	16	-	10	16	16	18	6	18
4	10	13	18	10	-	18	6	12	16	12
5	12	9	12	16	18	-	16	6	14	18
6	12	13	12	16	6	16	-	14	10	18
7	12	15	18	18	12	6	14	-	12	12
8	14	11	14	6	16	14	10	12	-	16
9	14	15	6	18	12	18	18	12	16	-

$\geq 84; (\frac{43}{191}, \frac{148}{191})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	5	4	4	5	4	4
1	1	-	4	4	4	3	3	4	5	5
2	3	4	-	4	6	7	7	2	5	5
3	3	4	4	-	2	7	7	6	5	5
4	5	4	6	2	-	5	5	4	5	5
5	4	3	7	7	5	-	4	5	6	2
6	4	3	7	7	5	4	-	5	2	6
7	5	4	2	6	4	5	5	-	5	5
8	4	5	5	5	5	6	2	5	-	4
9	4	5	5	5	5	2	6	5	4	-

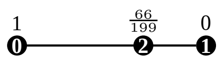
≥ 30; SMAPO# 1506

	0	1	2	3	4	5	6	7	8	9
0	-	2	6	5	8	9	9	8	9	9
1	2	-	6	7	6	7	7	10	11	11
2	6	6	-	7	10	13	13	4	9	9
3	5	7	7	-	3	12	12	9	10	10
4	8	6	10	3	-	9	11	6	7	11
5	9	7	13	12	9	-	8	9	12	4
6	9	7	13	12	11	8	-	11	4	12
7	8	10	4	9	6	9	11	-	9	13
8	9	11	9	10	7	12	4	9	-	8
9	9	11	9	10	11	4	12	13	8	-

≥ 56; SMAPO# 9011

	0	1	2	3	4	5	6	7	8	9
0	-	3	8	8	13	13	13	13	13	13
1	3	-	9	11	10	10	10	14	16	16
2	8	9	-	10	15	19	19	5	13	13
3	8	11	10	-	5	19	19	15	15	15
4	13	10	15	5	-	14	16	10	12	16
5	13	10	19	19	14	-	12	14	18	6
6	13	10	19	19	16	12	-	16	6	18
7	13	14	5	15	10	14	16	-	14	18
8	13	16	13	15	12	18	6	14	-	12
9	13	16	13	15	16	6	18	18	12	-

$\geq 84; (\frac{66}{199}, \frac{133}{199})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	5	4	4	5	4	4
1	1	-	4	4	4	3	3	4	5	5
2	3	4	-	4	6	7	7	2	5	5
3	3	4	4	-	2	7	7	6	5	5
4	5	4	6	2	-	5	5	4	5	5
5	4	3	7	7	5	-	4	5	6	2
6	4	3	7	7	5	4	-	5	2	6
7	5	4	2	6	4	5	5	-	5	5
8	4	5	5	5	5	6	2	5	-	4
9	4	5	5	5	5	2	6	5	4	-

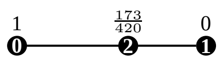
≥ 30; SMAPO# 1506

	0	1	2	3	4	5	6	7	8	9
0	-	2	6	5	8	9	9	8	9	11
1	2	-	6	7	6	7	7	10	11	11
2	6	6	-	7	10	13	13	4	9	9
3	5	7	7	-	3	12	12	9	8	10
4	8	6	10	3	-	9	11	6	7	11
5	9	7	13	12	9	-	8	9	12	4
6	9	7	13	12	11	8	-	11	4	12
7	8	10	4	9	6	9	11	-	9	13
8	9	11	9	8	7	12	4	9	-	8
9	11	11	9	10	11	4	12	13	8	-

≥ 56; SMAPO# 9012

	0	1	2	3	4	5	6	7	8	9
0	-	3	8	8	13	13	13	13	13	15
1	3	-	9	11	10	10	10	14	16	16
2	8	9	-	10	15	19	19	5	13	13
3	8	11	10	-	5	19	19	15	13	15
4	13	10	15	5	-	14	16	10	12	16
5	13	10	19	19	14	-	12	14	18	6
6	13	10	19	19	16	12	-	16	6	18
7	13	14	5	15	10	14	16	-	14	18
8	13	16	13	13	12	18	6	14	-	12
9	15	16	13	15	16	6	18	18	12	-

$\geq 84; (\frac{66}{199}, \frac{133}{199})$



TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	4	8	9	10	12	11	11	12	15
1	4	-	12	11	12	8	9	15	10	11
2	8	12	-	9	14	10	13	13	4	15
3	9	11	9	-	15	13	12	4	11	8
4	10	12	14	15	-	4	11	11	10	15
5	12	8	10	13	4	-	9	15	14	13
6	11	9	13	12	11	9	-	8	15	4
7	11	15	13	4	11	15	8	-	15	12
8	12	10	4	11	10	14	15	15	-	11
9	15	11	15	8	15	13	4	12	11	-

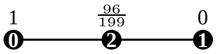
≥ 70; SMAPO# 12116

	0	1	2	3	4	5	6	7	8	9
0	-	5	11	12	11	15	16	16	17	22
1	5	-	14	17	16	10	13	19	14	17
2	11	14	-	13	20	14	19	19	6	23
3	12	17	13	-	21	17	18	6	17	12
4	11	16	20	21	-	6	15	15	14	21
5	15	10	14	17	6	-	15	21	20	21
6	16	13	19	18	15	15	-	12	23	6
7	16	19	19	6	15	21	12	-	23	18
8	17	14	6	17	14	20	23	23	-	17
9	22	17	23	12	21	21	6	18	17	-

≥ 100; SMAPO# 14843

	0	1	2	3	4	5	6	7	8	9
0	-	4	9	10	10	13	13	13	14	18
1	4	-	13	14	14	9	11	17	12	14
2	9	13	-	11	17	12	16	16	5	19
3	10	14	11	-	18	15	15	5	14	10
4	10	14	17	18	-	5	13	13	12	18
5	13	9	12	15	5	-	12	18	17	17
6	13	11	16	15	13	12	-	10	19	5
7	13	17	16	5	13	18	10	-	19	15
8	14	12	5	14	12	17	19	19	-	14
9	18	14	19	10	18	17	5	15	14	-

$\geq 84; (\frac{173}{420}, \frac{247}{420})$



TT-disjoint at 6

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	5	5	6	8	7	7	9
1	3	-	6	6	8	9	5	6	8	6
2	5	6	-	4	8	7	7	2	6	8
3	5	6	4	-	8	7	7	6	2	8
4	5	8	8	8	-	7	3	6	8	8
5	6	9	7	7	7	-	6	9	5	3
6	8	5	7	7	3	6	-	9	5	9
7	7	6	2	6	6	9	9	-	4	6
8	7	8	6	2	8	5	5	4	-	8
9	9	6	8	8	8	3	9	6	8	-

$\geq$ 42; SMAPO# 4540

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	5	5	5	8	8	8	7
1	2	-	7	7	7	7	6	8	8	5
2	5	7	-	6	10	8	7	3	9	8
3	5	7	6	-	10	8	9	9	3	8
4	5	7	10	10	-	8	3	7	7	6
5	5	7	8	8	8	-	5	9	5	2
6	8	6	7	9	3	5	-	10	6	7
7	8	8	3	9	7	9	10	-	6	7
8	8	8	9	3	7	5	6	6	-	7
9	7	5	8	8	6	2	7	7	7	-

$\geq$ 44; SMAPO# 5162

	0	1	2	3	4	5	6	7	8	9
0	-	5	10	10	10	11	15	15	15	16
1	5	-	13	13	15	16	10	14	16	11
2	10	13	-	10	18	15	13	5	15	16
3	10	13	10	-	18	15	15	15	5	16
4	10	15	18	18	-	15	5	13	15	14
5	11	16	15	15	15	-	10	18	10	5
6	15	10	13	15	5	10	-	18	10	15
7	15	14	5	15	13	18	18	-	10	13
8	15	16	15	5	15	10	10	10	-	15
9	16	11	16	16	14	5	15	13	15	-

$\geq 84; (\frac{96}{199}, \frac{103}{199})$

TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	3	3	4	4	4	5	5
1	2	-	4	5	5	4	4	4	3	3
2	4	4	-	5	3	4	2	4	3	5
3	3	5	5	-	4	3	3	3	2	4
4	3	5	3	4	-	3	5	1	4	2
5	4	4	4	3	3	-	4	2	3	1
6	4	4	2	3	5	4	-	4	5	3
7	4	4	4	3	1	2	4	-	3	3
8	5	3	3	2	4	3	5	3	-	4
9	5	3	5	4	2	1	3	3	4	-

$\geq$ 24; SMAPO# 662

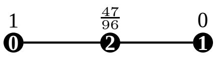
	0	1	2	3	4	5	6	7	8	9
0	-	4	7	8	9	9	10	11	11	13
1	4	-	11	10	11	13	8	13	7	9
2	7	11	-	11	8	12	3	12	8	12
3	8	10	11	-	13	9	8	11	3	11
4	9	11	8	13	-	12	11	4	12	8
5	9	13	12	9	12	-	11	8	8	4
6	10	8	3	8	11	11	-	13	11	9
7	11	13	12	11	4	8	13	-	8	12
8	11	7	8	3	12	8	11	8	-	12
9	13	9	12	11	8	4	9	12	12	-

$\geq$ 62; SMAPO# 10466

	0	1	2	3	4	5	6	7	8	9
0	-	5	10	10	11	12	13	14	15	17
1	5	-	15	15	16	17	12	17	10	12
2	10	15	-	16	11	16	5	16	11	17
3	10	15	16	-	17	12	11	14	5	15
4	11	16	11	17	-	15	16	5	16	10
5	12	17	16	12	15	-	15	10	11	5
6	13	12	5	11	16	15	-	17	16	12
7	14	17	16	14	5	10	17	-	11	15
8	15	10	11	5	16	11	16	11	-	16
9	17	12	17	15	10	5	12	15	16	-

$\geq 84; (\frac{50}{199}, \frac{149}{199})$

RHS = 86



TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	5	5	6	6	7	9	9
1	3	-	8	6	6	9	7	4	6	8
2	5	8	-	6	2	9	9	4	6	6
3	5	6	6	-	4	7	7	2	8	8
4	5	6	2	4	-	7	7	6	6	8
5	6	9	9	7	7	-	6	7	3	9
6	6	7	9	7	7	6	-	5	9	3
7	7	4	4	2	6	7	5	-	6	8
8	9	6	6	8	6	3	9	6	-	6
9	9	8	6	8	8	9	3	8	6	-

$\geq$ 42; SMAPO# 4551

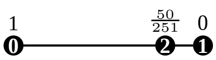
	0	1	2	3	4	5	6	7	8	9
0	-	3	5	5	6	6	6	8	9	9
1	3	-	6	8	7	7	9	5	6	8
2	5	6	-	8	3	9	9	5	6	6
3	5	8	8	-	5	7	9	3	8	8
4	6	7	3	5	-	6	8	8	7	9
5	6	7	9	7	6	-	6	8	3	9
6	6	9	9	9	8	6	-	8	9	3
7	8	5	5	3	8	8	8	-	7	9
8	9	6	6	8	7	3	9	7	-	6
9	9	8	6	8	9	9	3	9	6	-

$\geq$ 46; SMAPO# 5703

	0	1	2	3	4	5	6	7	8	9
0	-	5	9	9	10	11	11	14	17	17
1	5	-	14	14	13	16	16	9	12	16
2	9	14	-	14	5	18	18	9	12	12
3	9	14	14	-	9	14	16	5	16	16
4	10	13	5	9	-	13	15	14	13	17
5	11	16	18	14	13	-	12	15	6	18
6	11	16	18	16	15	12	-	13	18	6
7	14	9	9	5	14	15	13	-	13	17
8	17	12	12	16	13	6	18	13	-	12
9	17	16	12	16	17	18	6	17	12	-

$\geq 86; (\frac{47}{96}, \frac{49}{96})$

RHS = 88



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	0	1	2	3	2	3	3	4	5
1	0	-	1	2	3	2	3	3	4	5
2	1	1	-	3	2	3	4	2	3	4
3	2	2	3	-	1	4	3	3	2	5
4	3	3	2	1	-	3	4	2	1	4
5	2	2	3	4	3	-	5	1	2	3
6	3	3	4	3	4	5	-	4	5	2
7	3	3	2	3	2	1	4	-	3	4
8	4	4	3	2	1	2	5	3	-	3
9	5	5	4	5	4	3	2	4	3	-

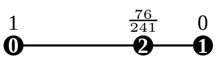
$\geq$ 18; SMAPO# 292

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	8	10	12	14	14	13	16
1	2	-	6	10	12	10	12	16	15	18
2	4	6	-	8	12	16	18	10	13	12
3	8	10	8	-	4	16	10	10	7	16
4	10	12	12	4	-	12	12	14	3	12
5	12	10	16	16	12	-	18	6	9	12
6	14	12	18	10	12	18	-	20	15	6
7	14	16	10	10	14	6	20	-	15	18
8	13	15	13	7	3	9	15	15	-	9
9	16	18	12	16	12	12	6	18	9	-

$\geq$ 72; SMAPO# 12645

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	9	13	14	17	17	17	21
1	2	-	7	11	15	12	15	19	19	23
2	5	7	-	10	14	19	22	12	16	16
3	9	11	10	-	4	19	12	12	8	20
4	13	15	14	4	-	15	16	16	4	16
5	14	12	19	19	15	-	23	7	11	15
6	17	15	22	12	16	23	-	24	20	8
7	17	19	12	12	16	7	24	-	18	22
8	17	19	16	8	4	11	20	18	-	12
9	21	23	16	20	16	15	8	22	12	-

$\geq 88; (\frac{50}{251}, \frac{201}{251})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	4	4	5	6	6
1	1	-	3	3	3	5	5	4	7	5
2	2	3	-	6	6	4	4	3	6	4
3	4	3	6	-	4	2	6	3	4	6
4	4	3	6	4	-	6	2	5	8	6
5	4	5	4	2	6	-	6	1	4	4
6	4	5	4	6	2	6	-	7	6	8
7	5	4	3	3	5	1	7	-	5	3
8	6	7	6	4	8	4	6	5	-	2
9	6	5	4	6	6	4	8	3	2	-

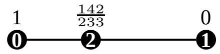
$\geq$ 28; SMAPO# 1389

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	9	8	8	8	11	12	14
1	2	-	6	7	6	8	10	9	14	12
2	4	6	-	13	12	10	8	7	16	10
3	9	7	13	-	9	3	9	6	9	15
4	8	6	12	9	-	10	4	11	16	14
5	8	8	10	3	10	-	12	3	12	12
6	8	10	8	9	4	12	-	15	12	18
7	11	9	7	6	11	3	15	-	15	9
8	12	14	16	9	16	12	12	15	-	6
9	14	12	10	15	14	12	18	9	6	-

$\geq$ 62; SMAPO# 10896

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	12	12	12	12	16	18	20
1	3	-	9	9	9	13	15	13	21	17
2	6	9	-	18	18	14	12	10	22	14
3	12	9	18	-	12	4	14	8	12	20
4	12	9	18	12	-	16	6	16	24	20
5	12	13	14	4	16	-	18	4	16	16
6	12	15	12	14	6	18	-	22	18	26
7	16	13	10	8	16	4	22	-	20	12
8	18	21	22	12	24	16	18	20	-	8
9	20	17	14	20	20	16	26	12	8	-

$\geq 88; (\frac{76}{241}, \frac{165}{241})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	4	6	4	6	4
1	1	-	3	3	3	5	5	3	5	5
2	2	3	-	6	6	4	4	6	4	4
3	4	3	6	-	4	2	2	4	6	6
4	4	3	6	4	-	2	6	4	2	6
5	4	5	4	2	2	-	4	6	4	4
6	6	5	4	2	6	4	-	6	4	8
7	4	3	6	4	4	6	6	-	6	2
8	6	5	4	6	2	4	4	6	-	8
9	4	5	4	6	6	4	8	2	8	-

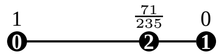
≥ 28; SMAPO# 1404

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	4	6	6	3	8	4	8
1	2	-	4	6	4	8	5	6	6	10
2	2	4	-	6	8	4	5	10	6	6
3	4	6	6	-	4	2	1	8	4	8
4	6	4	8	4	-	4	5	6	2	10
5	6	8	4	2	4	-	3	10	6	6
6	3	5	5	1	5	3	-	7	3	7
7	8	6	10	8	6	10	7	-	8	4
8	4	6	6	4	2	6	3	8	-	8
9	8	10	6	8	10	6	7	4	8	-

≥ 36; SMAPO# 3248

	0	1	2	3	4	5	6	7	8	9
0	-	4	4	12	14	14	15	16	16	16
1	4	-	8	12	10	18	15	12	16	20
2	4	8	-	16	18	10	11	20	12	12
3	12	12	16	-	12	6	5	16	16	20
4	14	10	18	12	-	8	17	14	6	22
5	14	18	10	6	8	-	11	22	14	14
6	15	15	11	5	17	11	-	19	11	23
7	16	12	20	16	14	22	19	-	20	8
8	16	16	12	16	6	14	11	20	-	24
9	16	20	12	20	22	14	23	8	24	-

$\geq 88; (\frac{142}{233}, \frac{91}{233})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	4	6	4	6	4
1	1	-	3	3	3	5	5	3	5	5
2	2	3	-	6	6	4	4	6	4	4
3	4	3	6	-	4	2	2	4	6	6
4	4	3	6	4	-	2	6	4	2	6
5	4	5	4	2	2	-	4	6	4	4
6	6	5	4	2	6	4	-	6	4	8
7	4	3	6	4	4	6	6	-	6	2
8	6	5	4	6	2	4	4	6	-	8
9	4	5	4	6	6	4	8	2	8	-

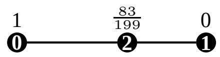
≥ 28; SMAPO# 1404

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	8	10	10	9	12	10	12
1	3	-	6	9	7	13	12	9	11	15
2	3	6	-	11	13	7	8	15	9	9
3	8	9	11	-	8	4	3	12	10	14
4	10	7	13	8	-	6	11	10	4	16
5	10	13	7	4	6	-	7	16	8	10
6	9	12	8	3	11	7	-	15	7	15
7	12	9	15	12	10	16	15	-	14	6
8	10	11	9	10	4	8	7	14	-	16
9	12	15	9	14	16	10	15	6	16	-

≥ 62; SMAPO# 10855

	0	1	2	3	4	5	6	7	8	9
0	-	4	4	12	14	14	15	16	16	16
1	4	-	8	12	10	18	17	12	16	20
2	4	8	-	16	18	10	11	20	12	12
3	12	12	16	-	12	6	5	16	16	20
4	14	10	18	12	-	8	17	14	6	22
5	14	18	10	6	8	-	11	22	12	14
6	15	17	11	5	17	11	-	21	11	23
7	16	12	20	16	14	22	21	-	20	8
8	16	16	12	16	6	12	11	20	-	24
9	16	20	12	20	22	14	23	8	24	-

$\geq 88; (\frac{71}{235}, \frac{164}{235})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	4	5	5	6	6	6	7
1	2	-	5	6	3	7	4	6	6	7
2	3	5	-	7	2	8	5	5	5	8
3	4	6	7	-	5	7	2	8	6	7
4	5	3	2	5	-	6	5	7	5	6
5	5	7	8	7	6	-	5	3	9	6
6	6	4	5	2	5	5	-	8	8	5
7	6	6	5	8	7	3	8	-	6	9
8	6	6	5	6	5	9	8	6	-	3
9	7	7	8	7	6	6	5	9	3	-

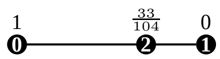
≥ 38; SMAPO# 3320

	0	1	2	3	4	5	6	7	8	9
0	-	2	6	5	7	7	8	9	9	9
1	2	-	8	7	5	9	6	7	9	9
2	6	8	-	9	3	11	6	7	7	11
3	5	7	9	-	6	8	3	10	8	10
4	7	5	3	6	-	10	9	8	10	8
5	7	9	11	8	10	-	7	4	12	8
6	8	6	6	3	9	7	-	11	11	7
7	9	7	7	10	8	4	11	-	8	12
8	9	9	7	8	10	12	11	8	-	4
9	9	9	11	10	8	8	7	12	4	-

≥ 52; SMAPO# 7820

	0	1	2	3	4	5	6	7	8	9
0	-	4	8	9	12	12	14	15	15	16
1	4	-	12	13	8	16	10	13	15	16
2	8	12	-	15	4	18	10	11	11	18
3	9	13	15	-	11	15	5	18	14	17
4	12	8	4	11	-	16	14	15	15	14
5	12	16	18	15	16	-	12	7	21	14
6	14	10	10	5	14	12	-	19	19	12
7	15	13	11	18	15	7	19	-	14	21
8	15	15	11	14	15	21	19	14	-	7
9	16	16	18	17	14	14	12	21	7	-

$\geq 88; (\frac{83}{199}, \frac{116}{199})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	4	4	5	5	4	4
1	1	-	4	4	3	5	4	4	3	5
2	3	4	-	4	7	5	6	2	7	5
3	3	4	4	-	7	5	2	6	7	5
4	4	3	7	7	-	6	5	5	4	2
5	4	5	5	5	6	-	5	5	2	4
6	5	4	6	2	5	5	-	4	5	5
7	5	4	2	6	5	5	4	-	5	5
8	4	3	7	7	4	2	5	5	-	6
9	4	5	5	5	2	4	5	5	6	-

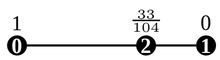
≥ 30; SMAPO# 1506

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	6	9	9	9	9	11	11
1	3	-	8	9	8	12	6	12	8	12
2	7	8	-	7	14	10	10	4	14	10
3	6	9	7	-	13	9	3	9	13	11
4	9	8	14	13	-	12	10	10	8	4
5	9	12	10	9	12	-	6	10	4	8
6	9	6	10	3	10	6	-	6	10	10
7	9	12	4	9	10	10	6	-	12	14
8	11	8	14	13	8	4	10	12	-	12
9	11	12	10	11	4	8	10	14	12	-

≥ 60; SMAPO# 10090

	0	1	2	3	4	5	6	7	8	9
0	-	4	9	9	13	13	14	14	15	15
1	4	-	11	13	11	17	10	16	11	17
2	9	11	-	10	20	14	15	5	20	14
3	9	13	10	-	20	14	5	15	20	16
4	13	11	20	20	-	18	15	15	12	6
5	13	17	14	14	18	-	11	15	6	12
6	14	10	15	5	15	11	-	10	15	15
7	14	16	5	15	15	15	10	-	17	19
8	15	11	20	20	12	6	15	17	-	18
9	15	17	14	16	6	12	15	19	18	-

$\geq 88; (\frac{33}{104}, \frac{71}{104})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	4	4	5	5	4	4
1	1	-	4	4	3	5	4	4	3	5
2	3	4	-	4	7	5	6	2	7	5
3	3	4	4	-	7	5	2	6	7	5
4	4	3	7	7	-	6	5	5	4	2
5	4	5	5	5	6	-	5	5	2	4
6	5	4	6	2	5	5	-	4	5	5
7	5	4	2	6	5	5	4	-	5	5
8	4	3	7	7	4	2	5	5	-	6
9	4	5	5	5	2	4	5	5	6	-

≥ 30; SMAPO# 1506

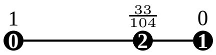
	0	1	2	3	4	5	6	7	8	9
0	-	3	7	6	9	9	9	9	11	11
1	3	-	8	9	8	12	6	12	8	12
2	7	8	-	7	14	10	10	4	14	10
3	6	9	7	-	13	9	3	9	13	11
4	9	8	14	13	-	12	10	10	8	4
5	9	12	10	9	12	-	6	12	4	8
6	9	6	10	3	10	6	-	6	10	8
7	9	12	4	9	10	12	6	-	12	14
8	11	8	14	13	8	4	10	12	-	12
9	11	12	10	11	4	8	8	14	12	-

≥ 60; SMAPO# 10089

	0	1	2	3	4	5	6	7	8	9
0	-	4	9	9	13	13	14	14	15	15
1	4	-	11	13	11	17	10	16	11	17
2	9	11	-	10	20	14	15	5	20	14
3	9	13	10	-	20	14	5	15	20	16
4	13	11	20	20	-	18	15	15	12	6
5	13	17	14	14	18	-	11	17	6	12
6	14	10	15	5	15	11	-	10	15	13
7	14	16	5	15	15	17	10	-	17	19
8	15	11	20	20	12	6	15	17	-	18
9	15	17	14	16	6	12	13	19	18	-

$\geq 88; (\frac{33}{104}, \frac{71}{104})$

RHS = 90



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	5	5	4	4	4	4
1	1	-	4	4	4	4	3	3	5	5
2	3	4	-	4	6	2	7	7	5	5
3	3	4	4	-	2	6	7	7	5	5
4	5	4	6	2	-	4	5	5	5	5
5	5	4	2	6	4	-	5	5	5	5
6	4	3	7	7	5	5	-	4	6	2
7	4	3	7	7	5	5	4	-	2	6
8	4	5	5	5	5	5	6	2	-	4
9	4	5	5	5	5	5	2	6	4	-

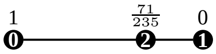
$\geq$ 30; SMAPO# 1506

	0	1	2	3	4	5	6	7	8	9
0	-	2	6	5	8	8	10	10	11	11
1	2	-	6	7	6	10	8	8	13	13
2	6	6	-	7	10	4	14	14	9	9
3	5	7	7	-	3	9	13	13	10	10
4	8	6	10	3	-	6	10	12	9	11
5	8	10	4	9	6	-	10	12	11	13
6	10	8	14	13	10	10	-	10	15	5
7	10	8	14	13	12	12	10	-	5	15
8	11	13	9	10	9	11	15	5	-	10
9	11	13	9	10	11	13	5	15	10	-

$\geq$ 62; SMAPO# 10666

	0	1	2	3	4	5	6	7	8	9
0	-	3	8	8	13	13	14	14	15	15
1	3	-	9	11	10	14	11	11	18	18
2	8	9	-	10	15	5	20	20	13	13
3	8	11	10	-	5	15	20	20	15	15
4	13	10	15	5	-	10	15	17	14	16
5	13	14	5	15	10	-	15	17	16	18
6	14	11	20	20	15	15	-	14	21	7
7	14	11	20	20	17	17	14	-	7	21
8	15	18	13	15	14	16	21	7	-	14
9	15	18	13	15	16	18	7	21	14	-

$\geq 90; (\frac{33}{104}, \frac{71}{104})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	4	6	4	6	4
1	1	-	3	3	5	3	5	3	5	5
2	2	3	-	6	4	6	4	6	4	4
3	4	3	6	-	2	4	2	4	6	6
4	4	5	4	2	-	2	4	6	4	4
5	4	3	6	4	2	-	6	4	2	6
6	6	5	4	2	4	6	-	6	4	8
7	4	3	6	4	6	4	6	-	6	2
8	6	5	4	6	4	2	4	6	-	8
9	4	5	4	6	4	6	8	2	8	-

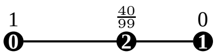
$\geq$ 28; SMAPO# 1404

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	8	10	11	9	12	10	12
1	3	-	6	9	13	8	10	9	13	15
2	3	6	-	11	7	14	8	15	9	9
3	8	9	11	-	4	9	3	12	10	14
4	10	13	7	4	-	7	7	16	12	10
5	11	8	14	9	7	-	12	11	5	17
6	9	10	8	3	7	12	-	11	7	15
7	12	9	15	12	16	11	11	-	14	6
8	10	13	9	10	12	5	7	14	-	16
9	12	15	9	14	10	17	15	6	16	-

$\geq$ 64; SMAPO# 11342

	0	1	2	3	4	5	6	7	8	9
0	-	4	4	12	14	15	15	16	16	16
1	4	-	8	12	18	11	15	12	18	20
2	4	8	-	16	10	19	11	20	12	12
3	12	12	16	-	6	13	5	16	16	20
4	14	18	10	6	-	9	11	22	16	14
5	15	11	19	13	9	-	18	15	7	23
6	15	15	11	5	11	18	-	17	11	23
7	16	12	20	16	22	15	17	-	20	8
8	16	18	12	16	16	7	11	20	-	24
9	16	20	12	20	14	23	23	8	24	-

$\geq 90; (\frac{71}{235}, \frac{164}{235})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	5	6	6	7	6	6
1	2	-	5	6	7	8	6	5	4	8
2	5	5	-	5	8	7	5	6	7	3
3	4	6	5	-	7	6	6	7	2	6
4	5	7	8	7	-	7	3	4	5	5
5	6	8	7	6	7	-	4	3	4	6
6	6	6	5	6	3	4	-	7	6	8
7	7	5	6	7	4	3	7	-	7	7
8	6	4	7	2	5	4	6	7	-	4
9	6	8	3	6	5	6	8	7	4	-

$\geq$ 38; SMAPO# 3287

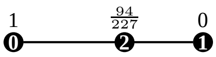
	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	6	6	9	9	11	11
1	3	-	8	10	9	9	8	6	8	10
2	7	8	-	8	11	9	8	8	12	4
3	7	10	8	-	11	9	10	10	4	12
4	6	9	11	11	-	8	3	5	7	7
5	6	9	9	9	8	-	5	3	7	9
6	9	8	8	10	3	5	-	8	8	10
7	9	6	8	10	5	3	8	-	10	10
8	11	8	12	4	7	7	8	10	-	8
9	11	10	4	12	7	9	10	10	8	-

$\geq$ 54; SMAPO# 8346

	0	1	2	3	4	5	6	7	8	9
0	-	5	11	11	11	12	15	16	17	17
1	5	-	12	16	16	17	14	11	12	18
2	11	12	-	12	18	15	12	13	18	6
3	11	16	12	-	18	15	16	17	6	18
4	11	16	18	18	-	15	6	9	12	12
5	12	17	15	15	15	-	9	6	11	15
6	15	14	12	16	6	9	-	15	14	18
7	16	11	13	17	9	6	15	-	17	17
8	17	12	18	6	12	11	14	17	-	12
9	17	18	6	18	12	15	18	17	12	-

$\geq 90; (\frac{40}{99}, \frac{59}{99})$

RHS = 92



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	4	4	5	5	5	5	7	7
1	1	-	5	5	4	4	6	6	6	6
2	4	5	-	6	9	9	7	7	9	3
3	4	5	6	-	9	9	7	7	3	9
4	5	4	9	9	-	4	2	6	6	6
5	5	4	9	9	4	-	6	2	8	8
6	5	6	7	7	2	6	-	4	8	8
7	5	6	7	7	6	2	4	-	6	6
8	7	6	9	3	6	8	8	6	-	6
9	7	6	3	9	6	8	8	6	6	-

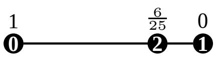
$\geq$ 38; SMAPO# 3475

	0	1	2	3	4	5	6	7	8	9
0	-	2	6	5	9	9	9	9	8	8
1	2	-	6	7	7	7	11	11	6	10
2	6	6	-	7	13	13	9	9	10	4
3	5	7	7	-	12	12	10	10	3	9
4	9	7	13	12	-	8	4	12	9	9
5	9	7	13	12	8	-	12	4	11	11
6	9	11	9	10	4	12	-	8	11	13
7	9	11	9	10	12	4	8	-	7	9
8	8	6	10	3	9	11	11	7	-	6
9	8	10	4	9	9	11	13	9	6	-

$\geq$ 56; SMAPO# 9011

	0	1	2	3	4	5	6	7	8	9
0	-	3	9	9	14	14	14	14	15	15
1	3	-	10	12	11	11	17	17	12	16
2	9	10	-	12	21	21	15	15	18	6
3	9	12	12	-	21	21	17	17	6	18
4	14	11	21	21	-	12	6	18	15	15
5	14	11	21	21	12	-	18	6	19	19
6	14	17	15	17	6	18	-	12	19	21
7	14	17	15	17	18	6	12	-	13	15
8	15	12	18	6	15	19	19	13	-	12
9	15	16	6	18	15	19	21	15	12	-

$\geq 92; (\frac{94}{227}, \frac{133}{227})$



TT-disjoint at 7

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	2	3	3	4	4	3
1	1	-	3	3	3	4	2	3	5	2
2	2	3	-	4	4	3	1	4	4	3
3	2	3	4	-	2	3	3	4	4	1
4	2	3	4	2	-	1	3	4	4	3
5	3	4	3	3	1	-	2	5	3	2
6	3	2	1	3	3	2	-	5	3	4
7	4	3	4	4	4	5	5	-	2	3
8	4	5	4	4	4	3	3	2	-	5
9	3	2	3	1	3	2	4	3	5	-

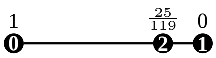
$\geq$ 20; SMAPO# 363

	0	1	2	3	4	5	6	7	8	9
0	-	5	8	9	10	11	13	13	12	14
1	5	-	13	14	11	14	8	10	13	9
2	8	13	-	13	14	11	5	9	14	10
3	9	14	13	-	9	14	12	12	13	5
4	10	11	14	9	-	5	11	9	14	14
5	11	14	11	14	5	-	10	14	9	9
6	13	8	5	12	11	10	-	14	9	13
7	13	10	9	12	9	14	14	-	5	9
8	12	13	14	13	14	9	9	5	-	12
9	14	9	10	5	14	9	13	9	12	-

$\geq$ 74; SMAPO# 12666

	0	1	2	3	4	5	6	7	8	9
0	-	6	10	11	12	14	16	16	16	17
1	6	-	16	17	14	18	10	12	18	11
2	10	16	-	17	18	14	6	12	18	13
3	11	17	17	-	11	17	15	15	17	6
4	12	14	18	11	-	6	14	12	18	17
5	14	18	14	17	6	-	12	18	12	11
6	16	10	6	15	14	12	-	18	12	17
7	16	12	12	15	12	18	18	-	6	11
8	16	18	18	17	18	12	12	6	-	17
9	17	11	13	6	17	11	17	11	17	-

$\geq 92; (\frac{6}{25}, \frac{19}{25})$



TT-disjoint at 5

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	4	3	4	4	4
1	1	-	3	4	2	5	4	3	3	5
2	2	3	-	3	1	4	3	4	4	4
3	3	4	3	-	4	5	2	5	3	3
4	3	2	1	4	-	3	4	5	5	3
5	4	5	4	5	3	-	3	4	2	4
6	3	4	3	2	4	3	-	3	5	5
7	4	3	4	5	5	4	3	-	4	2
8	4	3	4	3	5	2	5	4	-	4
9	4	5	4	3	3	4	5	2	4	-

$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	3	8	10	11	13	14	14	14	16
1	3	-	11	13	8	16	15	11	11	15
2	8	11	-	12	3	11	12	12	16	14
3	10	13	12	-	15	13	4	12	8	8
4	11	8	3	15	-	12	15	15	15	11
5	13	16	11	13	12	-	9	9	5	9
6	14	15	12	4	15	9	-	8	12	12
7	14	11	12	12	15	9	8	-	10	4
8	14	11	16	8	15	5	12	10	-	14
9	16	15	14	8	11	9	12	4	14	-

$\geq$ 70; SMAPO# 12163

	0	1	2	3	4	5	6	7	8	9
0	-	4	10	13	14	16	17	18	18	20
1	4	-	14	17	10	20	19	14	14	20
2	10	14	-	15	4	14	15	16	20	18
3	13	17	15	-	19	17	6	17	11	11
4	14	10	4	19	-	14	19	20	20	14
5	16	20	14	17	14	-	11	12	6	12
6	17	19	15	6	19	11	-	11	17	17
7	18	14	16	17	20	12	11	-	14	6
8	18	14	20	11	20	6	17	14	-	18
9	20	20	18	11	14	12	17	6	18	-

$\geq 92; (\frac{25}{119}, \frac{94}{119})$

TT-disjoint at 1

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	1	2	3	3	2	4	4
1	1	-	2	2	3	2	4	3	3	5
2	1	2	-	2	3	4	2	3	5	3
3	1	2	2	-	1	2	2	1	3	3
4	2	3	3	1	-	1	3	2	4	4
5	3	2	4	2	1	-	2	3	3	5
6	3	4	2	2	3	2	-	1	5	3
7	2	3	3	1	2	3	1	-	4	4
8	4	3	5	3	4	3	5	4	-	2
9	4	5	3	3	4	5	3	4	2	-

$\geq$ 18; SMAPO# 287

	0	1	2	3	4	5	6	7	8	9
0	-	4	3	10	12	12	12	14	12	12
1	4	-	7	6	12	10	16	10	10	16
2	3	7	-	11	13	15	9	15	15	9
3	10	6	11	-	6	12	14	8	16	10
4	12	12	13	6	-	6	16	14	18	12
5	12	10	15	12	6	-	10	16	12	18
6	12	16	9	14	16	10	-	6	18	12
7	14	10	15	8	14	16	6	-	20	18
8	12	10	15	16	18	12	18	20	-	6
9	12	16	9	10	12	18	12	18	6	-

$\geq$ 76; SMAPO# 13220

	0	1	2	3	4	5	6	7	8	9
0	-	4	4	11	14	15	15	16	16	16
1	4	-	8	7	14	11	19	12	12	20
2	4	8	-	13	16	19	11	18	20	12
3	11	7	13	-	7	14	16	9	19	13
4	14	14	16	7	-	7	19	16	22	16
5	15	11	19	14	7	-	12	19	15	23
6	15	19	11	16	19	12	-	7	23	15
7	16	12	18	9	16	19	7	-	24	22
8	16	12	20	19	22	15	23	24	-	8
9	16	20	12	13	16	23	15	22	8	-

$\geq 92; (\frac{44}{235}, \frac{191}{235})$

TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	4	4	4	4	6	6	4
1	1	-	3	3	3	5	3	5	5	5
2	2	3	-	6	6	4	6	4	4	4
3	4	3	6	-	4	2	4	2	6	6
4	4	3	6	4	-	2	4	6	2	6
5	4	5	4	2	2	-	6	4	4	4
6	4	3	6	4	4	6	-	6	6	2
7	6	5	4	2	6	4	6	-	4	8
8	6	5	4	6	2	4	6	4	-	8
9	4	5	4	6	6	4	2	8	8	-

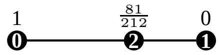
$\geq$ 28; SMAPO# 1404

	0	1	2	3	4	5	6	7	8	9
0	-	3	3	10	11	11	12	10	10	12
1	3	-	6	9	8	14	9	11	13	15
2	3	6	-	13	14	8	15	9	9	9
3	10	9	13	-	9	5	12	4	10	16
4	11	8	14	9	-	6	11	11	5	17
5	11	14	8	5	6	-	17	9	9	11
6	12	9	15	12	11	17	-	14	16	6
7	10	11	9	4	11	9	14	-	6	16
8	10	13	9	10	5	9	16	6	-	16
9	12	15	9	16	17	11	6	16	16	-

$\geq$ 66; SMAPO# 11643

	0	1	2	3	4	5	6	7	8	9
0	-	4	4	14	15	15	16	16	16	16
1	4	-	8	12	11	19	12	16	18	20
2	4	8	-	18	19	11	20	12	12	12
3	14	12	18	-	13	7	16	6	16	22
4	15	11	19	13	-	8	15	17	7	23
5	15	19	11	7	8	-	23	13	13	15
6	16	12	20	16	15	23	-	20	22	8
7	16	16	12	6	17	13	20	-	10	24
8	16	18	12	16	7	13	22	10	-	24
9	16	20	12	22	23	15	8	24	24	-

$\geq 92; (\frac{71}{244}, \frac{173}{244})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	4	6	5	5	6	7
1	1	-	3	2	5	5	4	6	7	6
2	2	3	-	5	4	4	7	7	6	5
3	3	2	5	-	7	7	4	4	5	6
4	4	5	4	7	-	8	3	7	6	9
5	6	5	4	7	8	-	7	3	8	5
6	5	4	7	4	3	7	-	8	9	6
7	5	6	7	4	7	3	8	-	5	8
8	6	7	6	5	6	8	9	5	-	3
9	7	6	5	6	9	5	6	8	3	-

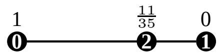
$\geq$ 36; SMAPO# 2790

	0	1	2	3	4	5	6	7	8	9
0	-	3	2	6	6	8	9	9	9	10
1	3	-	5	3	9	9	10	8	10	11
2	2	5	-	8	6	6	11	11	9	8
3	6	3	8	-	12	12	7	7	9	8
4	6	9	6	12	-	12	5	11	9	14
5	8	9	6	12	12	-	13	5	13	8
6	9	10	11	7	5	13	-	12	14	9
7	9	8	11	7	11	5	12	-	8	13
8	9	10	9	9	9	13	14	8	-	5
9	10	11	8	8	14	8	9	13	5	-

$\geq$ 58; SMAPO# 9594

	0	1	2	3	4	5	6	7	8	9
0	-	4	4	8	10	14	14	14	15	17
1	4	-	8	4	14	14	14	14	17	17
2	4	8	-	12	10	10	18	18	15	13
3	8	4	12	-	18	18	10	10	13	13
4	10	14	10	18	-	20	8	18	15	23
5	14	14	10	18	20	-	20	8	21	13
6	14	14	18	10	8	20	-	20	23	15
7	14	14	18	10	18	8	20	-	13	21
8	15	17	15	13	15	21	23	13	-	8
9	17	17	13	13	23	13	15	21	8	-

$\geq 92; (\frac{81}{212}, \frac{131}{212})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	5	5	4	4	4	4
1	1	-	4	4	4	4	5	5	3	3
2	3	4	-	4	6	2	5	5	7	7
3	3	4	4	-	2	6	5	5	7	7
4	5	4	6	2	-	4	5	5	5	5
5	5	4	2	6	4	-	5	5	5	5
6	4	5	5	5	5	5	-	4	6	2
7	4	5	5	5	5	5	4	-	2	6
8	4	3	7	7	5	5	6	2	-	4
9	4	3	7	7	5	5	2	6	4	-

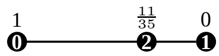
$\geq$ 30; SMAPO# 1506

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	6	9	9	10	10	11	11
1	3	-	6	9	6	10	13	13	8	8
2	7	6	-	7	10	4	9	9	14	14
3	6	9	7	-	3	9	12	12	13	13
4	9	6	10	3	-	6	9	11	10	12
5	9	10	4	9	6	-	11	13	10	12
6	10	13	9	12	9	11	-	10	15	5
7	10	13	9	12	11	13	10	-	5	15
8	11	8	14	13	10	10	15	5	-	10
9	11	8	14	13	12	12	5	15	10	-

$\geq$ 64; SMAPO# 11120

	0	1	2	3	4	5	6	7	8	9
0	-	4	9	9	14	14	14	14	15	15
1	4	-	9	13	10	14	18	18	11	11
2	9	9	-	10	15	5	13	13	20	20
3	9	13	10	-	5	15	17	17	20	20
4	14	10	15	5	-	10	14	16	15	17
5	14	14	5	15	10	-	16	18	15	17
6	14	18	13	17	14	16	-	14	21	7
7	14	18	13	17	16	18	14	-	7	21
8	15	11	20	20	15	15	21	7	-	14
9	15	11	20	20	17	17	7	21	14	-

$\geq 92; (\frac{11}{35}, \frac{24}{35})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	5	5	4	4	4	4
1	1	-	4	4	4	4	5	3	3	5
2	3	4	-	4	6	2	5	7	7	5
3	3	4	4	-	2	6	5	7	7	5
4	5	4	6	2	-	4	5	5	5	5
5	5	4	2	6	4	-	5	5	5	5
6	4	5	5	5	5	5	-	2	6	4
7	4	3	7	7	5	5	2	-	4	6
8	4	3	7	7	5	5	6	4	-	2
9	4	5	5	5	5	5	4	6	2	-

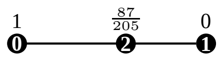
$\geq$ 30; SMAPO# 1506

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	6	9	9	10	11	11	12
1	3	-	6	9	6	10	13	8	8	13
2	7	6	-	7	10	4	9	14	14	9
3	6	9	7	-	3	9	10	13	13	12
4	9	6	10	3	-	6	11	10	12	9
5	9	10	4	9	6	-	13	10	12	11
6	10	13	9	10	11	13	-	5	15	10
7	11	8	14	13	10	10	5	-	10	15
8	11	8	14	13	12	12	15	10	-	5
9	12	13	9	12	9	11	10	15	5	-

$\geq$ 64; SMAPO# 11119

	0	1	2	3	4	5	6	7	8	9
0	-	4	9	9	14	14	14	15	15	16
1	4	-	9	13	10	14	18	11	11	18
2	9	9	-	10	15	5	13	20	20	13
3	9	13	10	-	5	15	15	20	20	17
4	14	10	15	5	-	10	16	15	17	14
5	14	14	5	15	10	-	18	15	17	16
6	14	18	13	15	16	18	-	7	21	14
7	15	11	20	20	15	15	7	-	14	21
8	15	11	20	20	17	17	21	14	-	7
9	16	18	13	17	14	16	14	21	7	-

$\geq 92; (\frac{11}{35}, \frac{24}{35})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	6	6	7	7	6	6
1	2	-	5	6	8	8	5	7	4	8
2	5	5	-	5	7	9	6	6	7	3
3	4	6	5	-	6	8	7	7	2	6
4	6	8	7	6	-	8	3	5	4	8
5	6	8	9	8	8	-	5	3	6	6
6	7	5	6	7	3	5	-	8	7	7
7	7	7	6	7	5	3	8	-	5	9
8	6	4	7	2	4	6	7	5	-	4
9	6	8	3	6	8	6	7	9	4	-

$\geq$ 40; SMAPO# 3906

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	6	6	9	9	11	11
1	3	-	8	10	9	9	6	8	8	10
2	7	8	-	8	9	11	8	8	12	4
3	7	10	8	-	9	11	10	10	4	12
4	6	9	9	9	-	8	3	5	7	9
5	6	9	11	11	8	-	5	3	7	7
6	9	6	8	10	3	5	-	8	10	10
7	9	8	8	10	5	3	8	-	8	10
8	11	8	12	4	7	7	10	8	-	8
9	11	10	4	12	9	7	10	10	8	-

$\geq$ 54; SMAPO# 8346

	0	1	2	3	4	5	6	7	8	9
0	-	5	11	11	12	12	16	16	17	17
1	5	-	12	16	17	17	11	15	12	18
2	11	12	-	12	15	19	13	13	18	6
3	11	16	12	-	15	19	17	17	6	18
4	12	17	15	15	-	16	6	10	11	17
5	12	17	19	19	16	-	10	6	13	13
6	16	11	13	17	6	10	-	16	17	17
7	16	15	13	17	10	6	16	-	13	19
8	17	12	18	6	11	13	17	13	-	12
9	17	18	6	18	17	13	17	19	12	-

$\geq 92; (\frac{87}{205}, \frac{118}{205})$

TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	2	2	3	3	5	4	3
1	1	-	4	3	3	2	2	4	3	2
2	3	4	-	5	3	4	4	2	5	4
3	2	3	5	-	4	3	1	3	2	3
4	2	3	3	4	-	3	3	5	2	1
5	3	2	4	3	3	-	4	4	1	2
6	3	2	4	1	3	4	-	4	3	4
7	5	4	2	3	5	4	4	-	3	4
8	4	3	5	2	2	1	3	3	-	3
9	3	2	4	3	1	2	4	4	3	-

$\geq$ 20; SMAPO# 363

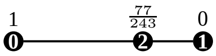
	0	1	2	3	4	5	6	7	8	9
0	-	5	8	8	9	11	13	11	12	14
1	5	-	13	13	14	10	8	12	13	9
2	8	13	-	14	11	9	9	5	14	12
3	8	13	14	-	15	13	5	9	10	12
4	9	14	11	15	-	14	10	14	9	5
5	11	10	9	13	14	-	14	14	5	9
6	13	8	9	5	10	14	-	14	9	13
7	11	12	5	9	14	14	14	-	9	13
8	12	13	14	10	9	5	9	9	-	14
9	14	9	12	12	5	9	13	13	14	-

$\geq$ 74; SMAPO# 12683

	0	1	2	3	4	5	6	7	8	9
0	-	6	10	10	11	14	16	16	16	17
1	6	-	16	16	17	12	10	16	16	11
2	10	16	-	18	13	12	12	6	18	15
3	10	16	18	-	19	16	6	12	12	15
4	11	17	13	19	-	17	13	19	11	6
5	14	12	12	16	17	-	18	18	6	11
6	16	10	12	6	13	18	-	18	12	17
7	16	16	6	12	19	18	18	-	12	17
8	16	16	18	12	11	6	12	12	-	17
9	17	11	15	15	6	11	17	17	17	-

$\geq 92; (\frac{24}{101}, \frac{77}{101})$

RHS = 94



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	0	2	3	4	4	6	6	6	6
1	0	-	2	3	4	4	6	6	6	6
2	2	2	-	3	6	6	4	4	6	6
3	3	3	3	-	5	5	5	5	3	3
4	4	4	6	5	-	4	6	2	4	6
5	4	4	6	5	4	-	6	6	6	2
6	6	6	4	5	6	6	-	4	2	6
7	6	6	4	5	2	6	4	-	6	4
8	6	6	6	3	4	6	2	6	-	4
9	6	6	6	3	6	2	6	4	4	-

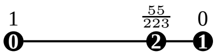
$\geq$ 28; SMAPO# 1264

	0	1	2	3	4	5	6	7	8	9
0	-	2	4	9	8	12	10	11	13	13
1	2	-	6	7	10	10	12	13	15	15
2	4	6	-	5	12	12	8	9	13	13
3	9	7	5	-	13	13	13	12	8	8
4	8	10	12	13	-	10	14	5	9	15
5	12	10	12	13	10	-	14	15	15	5
6	10	12	8	13	14	14	-	9	5	13
7	11	13	9	12	5	15	9	-	14	10
8	13	15	13	8	9	15	5	14	-	10
9	13	15	13	8	15	5	13	10	10	-

$\geq$ 68; SMAPO# 11792

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	12	12	16	16	17	19	19
1	2	-	7	10	14	14	18	19	21	21
2	5	7	-	7	17	17	11	12	18	18
3	12	10	7	-	18	18	18	17	11	11
4	12	14	17	18	-	14	20	7	13	21
5	16	14	17	18	14	-	20	21	21	7
6	16	18	11	18	20	20	-	13	7	19
7	17	19	12	17	7	21	13	-	20	14
8	19	21	18	11	13	21	7	20	-	14
9	19	21	18	11	21	7	19	14	14	-

$\geq 94; (\frac{77}{243}, \frac{166}{243})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	2	3	3	5	5	4
1	1	-	4	4	3	4	4	4	4	3
2	3	4	-	4	5	4	4	6	2	5
3	3	4	4	-	5	4	4	2	6	5
4	2	3	5	5	-	1	3	3	3	2
5	3	4	4	4	1	-	2	4	4	3
6	3	4	4	4	3	2	-	4	4	1
7	5	4	6	2	3	4	4	-	4	5
8	5	4	2	6	3	4	4	4	-	5
9	4	3	5	5	2	3	1	5	5	-

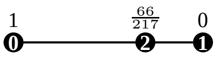
$\geq$ 24; SMAPO# 665

	0	1	2	3	4	5	6	7	8	9
0	-	3	8	7	10	11	12	11	11	13
1	3	-	9	10	11	14	15	8	14	10
2	8	9	-	9	16	11	12	13	5	15
3	7	10	9	-	15	12	13	4	12	14
4	10	11	16	15	-	5	14	11	11	9
5	11	14	11	12	5	-	9	10	16	14
6	12	15	12	13	14	9	-	9	15	5
7	11	8	13	4	11	10	9	-	8	14
8	11	14	5	12	11	16	15	8	-	14
9	13	10	15	14	9	14	5	14	14	-

$\geq$ 72; SMAPO# 12446

	0	1	2	3	4	5	6	7	8	9
0	-	4	10	10	12	14	15	16	16	17
1	4	-	12	14	14	18	19	12	18	13
2	10	12	-	12	20	14	15	18	6	19
3	10	14	12	-	20	16	17	6	18	19
4	12	14	20	20	-	6	17	14	14	11
5	14	18	14	16	6	-	11	14	20	17
6	15	19	15	17	17	11	-	13	19	6
7	16	12	18	6	14	14	13	-	12	19
8	16	18	6	18	14	20	19	12	-	19
9	17	13	19	19	11	17	6	19	19	-

$\geq 94; (\frac{55}{223}, \frac{168}{223})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	5	4	5	4	4	4
1	1	-	4	4	4	3	4	5	5	3
2	3	4	-	4	6	7	2	5	5	7
3	3	4	4	-	2	7	6	5	5	7
4	5	4	6	2	-	5	4	5	5	5
5	4	3	7	7	5	-	5	6	2	4
6	5	4	2	6	4	5	-	5	5	5
7	4	5	5	5	5	6	5	-	4	2
8	4	5	5	5	5	2	5	4	-	6
9	4	3	7	7	5	4	5	2	6	-

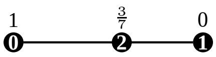
$\geq$ 30; SMAPO# 1506

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	6	9	10	9	11	11	12
1	3	-	8	9	6	9	12	14	14	9
2	7	8	-	7	10	15	4	10	10	15
3	6	9	7	-	3	14	9	11	11	14
4	9	6	10	3	-	11	6	8	10	11
5	10	9	15	14	11	-	11	15	5	10
6	9	12	4	9	6	11	-	12	14	13
7	11	14	10	11	8	15	12	-	10	5
8	11	14	10	11	10	5	14	10	-	15
9	12	9	15	14	11	10	13	5	15	-

$\geq$ 66; SMAPO# 11529

	0	1	2	3	4	5	6	7	8	9
0	-	4	9	9	14	14	14	15	15	16
1	4	-	11	13	10	12	16	19	19	12
2	9	11	-	10	15	21	5	14	14	21
3	9	13	10	-	5	21	15	16	16	21
4	14	10	15	5	-	16	10	13	15	16
5	14	12	21	21	16	-	16	21	7	14
6	14	16	5	15	10	16	-	17	19	18
7	15	19	14	16	13	21	17	-	14	7
8	15	19	14	16	15	7	19	14	-	21
9	16	12	21	21	16	14	18	7	21	-

$\geq 94; (\frac{66}{217}, \frac{151}{217})$



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	4	6	6	7	6	6
1	2	-	5	6	6	8	6	5	4	8
2	5	5	-	9	9	9	7	6	7	3
3	4	6	9	-	4	8	6	9	2	8
4	4	6	9	4	-	6	2	9	6	6
5	6	8	9	8	6	-	8	3	6	6
6	6	6	7	6	2	8	-	9	4	8
7	7	5	6	9	9	3	9	-	7	9
8	6	4	7	2	6	6	4	7	-	6
9	6	8	3	8	6	6	8	9	6	-

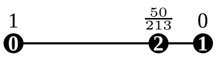
$\geq$ 40; SMAPO# 4059

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	6	7	6	9	9	10	10
1	3	-	7	9	10	9	10	6	7	9
2	6	7	-	12	13	10	9	7	10	4
3	6	9	12	-	7	10	11	11	4	10
4	7	10	13	7	-	9	4	12	11	9
5	6	9	10	10	9	-	11	3	6	6
6	9	10	9	11	4	11	-	12	7	13
7	9	6	7	11	12	3	12	-	9	9
8	10	7	10	4	11	6	7	9	-	8
9	10	9	4	10	9	6	13	9	8	-

$\geq$ 56; SMAPO# 8969

	0	1	2	3	4	5	6	7	8	9
0	-	5	10	10	11	12	15	16	16	16
1	5	-	11	15	16	17	16	11	11	17
2	10	11	-	20	21	18	15	12	16	6
3	10	15	20	-	11	18	17	20	6	18
4	11	16	21	11	-	15	6	21	17	15
5	12	17	18	18	15	-	19	6	12	12
6	15	16	15	17	6	19	-	21	11	21
7	16	11	12	20	21	6	21	-	16	18
8	16	11	16	6	17	12	11	16	-	14
9	16	17	6	18	15	12	21	18	14	-

$\geq 94; (\frac{3}{7}, \frac{4}{7})$



TT-disjoint at 4

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	4	3	3	4	4	4
1	1	-	3	4	3	2	4	5	5	3
2	2	3	-	3	4	1	3	4	4	4
3	3	4	3	-	5	4	2	5	3	3
4	4	3	4	5	-	5	3	4	2	4
5	3	2	1	4	5	-	4	3	3	5
6	3	4	3	2	3	4	-	3	5	5
7	4	5	4	5	4	3	3	-	4	2
8	4	5	4	3	2	3	5	4	-	4
9	4	3	4	3	4	5	5	2	4	-

$\geq$ 24; SMAPO# 662

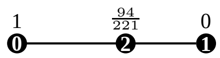
	0	1	2	3	4	5	6	7	8	9
0	-	5	8	9	12	13	13	13	13	15
1	5	-	13	14	11	8	14	14	14	10
2	8	13	-	11	10	5	11	11	15	13
3	9	14	11	-	13	12	4	12	8	8
4	12	11	10	13	-	15	9	9	5	9
5	13	8	5	12	15	-	14	10	10	14
6	13	14	11	4	9	14	-	8	12	12
7	13	14	11	12	9	10	8	-	10	4
8	13	14	15	8	5	10	12	10	-	14
9	15	10	13	8	9	14	12	4	14	-

$\geq$ 72; SMAPO# 12383

	0	1	2	3	4	5	6	7	8	9
0	-	6	10	12	15	16	16	17	17	19
1	6	-	16	18	13	10	18	19	19	13
2	10	16	-	14	13	6	14	15	19	17
3	12	18	14	-	17	16	6	17	11	11
4	15	13	13	17	-	19	11	12	6	12
5	16	10	6	16	19	-	18	13	13	19
6	16	18	14	6	11	18	-	11	17	17
7	17	19	15	17	12	13	11	-	14	6
8	17	19	19	11	6	13	17	14	-	18
9	19	13	17	11	12	19	17	6	18	-

$\geq 94; (\frac{50}{213}, \frac{163}{213})$

RHS = 96



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	4	6	7	6	8	6	7
1	2	-	5	6	6	9	4	6	8	9
2	5	5	-	5	9	6	7	9	3	6
3	4	6	5	-	8	7	2	8	6	7
4	6	6	9	8	-	9	6	6	6	3
5	7	9	6	7	9	-	5	3	7	6
6	6	4	7	2	6	5	-	8	4	7
7	8	6	9	8	6	3	8	-	8	9
8	6	8	3	6	6	7	4	8	-	9
9	7	9	6	7	3	6	7	9	9	-

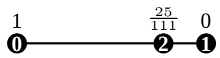
≥ 42; SMAPO# 4551

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	7	6	11	9	11	10
1	3	-	8	10	6	9	8	6	10	9
2	7	8	-	8	12	9	12	12	4	9
3	7	10	8	-	12	9	4	12	12	11
4	7	6	12	12	-	9	8	6	8	3
5	6	9	9	9	9	-	7	3	9	6
6	11	8	12	4	8	7	-	10	8	9
7	9	6	12	12	6	3	10	-	10	9
8	11	10	4	12	8	9	8	10	-	11
9	10	9	9	11	3	6	9	9	11	-

≥ 56; SMAPO# 8911

	0	1	2	3	4	5	6	7	8	9
0	-	5	11	11	13	13	17	17	17	17
1	5	-	12	16	12	18	12	12	18	18
2	11	12	-	12	20	14	18	20	6	14
3	11	16	12	-	20	16	6	20	18	18
4	13	12	20	20	-	18	14	12	14	6
5	13	18	14	16	18	-	12	6	16	12
6	17	12	18	6	14	12	-	18	12	16
7	17	12	20	20	12	6	18	-	18	18
8	17	18	6	18	14	16	12	18	-	20
9	17	18	14	18	6	12	16	18	20	-

$\geq 96; (\frac{94}{221}, \frac{127}{221})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	3	3	4	5	5	4
1	1	-	3	4	4	2	3	4	4	3
2	2	3	-	5	5	1	4	3	3	4
3	3	4	5	-	4	4	3	2	4	5
4	3	4	5	4	-	4	5	4	2	3
5	3	2	1	4	4	-	3	4	4	3
6	4	3	4	3	5	3	-	5	3	2
7	5	4	3	2	4	4	5	-	4	3
8	5	4	3	4	2	4	3	4	-	5
9	4	3	4	5	3	3	2	3	5	-

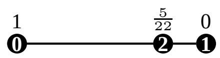
≥ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	5	8	10	9	13	13	13	13	15
1	5	-	13	15	14	8	14	14	14	10
2	8	13	-	16	15	5	15	11	11	13
3	10	15	16	-	9	11	11	5	11	15
4	9	14	15	9	-	12	12	14	4	8
5	13	8	5	11	12	-	10	16	10	12
6	13	14	15	11	12	10	-	14	8	4
7	13	14	11	5	14	16	14	-	10	10
8	13	14	11	11	4	10	8	10	-	12
9	15	10	13	15	8	12	4	10	12	-

≥ 74; SMAPO# 12727

	0	1	2	3	4	5	6	7	8	9
0	-	6	10	12	12	16	17	18	18	19
1	6	-	16	18	18	10	17	18	18	13
2	10	16	-	20	20	6	19	14	14	17
3	12	18	20	-	12	14	13	6	14	19
4	12	18	20	12	-	16	17	18	6	11
5	16	10	6	14	16	-	13	20	14	15
6	17	17	19	13	17	13	-	19	11	6
7	18	18	14	6	18	20	19	-	14	13
8	18	18	14	14	6	14	11	14	-	17
9	19	13	17	19	11	15	6	13	17	-

$\geq 96; (\frac{25}{111}, \frac{86}{111})$



TT-disjoint at 7

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	5	5	4	4
1	2	-	5	5	3	3	3	3	4	4
2	3	5	-	4	4	4	2	4	5	3
3	3	5	4	-	4	4	4	2	3	5
4	3	3	4	4	-	2	4	4	3	1
5	3	3	4	4	2	-	4	4	1	3
6	5	3	2	4	4	4	-	4	3	5
7	5	3	4	2	4	4	4	-	5	3
8	4	4	5	3	3	1	3	5	-	2
9	4	4	3	5	1	3	5	3	2	-

≥ 24; SMAPO# 662

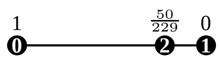
	0	1	2	3	4	5	6	7	8	9
0	-	4	8	8	9	10	12	13	13	14
1	4	-	12	12	13	12	8	9	15	14
2	8	12	-	14	15	14	4	9	15	10
3	8	12	14	-	15	16	10	5	11	14
4	9	13	15	15	-	9	13	12	14	5
5	10	12	14	16	9	-	12	11	5	14
6	12	8	4	10	13	12	-	9	11	14
7	13	9	9	5	12	11	9	-	16	11
8	13	15	15	11	14	5	11	16	-	9
9	14	14	10	14	5	14	14	11	9	-

≥ 74; SMAPO# 12730

	0	1	2	3	4	5	6	7	8	9
0	-	6	11	11	12	13	17	17	17	18
1	6	-	17	17	16	15	11	11	19	18
2	11	17	-	18	19	18	6	12	20	13
3	11	17	18	-	19	20	14	6	14	19
4	12	16	19	19	-	11	17	15	17	6
5	13	15	18	20	11	-	16	14	6	17
6	17	11	6	14	17	16	-	12	14	19
7	17	11	12	6	15	14	12	-	20	13
8	17	19	20	14	17	6	14	20	-	11
9	18	18	13	19	6	17	19	13	11	-

$\geq 96; (\frac{5}{22}, \frac{17}{22})$

RHS = 98



TT-disjoint at 7

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	5	5	4	4
1	2	-	5	5	3	3	3	3	4	4
2	3	5	-	4	4	4	2	4	3	5
3	3	5	4	-	4	4	4	2	5	3
4	3	3	4	4	-	2	4	4	3	1
5	3	3	4	4	2	-	4	4	1	3
6	5	3	2	4	4	4	-	4	5	3
7	5	3	4	2	4	4	4	-	3	5
8	4	4	3	5	3	1	5	3	-	2
9	4	4	5	3	1	3	3	5	2	-

≥ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	4	8	8	9	9	12	13	14	14
1	4	-	12	12	13	13	8	9	14	16
2	8	12	-	14	15	15	4	9	10	16
3	8	12	14	-	17	17	10	5	14	12
4	9	13	15	17	-	10	13	12	15	5
5	9	13	15	17	10	-	13	14	5	15
6	12	8	4	10	13	13	-	9	14	12
7	13	9	9	5	12	14	9	-	11	17
8	14	14	10	14	15	5	14	11	-	10
9	14	16	16	12	5	15	12	17	10	-

≥ 76; SMAPO# 13090

	0	1	2	3	4	5	6	7	8	9
0	-	6	11	11	12	12	17	17	18	18
1	6	-	17	17	16	16	11	11	18	20
2	11	17	-	18	19	19	6	12	13	21
3	11	17	18	-	21	21	14	6	19	15
4	12	16	19	21	-	12	17	15	18	6
5	12	16	19	21	12	-	17	17	6	18
6	17	11	6	14	17	17	-	12	19	15
7	17	11	12	6	15	17	12	-	13	21
8	18	18	13	19	18	6	19	13	-	12
9	18	20	21	15	6	18	15	21	12	-

$\geq 98; (\frac{50}{229}, \frac{179}{229})$

RHS = 100

TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	2	4	3	4
1	1	-	3	3	4	4	3	3	2	3
2	2	3	-	4	3	5	4	2	3	4
3	2	3	4	-	5	3	4	4	3	2
4	3	4	3	5	-	2	3	5	4	3
5	3	4	5	3	2	-	3	3	4	5
6	2	3	4	4	3	3	-	2	1	2
7	4	3	2	4	5	3	2	-	3	4
8	3	2	3	3	4	4	1	3	-	3
9	4	3	4	2	3	5	2	4	3	-

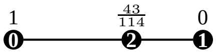
≥ 22; SMAPO# 464

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	9	8	11	13	12	13	13
1	3	-	10	10	11	14	16	9	10	16
2	7	10	-	16	9	14	16	5	16	16
3	9	10	16	-	17	12	18	13	12	6
4	8	11	9	17	-	5	11	14	17	11
5	11	14	14	12	5	-	14	9	16	16
6	13	16	16	18	11	14	-	11	6	12
7	12	9	5	13	14	9	11	-	13	15
8	13	10	16	12	17	16	6	13	-	18
9	13	16	16	6	11	16	12	15	18	-

≥ 80; SMAPO# 13606

	0	1	2	3	4	5	6	7	8	9
0	-	4	9	10	11	14	15	16	16	17
1	4	-	13	12	15	18	19	12	12	19
2	9	13	-	19	12	19	20	7	19	20
3	10	12	19	-	21	14	21	16	14	7
4	11	15	12	21	-	7	14	19	21	14
5	14	18	19	14	7	-	17	12	20	21
6	15	19	20	21	14	17	-	13	7	14
7	16	12	7	16	19	12	13	-	16	19
8	16	12	19	14	21	20	7	16	-	21
9	17	19	20	7	14	21	14	19	21	-

$\geq 100; (\frac{43}{226}, \frac{183}{226})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	2	3	4	4
1	1	-	3	3	4	4	3	2	3	3
2	2	3	-	4	3	5	4	3	2	4
3	2	3	4	-	5	3	4	3	4	2
4	3	4	3	5	-	2	3	4	5	3
5	3	4	5	3	2	-	3	4	3	5
6	2	3	4	4	3	3	-	1	2	2
7	3	2	3	3	4	4	1	-	3	3
8	4	3	2	4	5	3	2	3	-	4
9	4	3	4	2	3	5	2	3	4	-

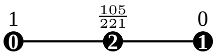
$\geq$ 22; SMAPO# 464

	0	1	2	3	4	5	6	7	8	9
0	-	2	5	9	5	8	10	9	8	10
1	2	-	7	7	7	10	12	7	6	12
2	5	7	-	14	6	9	11	14	3	11
3	9	7	14	-	14	11	15	10	11	5
4	5	7	6	14	-	3	9	14	9	9
5	8	10	9	11	3	-	10	13	6	12
6	10	12	11	15	9	10	-	5	8	10
7	9	7	14	10	14	13	5	-	11	15
8	8	6	3	11	9	6	8	11	-	10
9	10	12	11	5	9	12	10	15	10	-

$\geq$ 60; SMAPO# 10227

	0	1	2	3	4	5	6	7	8	9
0	-	4	9	11	11	14	14	15	16	18
1	4	-	13	11	15	18	18	11	12	18
2	9	13	-	20	12	19	19	20	7	19
3	11	11	20	-	22	15	21	14	17	7
4	11	15	12	22	-	7	15	22	19	15
5	14	18	19	15	7	-	16	21	12	22
6	14	18	19	21	15	16	-	7	12	14
7	15	11	20	14	22	21	7	-	17	21
8	16	12	7	17	19	12	12	17	-	18
9	18	18	19	7	15	22	14	21	18	-

$\geq 100; (\frac{43}{114}, \frac{71}{114})$



TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	5	7	7	7	7	11	11
1	3	-	6	8	10	8	4	6	8	10
2	5	6	-	6	8	8	2	4	8	8
3	5	8	6	-	10	10	4	2	6	6
4	7	10	8	10	-	8	8	8	4	12
5	7	8	8	10	8	-	6	8	12	4
6	7	4	2	4	8	6	-	6	8	8
7	7	6	4	2	8	8	6	-	8	8
8	11	8	8	6	4	12	8	8	-	8
9	11	10	8	6	12	4	8	8	8	-

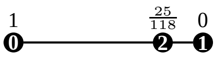
$\geq$ 48; SMAPO# 6681

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	6	7	7	8	9	11	11
1	3	-	8	7	8	10	5	8	8	8
2	5	8	-	9	8	10	3	6	10	8
3	6	7	9	-	11	11	6	3	7	7
4	7	8	8	11	-	8	9	8	4	12
5	7	10	10	11	8	-	9	10	12	4
6	8	5	3	6	9	9	-	9	9	9
7	9	8	6	3	8	10	9	-	10	10
8	11	8	10	7	4	12	9	10	-	8
9	11	8	8	7	12	4	9	10	8	-

$\geq$ 54; SMAPO# 8342

	0	1	2	3	4	5	6	7	8	9
0	-	5	9	10	13	13	14	15	21	21
1	5	-	14	15	18	18	9	14	16	18
2	9	14	-	15	16	18	5	10	18	16
3	10	15	15	-	21	21	10	5	13	13
4	13	18	16	21	-	16	17	16	8	24
5	13	18	18	21	16	-	15	18	24	8
6	14	9	5	10	17	15	-	15	17	17
7	15	14	10	5	16	18	15	-	18	18
8	21	16	18	13	8	24	17	18	-	16
9	21	18	16	13	24	8	17	18	16	-

$\geq 100; (\frac{105}{221}, \frac{116}{221})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	4	3	5	4	4	4	5
1	2	-	5	4	5	3	4	4	4	3
2	3	5	-	5	4	2	3	3	3	4
3	4	4	5	-	3	3	2	4	4	5
4	3	5	4	3	-	4	5	3	1	2
5	5	3	2	3	4	-	5	3	3	4
6	4	4	3	2	5	5	-	4	4	3
7	4	4	3	4	3	3	4	-	2	1
8	4	4	3	4	1	3	4	2	-	3
9	5	3	4	5	2	4	3	1	3	-

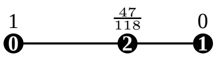
$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	4	8	9	9	12	14	14	14	17
1	4	-	12	11	13	8	10	12	14	13
2	8	12	-	15	15	4	10	10	10	15
3	9	11	15	-	10	11	5	13	15	18
4	9	13	15	10	-	13	13	15	5	10
5	12	8	4	11	13	-	14	12	14	17
6	14	10	10	5	13	14	-	18	18	13
7	14	12	10	13	15	12	18	-	10	5
8	14	14	10	15	5	14	18	10	-	15
9	17	13	15	18	10	17	13	5	15	-

$\geq$ 78; SMAPO# 13417

	0	1	2	3	4	5	6	7	8	9
0	-	6	11	12	12	17	18	18	18	22
1	6	-	17	14	18	11	14	16	18	16
2	11	17	-	19	19	6	13	13	13	19
3	12	14	19	-	12	13	6	16	18	22
4	12	18	19	12	-	17	18	18	6	12
5	17	11	6	13	17	-	19	15	17	21
6	18	14	13	6	18	19	-	22	22	16
7	18	16	13	16	18	15	22	-	12	6
8	18	18	13	18	6	17	22	12	-	18
9	22	16	19	22	12	21	16	6	18	-

$\geq 100; (\frac{25}{118}, \frac{93}{118})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	6	5	8	7	7	8	8
1	3	-	8	7	8	5	6	8	5	7
2	5	8	-	9	8	3	6	6	7	7
3	6	7	9	-	5	6	3	7	8	6
4	5	8	8	5	-	7	8	2	5	7
5	8	5	3	6	7	-	9	7	8	6
6	7	6	6	3	8	9	-	6	5	7
7	7	8	6	7	2	7	6	-	7	5
8	8	5	7	8	5	8	5	7	-	2
9	8	7	7	6	7	6	7	5	2	-

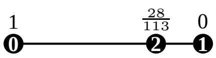
$\geq$ 42; SMAPO# 4504

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	7	7	9	11	11	11	11
1	3	-	9	10	10	6	8	12	8	8
2	6	9	-	13	13	3	9	9	11	11
3	7	10	13	-	8	10	4	12	12	8
4	7	10	13	8	-	10	10	4	8	12
5	9	6	3	10	10	-	12	12	12	8
6	11	8	9	4	10	12	-	10	8	12
7	11	12	9	12	4	12	10	-	12	8
8	11	8	11	12	8	12	8	12	-	4
9	11	8	11	8	12	8	12	8	4	-

$\geq$ 60; SMAPO# 9993

	0	1	2	3	4	5	6	7	8	9
0	-	6	11	12	12	17	18	18	19	19
1	6	-	17	16	18	11	14	20	13	15
2	11	17	-	21	21	6	15	15	18	18
3	12	16	21	-	12	15	6	18	19	13
4	12	18	21	12	-	17	18	6	13	19
5	17	11	6	15	17	-	21	19	20	14
6	18	14	15	6	18	21	-	16	13	19
7	18	20	15	18	6	19	16	-	19	13
8	19	13	18	19	13	20	13	19	-	6
9	19	15	18	13	19	14	19	13	6	-

$\geq 100; (\frac{47}{118}, \frac{71}{118})$



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	5	5	4	5	4
1	2	-	5	5	5	5	3	4	3	4
2	3	5	-	6	6	4	2	5	4	5
3	3	5	6	-	4	6	4	5	2	3
4	3	5	6	4	-	2	4	3	4	5
5	5	5	4	6	2	-	6	5	4	3
6	5	3	2	4	4	6	-	5	4	3
7	4	4	5	5	3	5	5	-	3	2
8	5	3	4	2	4	4	4	3	-	5
9	4	4	5	3	5	3	3	2	5	-

$\geq$ 28; SMAPO# 1227

	0	1	2	3	4	5	6	7	8	9
0	-	4	8	10	10	11	12	13	13	14
1	4	-	12	14	14	13	8	9	9	14
2	8	12	-	14	14	9	4	13	11	14
3	10	14	14	-	10	15	10	13	5	12
4	10	14	14	10	-	5	12	9	15	14
5	11	13	9	15	5	-	13	14	10	9
6	12	8	4	10	12	13	-	15	15	10
7	13	9	13	13	9	14	15	-	10	5
8	13	9	11	5	15	10	15	10	-	15
9	14	14	14	12	14	9	10	5	15	-

$\geq$ 74; SMAPO# 12704

	0	1	2	3	4	5	6	7	8	9
0	-	6	11	12	13	16	17	17	18	18
1	6	-	17	18	19	18	11	13	12	18
2	11	17	-	19	20	13	6	18	15	19
3	12	18	19	-	13	20	13	17	6	14
4	13	19	20	13	-	7	16	12	19	19
5	16	18	13	20	7	-	19	19	14	12
6	17	11	6	13	16	19	-	20	19	13
7	17	13	18	17	12	19	20	-	13	7
8	18	12	15	6	19	14	19	13	-	20
9	18	18	19	14	19	12	13	7	20	-

$\geq 100; (\frac{28}{113}, \frac{85}{113})$

TT-disjoint at 0

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	4	4	4	3	4	5	5
1	2	-	5	4	4	4	5	4	3	3
2	3	5	-	5	3	3	4	3	2	4
3	4	4	5	-	2	4	3	4	3	5
4	4	4	3	2	-	4	5	4	5	3
5	4	4	3	4	4	-	3	2	3	1
6	3	5	4	3	5	3	-	1	4	2
7	4	4	3	4	4	2	1	-	3	3
8	5	3	2	3	5	3	4	3	-	4
9	5	3	4	5	3	1	2	3	4	-

$\geq$ 24; SMAPO# 662

	0	1	2	3	4	5	6	7	8	9
0	-	5	9	9	11	11	12	13	13	16
1	5	-	12	14	10	16	15	16	8	11
2	9	12	-	14	10	12	17	14	4	15
3	9	14	14	-	4	16	11	16	10	15
4	11	10	10	4	-	14	15	16	14	11
5	11	16	12	16	14	-	15	10	10	5
6	12	15	17	11	15	15	-	5	15	10
7	13	16	14	16	16	10	5	-	10	15
8	13	8	4	10	14	10	15	10	-	15
9	16	11	15	15	11	5	10	15	15	-

$\geq$ 78; SMAPO# 13320

	0	1	2	3	4	5	6	7	8	9
0	-	6	11	12	14	14	14	16	17	20
1	6	-	17	18	14	20	20	20	11	14
2	11	17	-	19	13	15	21	17	6	19
3	12	18	19	-	6	20	14	20	13	20
4	14	14	13	6	-	18	20	20	19	14
5	14	20	15	20	18	-	18	12	13	6
6	14	20	21	14	20	18	-	6	19	12
7	16	20	17	20	20	12	6	-	13	18
8	17	11	6	13	19	13	19	13	-	19
9	20	14	19	20	14	6	12	18	19	-

$\geq 100; (\frac{25}{119}, \frac{94}{119})$

RHS = 106

TT-disjoint at 4

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	3	3	4	5
1	1	-	3	3	4	4	2	2	3	4
2	2	3	-	4	3	3	1	3	2	5
3	2	3	4	-	5	3	3	1	2	3
4	3	4	3	5	-	4	4	4	5	2
5	3	4	3	3	4	-	2	4	1	4
6	3	2	1	3	4	2	-	2	3	4
7	3	2	3	1	4	4	2	-	3	4
8	4	3	2	2	5	1	3	3	-	3
9	5	4	5	3	2	4	4	4	3	-

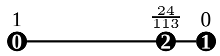
$\geq$ 20; SMAPO# 363

	0	1	2	3	4	5	6	7	8	9
0	-	6	10	10	11	12	16	16	16	15
1	6	-	16	16	13	14	10	10	14	15
2	10	16	-	16	11	16	6	14	10	15
3	10	16	16	-	17	12	14	6	12	11
4	11	13	11	17	-	11	15	11	17	6
5	12	14	16	12	11	-	10	16	6	17
6	16	10	6	14	15	10	-	12	16	15
7	16	10	14	6	11	16	12	-	14	17
8	16	14	10	12	17	6	16	14	-	11
9	15	15	15	11	6	17	15	17	11	-

$\geq$ 88; SMAPO# 14125

	0	1	2	3	4	5	6	7	8	9
0	-	7	12	12	13	15	19	19	20	20
1	7	-	19	19	16	18	12	12	17	19
2	12	19	-	20	13	19	7	17	12	20
3	12	19	20	-	21	15	17	7	14	14
4	13	16	13	21	-	14	18	14	21	7
5	15	18	19	15	14	-	12	20	7	21
6	19	12	7	17	18	12	-	14	19	19
7	19	12	17	7	14	20	14	-	17	21
8	20	17	12	14	21	7	19	17	-	14
9	20	19	20	14	7	21	19	21	14	-

$\geq 106; (\frac{24}{113}, \frac{89}{113})$



TT-disjoint at 4

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	3	3	3	3	4	5
1	1	-	3	3	4	4	2	2	3	4
2	2	3	-	4	3	3	1	3	2	5
3	2	3	4	-	5	3	3	1	2	3
4	3	4	3	5	-	4	4	4	5	2
5	3	4	3	3	4	-	2	4	1	4
6	3	2	1	3	4	2	-	2	3	4
7	3	2	3	1	4	4	2	-	3	4
8	4	3	2	2	5	1	3	3	-	3
9	5	4	5	3	2	4	4	4	3	-

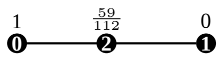
$\geq$ 20; SMAPO# 363

	0	1	2	3	4	5	6	7	8	9
0	-	6	10	10	11	14	16	16	16	15
1	6	-	16	16	13	14	10	10	14	15
2	10	16	-	16	11	16	6	14	10	15
3	10	16	16	-	17	12	12	6	12	11
4	11	13	11	17	-	11	15	11	17	6
5	14	14	16	12	11	-	10	16	6	17
6	16	10	6	12	15	10	-	12	16	15
7	16	10	14	6	11	16	12	-	14	17
8	16	14	10	12	17	6	16	14	-	11
9	15	15	15	11	6	17	15	17	11	-

$\geq$ 88; SMAPO# 14126

	0	1	2	3	4	5	6	7	8	9
0	-	7	12	12	13	17	19	19	20	20
1	7	-	19	19	16	18	12	12	17	19
2	12	19	-	20	13	19	7	17	12	20
3	12	19	20	-	21	15	15	7	14	14
4	13	16	13	21	-	14	18	14	21	7
5	17	18	19	15	14	-	12	20	7	21
6	19	12	7	15	18	12	-	14	19	19
7	19	12	17	7	14	20	14	-	17	21
8	20	17	12	14	21	7	19	17	-	14
9	20	19	20	14	7	21	19	21	14	-

$\geq 106; (\frac{24}{113}, \frac{89}{113})$



TT-disjoint at 4

	0	1	2	3	4	5	6	7	8	9
0	-	3	6	6	7	9	9	9	10	11
1	3	-	9	9	10	8	6	6	7	10
2	6	9	-	10	7	9	3	9	6	11
3	6	9	10	-	11	7	7	3	6	7
4	7	10	7	11	-	8	10	8	11	4
5	9	8	9	7	8	-	6	10	3	10
6	9	6	3	7	10	6	-	8	9	10
7	9	6	9	3	8	10	8	-	7	10
8	10	7	6	6	11	3	9	7	-	7
9	11	10	11	7	4	10	10	10	7	-

$\geq$ 52; SMAPO# 7796

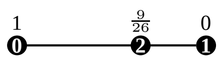
	0	1	2	3	4	5	6	7	8	9
0	-	4	6	6	7	8	10	10	10	9
1	4	-	10	10	9	8	6	6	8	9
2	6	10	-	10	7	10	4	8	6	9
3	6	10	10	-	11	8	8	4	8	7
4	7	9	7	11	-	7	9	7	11	4
5	8	8	10	8	7	-	6	10	4	11
6	10	6	4	8	9	6	-	8	10	9
7	10	6	8	4	7	10	8	-	8	11
8	10	8	6	8	11	4	10	8	-	7
9	9	9	9	7	4	11	9	11	7	-

$\geq$ 56; SMAPO# 8797

	0	1	2	3	4	5	6	7	8	9
0	-	7	12	12	13	17	19	19	20	20
1	7	-	19	19	18	16	12	12	15	19
2	12	19	-	20	13	19	7	17	12	20
3	12	19	20	-	21	15	15	7	14	14
4	13	18	13	21	-	14	18	14	21	7
5	17	16	19	15	14	-	12	20	7	21
6	19	12	7	15	18	12	-	16	19	19
7	19	12	17	7	14	20	16	-	15	21
8	20	15	12	14	21	7	19	15	-	14
9	20	19	20	14	7	21	19	21	14	-

$\geq 106; (\frac{59}{112}, \frac{53}{112})$

RHS = 110



TT-disjoint at 1

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	4	4	6	6	8	8	7
1	2	-	4	4	6	8	6	6	10	5
2	2	4	-	6	6	4	8	10	6	9
3	4	4	6	-	2	4	4	8	8	7
4	4	6	6	2	-	2	6	8	8	5
5	6	8	4	4	2	-	8	10	6	5
6	6	6	8	4	6	8	-	10	8	3
7	8	6	10	8	8	10	10	-	4	7
8	8	10	6	8	8	6	8	4	-	11
9	7	5	9	7	5	5	3	7	11	-

$\geq$ 40; SMAPO# 4336

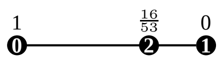
	0	1	2	3	4	5	6	7	8	9
0	-	4	3	10	10	10	14	12	12	14
1	4	-	7	6	10	14	10	10	16	12
2	3	7	-	11	11	7	15	15	9	17
3	10	6	11	-	4	8	8	16	14	14
4	10	10	11	4	-	4	12	16	14	10
5	10	14	7	8	4	-	16	16	10	10
6	14	10	15	8	12	16	-	20	14	6
7	12	10	15	16	16	16	20	-	6	14
8	12	16	9	14	14	10	14	6	-	20
9	14	12	17	14	10	10	6	14	20	-

$\geq$ 72; SMAPO# 12640

	0	1	2	3	4	5	6	7	8	9
0	-	5	5	14	14	16	20	20	20	21
1	5	-	10	9	15	21	15	15	25	16
2	5	10	-	17	17	11	23	25	15	26
3	14	9	17	-	6	12	12	24	22	21
4	14	15	17	6	-	6	18	24	22	15
5	16	21	11	12	6	-	24	26	16	15
6	20	15	23	12	18	24	-	30	22	9
7	20	15	25	24	24	26	30	-	10	21
8	20	25	15	22	22	16	22	10	-	31
9	21	16	26	21	15	15	9	21	31	-

$\geq 110; (\frac{9}{26}, \frac{17}{26})$

RHS = 112



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	4	4	5	4	6	6	7
1	1	-	4	5	3	6	5	5	5	6
2	3	4	-	7	5	8	7	5	3	6
3	4	5	7	-	6	5	8	8	8	3
4	4	3	5	6	-	7	2	6	6	7
5	5	6	8	5	7	-	5	3	5	8
6	4	5	7	8	2	5	-	6	4	5
7	6	5	5	8	6	3	6	-	8	5
8	6	5	3	8	6	5	4	8	-	7
9	7	6	6	3	7	8	5	5	7	-

$\geq$ 36; SMAPO# 2766

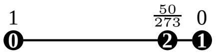
	0	1	2	3	4	5	6	7	8	9
0	-	2	8	8	10	9	12	12	12	13
1	2	-	8	10	8	11	14	10	14	11
2	8	8	-	16	12	17	18	12	6	13
3	8	10	16	-	16	9	18	14	18	5
4	10	8	12	16	-	17	6	14	18	15
5	9	11	17	9	17	-	11	5	11	14
6	12	14	18	18	6	11	-	14	12	13
7	12	10	12	14	14	5	14	-	16	9
8	12	14	6	18	18	11	12	16	-	15
9	13	11	13	5	15	14	13	9	15	-

$\geq$ 78; SMAPO# 13443

	0	1	2	3	4	5	6	7	8	9
0	-	3	10	12	14	14	16	18	18	20
1	3	-	11	15	11	17	19	15	19	17
2	10	11	-	22	16	24	24	16	8	18
3	12	15	22	-	22	14	26	22	26	8
4	14	11	16	22	-	24	8	20	24	22
5	14	17	24	14	24	-	16	8	16	22
6	16	19	24	26	8	16	-	20	16	18
7	18	15	16	22	20	8	20	-	24	14
8	18	19	8	26	24	16	16	24	-	22
9	20	17	18	8	22	22	18	14	22	-

$\geq 112; (\frac{16}{53}, \frac{37}{53})$

RHS = 116



TT-disjoint at 7

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	5	5	4	4
1	2	-	5	5	3	3	3	3	4	4
2	3	5	-	4	4	4	2	4	3	5
3	3	5	4	-	4	4	4	2	5	3
4	3	3	4	4	-	2	4	4	3	1
5	3	3	4	4	2	-	4	4	1	3
6	5	3	2	4	4	4	-	4	5	3
7	5	3	4	2	4	4	4	-	3	5
8	4	4	3	5	3	1	5	3	-	2
9	4	4	5	3	1	3	3	5	2	-

$\geq$ 24; SMAPO# 662

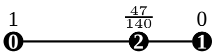
	0	1	2	3	4	5	6	7	8	9
0	-	5	10	11	12	12	15	17	18	18
1	5	-	15	16	15	17	10	12	17	19
2	10	15	-	17	18	18	5	11	12	20
3	11	16	17	-	21	21	12	6	17	15
4	12	15	18	21	-	12	15	15	18	6
5	12	17	18	21	12	-	17	17	6	18
6	15	10	5	12	15	17	-	12	17	15
7	17	12	11	6	15	17	12	-	13	21
8	18	17	12	17	18	6	17	13	-	12
9	18	19	20	15	6	18	15	21	12	-

$\geq$ 94; SMAPO# 14584

	0	1	2	3	4	5	6	7	8	9
0	-	7	13	14	15	15	20	21	22	22
1	7	-	20	21	18	20	13	14	21	23
2	13	20	-	21	22	22	7	14	15	25
3	14	21	21	-	25	25	16	7	22	18
4	15	18	22	25	-	14	19	18	21	7
5	15	20	22	25	14	-	21	20	7	21
6	20	13	7	16	19	21	-	15	22	18
7	21	14	14	7	18	20	15	-	15	25
8	22	21	15	22	21	7	22	15	-	14
9	22	23	25	18	7	21	18	25	14	-

$\geq 116; (\frac{50}{273}, \frac{223}{273})$

RHS = 118



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	6	5	8	7	7	8	8
1	3	-	8	7	8	5	6	8	5	7
2	5	8	-	9	8	3	6	6	7	7
3	6	7	9	-	5	6	3	7	8	6
4	5	8	8	5	-	7	8	2	5	7
5	8	5	3	6	7	-	9	7	8	6
6	7	6	6	3	8	9	-	6	5	7
7	7	8	6	7	2	7	6	-	7	5
8	8	5	7	8	5	8	5	7	-	2
9	8	7	7	6	7	6	7	5	2	-

$\geq$ 42; SMAPO# 4504

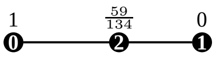
	0	1	2	3	4	5	6	7	8	9
0	-	4	8	9	9	12	14	14	14	15
1	4	-	12	13	13	8	10	16	10	11
2	8	12	-	17	17	4	12	12	14	13
3	9	13	17	-	10	13	5	15	15	10
4	9	13	17	10	-	13	13	5	11	16
5	12	8	4	13	13	-	16	16	16	11
6	14	10	12	5	13	16	-	14	10	15
7	14	16	12	15	5	16	14	-	16	11
8	14	10	14	15	11	16	10	16	-	5
9	15	11	13	10	16	11	15	11	5	-

$\geq$ 78; SMAPO# 13346

	0	1	2	3	4	5	6	7	8	9
0	-	7	13	14	14	20	21	21	22	23
1	7	-	20	19	21	13	16	24	15	18
2	13	20	-	25	25	7	18	18	21	20
3	14	19	25	-	14	18	7	21	22	15
4	14	21	25	14	-	20	21	7	16	23
5	20	13	7	18	20	-	25	23	24	17
6	21	16	18	7	21	25	-	20	15	22
7	21	24	18	21	7	23	20	-	23	16
8	22	15	21	22	16	24	15	23	-	7
9	23	18	20	15	23	17	22	16	7	-

$\geq 118; (\frac{47}{140}, \frac{93}{140})$

RHS = 122



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	3	7	7	6	6	9	9	11	11
1	3	-	8	10	9	9	8	6	8	10
2	7	8	-	8	11	9	8	8	12	4
3	7	10	8	-	11	9	10	10	4	12
4	6	9	11	11	-	8	3	5	7	7
5	6	9	9	9	8	-	5	3	7	9
6	9	8	8	10	3	5	-	8	8	10
7	9	6	8	10	5	3	8	-	10	10
8	11	8	12	4	7	7	8	10	-	8
9	11	10	4	12	7	9	10	10	8	-

$\geq$ 54; SMAPO# 8346

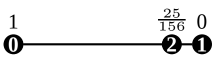
	0	1	2	3	4	5	6	7	8	9
0	-	4	9	8	9	10	12	13	12	12
1	4	-	9	12	13	14	12	9	8	14
2	9	9	-	9	14	11	9	10	13	5
3	8	12	9	-	13	12	12	13	4	12
4	9	13	14	13	-	13	5	8	9	9
5	10	14	11	12	13	-	8	5	8	12
6	12	12	9	12	5	8	-	13	10	14
7	13	9	10	13	8	5	13	-	13	13
8	12	8	13	4	9	8	10	13	-	8
9	12	14	5	12	9	12	14	13	8	-

$\geq$ 70; SMAPO# 12051

	0	1	2	3	4	5	6	7	8	9
0	-	7	15	15	15	16	21	22	23	23
1	7	-	16	22	22	23	20	15	16	24
2	15	16	-	16	24	19	16	17	24	8
3	15	22	16	-	24	21	22	23	8	24
4	15	22	24	24	-	21	8	13	16	16
5	16	23	19	21	21	-	13	8	15	21
6	21	20	16	22	8	13	-	21	18	24
7	22	15	17	23	13	8	21	-	23	23
8	23	16	24	8	16	15	18	23	-	16
9	23	24	8	24	16	21	24	23	16	-

$\geq 122; (\frac{59}{134}, \frac{75}{134})$

RHS = 132



TT-disjoint at 7

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	3	3	3	5	5	4	4
1	2	-	5	5	3	3	3	3	4	4
2	3	5	-	4	4	4	2	4	3	5
3	3	5	4	-	4	4	4	2	5	3
4	3	3	4	4	-	2	4	4	3	1
5	3	3	4	4	2	-	4	4	1	3
6	5	3	2	4	4	4	-	4	5	3
7	5	3	4	2	4	4	4	-	3	5
8	4	4	3	5	3	1	5	3	-	2
9	4	4	5	3	1	3	3	5	2	-

$\geq$ 24; SMAPO# 662

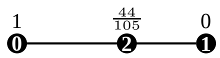
	0	1	2	3	4	5	6	7	8	9
0	-	6	12	12	14	14	18	19	21	21
1	6	-	18	18	18	20	12	13	21	23
2	12	18	-	20	22	22	6	13	15	23
3	12	18	20	-	24	24	14	7	21	17
4	14	18	22	24	-	14	18	17	21	7
5	14	20	22	24	14	-	20	19	7	21
6	18	12	6	14	18	20	-	13	21	17
7	19	13	13	7	17	19	13	-	16	24
8	21	21	15	21	21	7	21	16	-	14
9	21	23	23	17	7	21	17	24	14	-

$\geq$ 110; SMAPO# 15089

	0	1	2	3	4	5	6	7	8	9
0	-	8	15	15	17	17	23	23	25	25
1	8	-	23	23	21	23	15	15	25	27
2	15	23	-	24	26	26	8	16	18	28
3	15	23	24	-	28	28	18	8	26	20
4	17	21	26	28	-	16	22	20	24	8
5	17	23	26	28	16	-	24	22	8	24
6	23	15	8	18	22	24	-	16	26	20
7	23	15	16	8	20	22	16	-	18	28
8	25	25	18	26	24	8	26	18	-	16
9	25	27	28	20	8	24	20	28	16	-

$\geq 132; (\frac{25}{156}, \frac{131}{156})$

RHS = 136



TT-disjoint at 2

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	3	5	4	4	5	4	4
1	1	-	4	4	4	3	3	4	5	5
2	3	4	-	4	6	7	7	2	5	5
3	3	4	4	-	2	7	7	6	5	5
4	5	4	6	2	-	5	5	4	5	5
5	4	3	7	7	5	-	4	5	6	2
6	4	3	7	7	5	4	-	5	2	6
7	5	4	2	6	4	5	5	-	5	5
8	4	5	5	5	5	6	2	5	-	4
9	4	5	5	5	5	2	6	5	4	-

$\geq$ 30; SMAPO# 1506

	0	1	2	3	4	5	6	7	8	9
0	-	3	9	7	11	13	13	11	13	13
1	3	-	8	10	8	10	10	14	16	16
2	9	8	-	10	14	18	18	6	12	12
3	7	10	10	-	4	16	16	12	14	14
4	11	8	14	4	-	12	16	8	10	14
5	13	10	18	16	12	-	12	12	18	6
6	13	10	18	16	16	12	-	16	6	18
7	11	14	6	12	8	12	16	-	14	18
8	13	16	12	14	10	18	6	14	-	12
9	13	16	12	14	14	6	18	18	12	-

$\geq$ 80; SMAPO# 13644

	0	1	2	3	4	5	6	7	8	9
0	-	5	13	13	21	21	21	21	21	21
1	5	-	14	18	16	16	16	22	26	26
2	13	14	-	16	24	30	30	8	20	20
3	13	18	16	-	8	30	30	24	24	24
4	21	16	24	8	-	22	26	16	20	24
5	21	16	30	30	22	-	20	22	30	10
6	21	16	30	30	26	20	-	26	10	30
7	21	22	8	24	16	22	26	-	24	28
8	21	26	20	24	20	30	10	24	-	20
9	21	26	20	24	24	10	30	28	20	-

$\geq 136; (\frac{44}{105}, \frac{61}{105})$

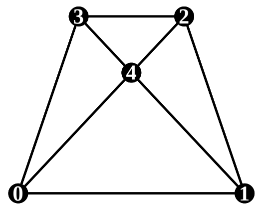
A.2 codim$(F) = 3$

Similar to the previous section, the results are sorted by right hand sides. For each non-NR facet we give a figure of the corresponding tilting complex $\mathcal{T}$ followed by several tables representing the TT-type facets. The tables are listed in the order of the vertex numbering in the figure.

For non-NR facets the tuple to the right of the right hand side specifies how the inequality defining the non-NR facet can be represented as convex combination of the inequalities in standard scaling defining the NR-facets. The order of the coefficients conforms with the order of the vertices of $F^{\diamond}$ in the corresponding figure.

The tilting complex for right hand side 47 is especially interesting and therefore provides an extended description.

RHS = 29



TT-disjoint at 8, 9

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	2	2	3	4	4	3
1	1	-	3	2	3	3	2	3	5	4
2	2	3	-	1	4	4	3	4	4	3
3	3	2	1	-	3	3	4	5	3	2
4	2	3	4	3	-	2	1	4	4	3
5	2	3	4	3	2	-	3	4	4	1
6	3	2	3	4	1	3	-	3	5	2
7	4	3	4	5	4	4	3	-	2	5
8	4	5	4	3	4	4	5	2	-	3
9	3	4	3	2	3	1	2	5	3	-

$\geq$ 20; SMAPO# 363

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	2	3	3	3	4	5
1	1	-	3	2	3	4	2	2	3	4
2	2	3	-	1	4	5	3	3	2	3
3	3	2	1	-	3	4	4	4	3	4
4	2	3	4	3	-	3	1	3	2	5
5	3	4	5	4	3	-	4	4	5	2
6	3	2	3	4	1	4	-	2	3	4
7	3	2	3	4	3	4	2	-	1	4
8	4	3	2	3	2	5	3	1	-	3
9	5	4	3	4	5	2	4	4	3	-

$\geq$ 20; SMAPO# 363

	0	1	2	3	4	5	6	7	8	9
0	-	1	4	5	6	6	7	6	9	9
1	1	-	5	4	7	7	6	5	8	8
2	4	5	-	1	8	8	7	8	5	5
3	5	4	1	-	7	9	8	7	6	6
4	6	7	8	7	-	6	3	8	5	9
5	6	7	8	9	6	-	9	8	9	3
6	7	6	7	8	3	9	-	5	8	6
7	6	5	8	7	8	8	5	-	3	7
8	9	8	5	6	5	9	8	3	-	6
9	9	8	5	6	9	3	6	7	6	-

$\geq$ 40; SMAPO# 4035

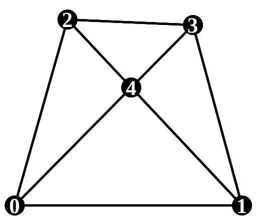
	0	1	2	3	4	5	6	7	8	9
0	-	1	4	5	6	5	7	7	8	8
1	1	-	5	4	7	6	6	6	9	9
2	4	5	-	1	8	7	7	9	6	6
3	5	4	1	-	7	8	8	8	5	5
4	6	7	8	7	-	5	3	9	6	8
5	5	6	7	8	5	-	8	8	7	3
6	7	6	7	8	3	8	-	6	9	5
7	7	6	9	8	9	8	6	-	3	9
8	8	9	6	5	6	7	9	3	-	6
9	8	9	6	5	8	3	5	9	6	-

$\geq$ 40; SMAPO# 4035

	0	1	2	3	4	5	6	7	8	9
0	-	1	3	4	4	4	5	5	6	6
1	1	-	4	3	5	5	4	4	6	6
2	3	4	-	1	6	6	5	6	4	4
3	4	3	1	-	5	6	6	6	4	4
4	4	5	6	5	-	4	2	6	4	6
5	4	5	6	6	4	-	6	6	6	2
6	5	4	5	6	2	6	-	4	6	4
7	5	4	6	6	6	6	4	-	2	6
8	6	6	4	4	4	6	6	2	-	4
9	6	6	4	4	6	2	4	6	4	-

$\geq 29; (\frac{48}{151}, 0, \frac{103}{151}, 0)$

RHS = 38



TT-disjoint at 1, 4

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	2	1	1	3	2	2
1	1	-	2	1	3	2	2	4	3	3
2	1	2	-	3	1	2	2	4	3	3
3	2	1	3	-	2	1	3	3	2	2
4	2	3	1	2	-	1	3	3	2	2
5	1	2	2	1	1	-	2	2	3	3
6	1	2	2	3	3	2	-	2	1	3
7	3	4	4	3	3	2	2	-	3	1
8	2	3	3	2	2	3	1	3	-	2
9	2	3	3	2	2	3	3	1	2	-

$\geq$ 14; SMAPO# 134

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	2	2	2	2	3	3
1	1	-	2	1	3	3	3	3	2	2
2	1	2	-	3	1	3	3	3	2	2
3	2	1	3	-	2	2	4	4	3	3
4	2	3	1	2	-	2	4	4	3	3
5	2	3	3	2	2	-	2	2	3	3
6	2	3	3	4	4	2	-	2	1	3
7	2	3	3	4	4	2	2	-	3	1
8	3	2	2	3	3	3	1	3	-	2
9	3	2	2	3	3	3	3	1	2	-

$\geq$ 16; SMAPO# 200

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	3	3	3	5	5	5
1	1	-	2	1	4	2	4	6	4	4
2	1	2	-	3	2	2	4	6	6	6
3	2	1	3	-	3	1	5	5	5	5
4	3	4	2	3	-	2	6	6	4	4
5	3	2	2	1	2	-	4	4	6	6
6	3	4	4	5	6	4	-	4	2	6
7	5	6	6	5	6	4	4	-	6	2
8	5	4	6	5	4	6	2	6	-	4
9	5	4	6	5	4	6	6	2	4	-

$\geq$ 24; SMAPO# 771

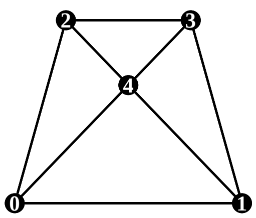
	0	1	2	3	4	5	6	7	8	9
0	-	2	1	2	2	4	4	4	6	6
1	2	-	3	2	4	4	6	6	4	4
2	1	3	-	3	1	3	5	5	5	5
3	2	2	3	-	2	2	6	6	6	6
4	2	4	1	2	-	2	6	6	4	4
5	4	4	3	2	2	-	4	4	6	6
6	4	6	5	6	6	4	-	4	2	6
7	4	6	5	6	6	4	4	-	6	2
8	6	4	5	6	4	6	2	6	-	4
9	6	4	5	6	4	6	6	2	4	-

$\geq$ 26; SMAPO# 949

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	4	4	5	5	7	8	8
1	2	-	4	2	6	5	7	9	6	6
2	2	4	-	6	2	5	7	9	8	8
3	4	2	6	-	4	3	9	9	8	8
4	4	6	2	4	-	3	9	9	6	6
5	5	5	5	3	3	-	6	6	9	9
6	5	7	7	9	9	6	-	6	3	9
7	7	9	9	9	9	6	6	-	9	3
8	8	6	8	8	6	9	3	9	-	6
9	8	6	8	8	6	9	9	3	6	-

$\geq 38; (0, \frac{19}{52}, \frac{33}{52}, 0)$

RHS = 40



TT-disjoint at 0, 1

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	2	3	2	2	3	3
1	1	-	2	1	3	2	3	3	4	2
2	1	2	-	1	3	2	3	3	4	2
3	2	1	1	-	2	3	2	2	3	3
4	2	3	3	2	-	1	4	4	3	3
5	3	2	2	3	1	-	3	3	4	2
6	2	3	3	2	4	3	-	2	1	3
7	2	3	3	2	4	3	2	-	3	1
8	3	4	4	3	3	4	1	3	-	2
9	3	2	2	3	3	2	3	1	2	-

$\geq$ 16; SMAPO# 214

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	2	3	3	3	2	4
1	1	-	2	1	3	2	2	2	3	3
2	1	2	-	1	3	2	2	2	3	3
3	2	1	1	-	2	3	3	3	2	4
4	2	3	3	2	-	1	3	3	2	4
5	3	2	2	3	1	-	4	4	3	3
6	3	2	2	3	3	4	-	2	1	3
7	3	2	2	3	3	4	2	-	3	1
8	2	3	3	2	2	3	1	3	-	2
9	4	3	3	4	4	3	3	1	2	-

$\geq$ 16; SMAPO# 214

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	3	3	4	4	4	6	6
1	2	-	2	1	3	2	4	4	6	4
2	2	2	-	1	3	4	6	6	6	4
3	3	1	1	-	4	3	5	5	5	5
4	3	3	3	4	-	1	7	7	5	5
5	4	2	4	3	1	-	6	6	6	4
6	4	4	6	5	7	6	-	4	2	6
7	4	4	6	5	7	6	4	-	6	2
8	6	6	6	5	5	6	2	6	-	4
9	6	4	4	5	5	4	6	2	4	-

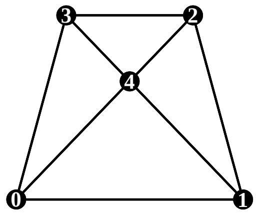
$\geq$ 26; SMAPO# 1036

	0	1	2	3	4	5	6	7	8	9
0	-	2	1	2	2	3	4	4	4	6
1	2	-	3	2	4	3	4	4	6	6
2	1	3	-	1	3	4	5	5	5	5
3	2	2	1	-	4	3	6	6	4	6
4	2	4	3	4	-	1	6	6	4	6
5	3	3	4	3	1	-	7	7	5	5
6	4	4	5	6	6	7	-	4	2	6
7	4	4	5	6	6	7	4	-	6	2
8	4	6	5	4	4	5	2	6	-	4
9	6	6	5	6	6	5	6	2	4	-

$\geq$ 26; SMAPO# 1036

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	4	4	6	6	6	7	9
1	2	-	4	2	6	4	6	6	9	7
2	2	4	-	2	6	6	8	8	9	7
3	4	2	2	-	6	6	8	8	7	9
4	4	6	6	6	-	2	10	10	7	9
5	6	4	6	6	2	-	10	10	9	7
6	6	6	8	8	10	10	-	6	3	9
7	6	6	8	8	10	10	6	-	9	3
8	7	9	9	7	7	9	3	9	-	6
9	9	7	7	9	9	7	9	3	6	-

$\geq 40; (0, \frac{39}{110}, \frac{71}{110}, 0)$



TT-disjoint at 0, 5

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	2	3	3	3	4	4
1	1	-	3	3	1	2	2	2	3	3
2	2	3	-	2	2	3	3	3	2	2
3	2	3	2	-	2	1	3	3	4	4
4	2	1	2	2	-	1	3	3	2	2
5	3	2	3	1	1	-	2	2	3	3
6	3	2	3	3	3	2	-	2	1	3
7	3	2	3	3	3	2	2	-	3	1
8	4	3	2	4	2	3	1	3	-	2
9	4	3	2	4	2	3	3	1	2	-

$\geq$ 16; SMAPO# 200

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	3	3	2	2	3	3
1	1	-	2	3	2	2	3	3	4	4
2	1	2	-	1	2	2	3	3	2	2
3	2	3	1	-	3	1	2	2	3	3
4	3	2	2	3	-	2	3	3	2	2
5	3	2	2	1	2	-	3	3	4	4
6	2	3	3	2	3	3	-	2	1	3
7	2	3	3	2	3	3	2	-	3	1
8	3	4	2	3	2	4	1	3	-	2
9	3	4	2	3	2	4	3	1	2	-

$\geq$ 16; SMAPO# 200

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	3	4	4	4	4	6	6
1	2	-	2	3	2	2	6	6	6	6
2	2	2	-	1	4	2	6	6	4	4
3	3	3	1	-	3	1	5	5	5	5
4	4	2	4	3	-	2	6	6	4	4
5	4	2	2	1	2	-	4	4	6	6
6	4	6	6	5	6	4	-	4	2	6
7	4	6	6	5	6	4	4	-	6	2
8	6	6	4	5	4	6	2	6	-	4
9	6	6	4	5	4	6	6	2	4	-

$\geq$ 26; SMAPO# 949

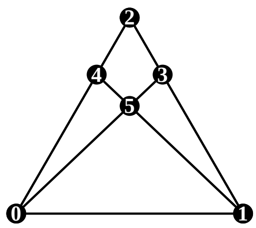
	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	2	4	4	4	6	6
1	1	-	3	3	1	3	5	5	5	5
2	2	3	-	2	4	4	6	6	4	4
3	2	3	2	-	2	2	6	6	6	6
4	2	1	4	2	-	2	6	6	4	4
5	4	3	4	2	2	-	4	4	6	6
6	4	5	6	6	6	4	-	4	2	6
7	4	5	6	6	6	4	4	-	6	2
8	6	5	4	6	4	6	2	6	-	4
9	6	5	4	6	4	6	6	2	4	-

$\geq$ 26; SMAPO# 949

	0	1	2	3	4	5	6	7	8	9
0	-	2	3	4	5	6	6	6	9	9
1	2	-	5	6	3	4	8	8	9	9
2	3	5	-	3	6	5	9	9	6	6
3	4	6	3	-	5	2	8	8	9	9
4	5	3	6	5	-	3	9	9	6	6
5	6	4	5	2	3	-	6	6	9	9
6	6	8	9	8	9	6	-	6	3	9
7	6	8	9	8	9	6	6	-	9	3
8	9	9	6	9	6	9	3	9	-	6
9	9	9	6	9	6	9	9	3	6	-

$\geq 40; (\frac{19}{53}, 0, \frac{34}{53}, 0)$

RHS = 47



TT-disjoint at 3, 4

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	4	3	3	4	4	4
1	1	-	3	4	5	2	2	3	3	3
2	2	3	-	3	4	3	1	4	2	2
3	3	4	3	-	3	2	2	5	1	3
4	4	5	4	3	-	5	3	2	4	4
5	3	2	3	2	5	-	4	3	3	1
6	3	2	1	2	3	4	-	5	3	3
7	4	3	4	5	2	3	5	-	4	4
8	4	3	2	1	4	3	3	4	-	2
9	4	3	2	3	4	1	3	4	2	-

$\geq$ 20; SMAPO# 363

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	2	3	3	3	5	4
1	1	-	3	4	3	2	2	2	4	3
2	2	3	-	5	4	3	1	3	3	2
3	3	4	5	-	3	4	4	4	2	5
4	2	3	4	3	-	3	3	1	5	2
5	3	2	3	4	3	-	4	2	4	1
6	3	2	1	4	3	4	-	4	4	3
7	3	2	3	4	1	2	4	-	4	3
8	5	4	3	2	5	4	4	4	-	3
9	4	3	2	5	2	1	3	3	3	-

$\geq$ 20; SMAPO# 363

	0	1	2	3	4	5	6	7	8	9
0	-	4	6	7	7	8	10	11	9	10
1	4	-	10	11	11	8	6	7	9	10
2	6	10	-	11	11	10	4	9	7	8
3	7	11	11	-	8	7	7	10	4	11
4	7	11	11	8	-	11	7	4	10	7
5	8	8	10	7	11	-	10	7	11	4
6	10	6	4	7	7	10	-	9	11	8
7	11	7	9	10	4	7	9	-	10	11
8	9	9	7	4	10	11	11	10	-	7
9	10	10	8	11	7	4	8	11	7	-

$\geq$ 58; SMAPO# 9365

	0	1	2	3	4	5	6	7	8	9
0	-	5	8	9	9	11	13	14	14	14
1	5	-	13	14	14	10	8	9	13	13
2	8	13	-	15	15	13	5	12	10	10
3	9	14	15	-	10	10	10	13	5	15
4	9	14	15	10	-	14	10	5	15	9
5	11	10	13	10	14	-	14	9	15	5
6	13	8	5	10	10	14	-	13	15	11
7	14	9	12	13	5	9	13	-	14	14
8	14	13	10	5	15	15	15	14	-	10
9	14	13	10	15	9	5	11	14	10	-

$\geq 76; (0, \frac{16}{55}, \frac{39}{55})$

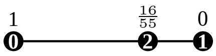
	0	1	2	3	4	5	6	7	8	9
0	-	5	8	10	10	11	13	15	13	14
1	5	-	13	15	15	10	8	10	12	13
2	8	13	-	14	14	13	5	13	9	10
3	10	15	14	-	10	9	9	15	5	14
4	10	15	14	10	-	15	9	5	13	10
5	11	10	13	9	15	-	14	10	14	5
6	13	8	5	9	9	14	-	14	14	11
7	15	10	13	15	5	10	14	-	14	15
8	13	12	9	5	13	14	14	14	-	9
9	14	13	10	14	10	5	11	15	9	-

$\geq 76; (\frac{16}{55}, 0, \frac{39}{55})$

	0	1	2	3	4	5	6	7	8	9
0	-	3	5	6	6	7	8	9	9	9
1	3	-	8	9	9	6	5	6	8	8
2	5	8	-	9	9	8	3	8	6	6
3	6	9	9	-	6	6	6	9	3	9
4	6	9	9	6	-	9	6	3	9	6
5	7	6	8	6	9	-	9	6	9	3
6	8	5	3	6	6	9	-	9	9	7
7	9	6	8	9	3	6	9	-	9	9
8	9	8	6	3	9	9	9	9	-	6
9	9	8	6	9	6	3	7	9	6	-

$\geq 47; (\frac{16}{71}, \frac{16}{71}, \frac{39}{71})$

The sides $(0, 2)$ and $(1, 2)$ of $F^\diamond$ correspond to one-dimensional tilting complexes. The lower two vertices $\boldsymbol{v}_0, \boldsymbol{v}_1$ of $F^\diamond$ represent NR-facets H_0, H_1 with SMAPO number 363, while the upper vertex $\boldsymbol{v}_2$ represents an NR-facet H_2 with SMAPO number 9365. In Section A.1 we have exactly one tilting complex with those two SMAPO numbers involved, namely



TT-disjoint at 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	3	2	3	3	3	5	4
1	1	-	3	4	3	2	2	2	4	3
2	2	3	-	5	4	3	1	3	3	2
3	3	4	5	-	3	4	4	4	2	5
4	2	3	4	3	-	3	3	1	5	2
5	3	2	3	4	3	-	4	2	4	1
6	3	2	1	4	3	4	-	4	4	3
7	3	2	3	4	1	2	4	-	4	3
8	5	4	3	2	5	4	4	4	-	3
9	4	3	2	5	2	1	3	3	3	-

$\geq$ 20; SMAPO# 363

	0	1	2	3	4	5	6	7	8	9
0	-	4	6	7	7	8	10	11	9	10
1	4	-	10	11	11	8	6	7	9	10
2	6	10	-	11	11	10	4	9	7	8
3	7	11	11	-	8	7	7	10	4	11
4	7	11	11	8	-	11	7	4	10	7
5	8	8	10	7	11	-	10	7	11	4
6	10	6	4	7	7	10	-	9	11	8
7	11	7	9	10	4	7	9	-	10	11
8	9	9	7	4	10	11	11	10	-	7
9	10	10	8	11	7	4	8	11	7	-

$\geq$ 58; SMAPO# 9365

	0	1	2	3	4	5	6	7	8	9
0	-	5	8	9	9	11	13	14	14	14
1	5	-	13	14	14	10	8	9	13	13
2	8	13	-	15	15	13	5	12	10	10
3	9	14	15	-	10	10	10	13	5	15
4	9	14	15	10	-	14	10	5	15	9
5	11	10	13	10	14	-	14	9	15	5
6	13	8	5	10	10	14	-	13	15	11
7	14	9	12	13	5	9	13	-	14	14
8	14	13	10	5	15	15	15	14	-	10
9	14	13	10	15	9	5	11	14	10	-

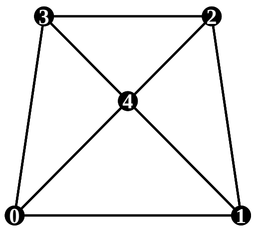
$\geq 76; (\frac{16}{55}, \frac{39}{55})$

This one-dimensional tilting complex is equal to the side $(1, 2)$. Actually, it is also equal to $(0, 2)$, but only modulo permutation of nodes, since the coefficients of H_0 do not fit. In order to see this, we have to remember that the SMAPO numbers represent whole classes of facets, whose members are equal modulo permutation of nodes. So, we have to find a node permutation that maps H_1 to H_0 and leaves H_2 unchanged. This is accomplished by

$$\pi := \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 2 & 1 & 4 & 3 & 9 & 0 & 8 & 7 & 5 \end{pmatrix}.$$

Note that this permutation swaps the nodes 3 and 4. Together with the fact that H_1 and H_2 are TT-disjoint at node 3, this explains why H_0, H_1 and H_2 are TT-disjoint at nodes 3 and 4.

RHS = 68



TT-disjoint at 2, 3

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	5	4	6	4	4	6
1	1	-	2	1	6	5	5	5	5	5
2	1	2	-	3	4	3	7	5	5	7
3	2	1	3	-	7	6	4	4	4	4
4	5	6	4	7	-	5	3	7	7	9
5	4	5	3	6	5	-	8	2	6	4
6	6	5	7	4	3	8	-	8	8	6
7	4	5	5	4	7	2	8	-	4	6
8	4	5	5	4	7	6	8	4	-	2
9	6	5	7	4	9	4	6	6	2	-

$\geq$ 30; SMAPO# 1661

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	5	5	6	5	5	5
1	1	-	2	1	6	6	5	4	4	4
2	1	2	-	3	4	4	7	4	4	6
3	2	1	3	-	7	7	4	5	5	3
4	5	6	4	7	-	6	3	8	8	8
5	5	6	4	7	6	-	9	2	6	4
6	6	5	7	4	3	9	-	7	7	5
7	5	4	4	5	8	2	7	-	4	6
8	5	4	4	5	8	6	7	4	-	2
9	5	4	6	3	8	4	5	6	2	-

$\geq$ 30; SMAPO# 1661

	0	1	2	3	4	5	6	7	8	9
0	-	1	2	2	5	7	6	8	8	7
1	1	-	3	1	6	8	5	7	7	6
2	2	3	-	4	5	7	8	6	6	9
3	2	1	4	-	7	9	4	6	6	5
4	5	6	5	7	-	8	3	11	11	10
5	7	8	7	9	8	-	11	3	9	6
6	6	5	8	4	3	11	-	10	10	7
7	8	7	6	6	11	3	10	-	6	9
8	8	7	6	6	11	9	10	6	-	3
9	7	6	9	5	10	6	7	9	3	-

$\geq$ 40; SMAPO# 4376

	0	1	2	3	4	5	6	7	8	9
0	-	1	1	3	5	6	6	7	7	8
1	1	-	2	2	6	7	5	8	8	7
2	1	2	-	4	4	5	7	6	6	9
3	3	2	4	-	8	9	5	6	6	7
4	5	6	4	8	-	7	3	10	10	11
5	6	7	5	9	7	-	10	3	9	6
6	6	5	7	5	3	10	-	11	11	8
7	7	8	6	6	10	3	11	-	6	9
8	7	8	6	6	10	9	11	6	-	3
9	8	7	9	7	11	6	8	9	3	-

$\geq$ 40; SMAPO# 4376

	0	1	2	3	4	5	6	7	8	9
0	-	2	2	4	10	11	12	12	12	13
1	2	-	4	2	12	13	10	12	12	11
2	2	4	-	6	8	9	14	10	10	15
3	4	2	6	-	14	15	8	10	10	9
4	10	12	8	14	-	13	6	18	18	19
5	11	13	9	15	13	-	19	5	15	10
6	12	10	14	8	6	19	-	18	18	13
7	12	12	10	10	18	5	18	-	10	15
8	12	12	10	10	18	15	18	10	-	5
9	13	11	15	9	19	10	13	15	5	-

$\geq 68; (\frac{20}{47}, 0, \frac{27}{47}, 0)$

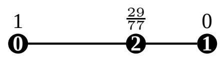
Appendix B

Visualization of tilting complexes for GTSP(12)

B.1 codim$(F) = 2$

The structure of the data conforms with the conventions described in Section A.1. We only listed those one-dimensional tilting complexes, for which the involved NR-facets are TT-disjoint at *more* than one node. The complete results are located in the directory `Data/gtsp12/codim2` on the attached CD.

RHS = 62



TT-disjoint at 4, 5

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	1	3	3	4	4	4	3	3	3	4
1	1	-	2	4	4	3	5	3	4	4	4	5
2	1	2	-	4	4	5	3	5	4	4	4	3
3	3	4	4	-	4	1	1	3	4	2	2	3
4	3	4	4	4	-	5	3	3	2	4	4	5
5	4	3	5	1	5	-	2	4	3	3	3	4
6	4	5	3	1	3	2	-	4	5	3	3	4
7	4	3	5	3	3	4	4	-	5	3	1	2
8	3	4	4	4	2	3	5	5	-	4	4	3
9	3	4	4	2	4	3	3	3	4	-	2	1
10	3	4	4	2	4	3	3	1	4	2	-	3
11	4	5	3	3	5	4	4	2	3	1	3	-

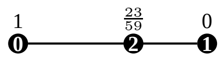
≥ 24

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	1	3	5	5	4	5	6	6	6	7
1	1	-	2	4	4	4	5	4	7	7	7	8
2	1	2	-	4	6	6	3	6	5	7	7	6
3	3	4	4	-	6	2	1	6	5	7	7	6
4	5	4	6	6	-	8	5	6	3	7	9	10
5	5	4	6	2	8	-	3	8	5	5	5	8
6	4	5	3	1	5	3	-	7	6	6	6	7
7	5	4	6	6	6	8	7	-	7	9	3	6
8	6	7	5	5	3	5	6	7	-	10	10	7
9	6	7	7	7	7	5	6	9	10	-	6	3
10	6	7	7	7	9	5	6	3	10	6	-	9
11	7	8	6	6	10	8	7	6	7	3	9	-

≥ 42

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	2	2	6	7	8	8	9	9	9	9	11
1	2	-	4	8	7	6	10	7	11	11	11	13
2	2	4	-	8	9	10	6	11	9	11	11	9
3	6	8	8	-	9	2	2	9	9	9	9	9
4	7	7	9	9	-	11	7	8	4	10	12	14
5	8	6	10	2	11	-	4	11	7	7	7	11
6	8	10	6	2	7	4	-	11	11	9	9	11
7	9	7	11	9	8	11	11	-	12	12	4	8
8	9	11	9	9	4	7	11	12	-	14	14	10
9	9	11	11	9	10	7	9	12	14	-	8	4
10	9	11	11	9	12	7	9	4	14	8	-	12
11	11	13	9	9	14	11	11	8	10	4	12	-

$\geq 62; (\frac{29}{77}, \frac{48}{77})$



TT-disjoint at 2, 5

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	0	2	3	4	3	5	3	3	5	4	4
1	0	-	2	3	4	3	5	3	3	5	4	4
2	2	2	-	5	4	5	3	3	3	3	4	4
3	3	3	5	-	1	4	2	4	4	4	3	5
4	4	4	4	1	-	3	1	3	3	3	4	4
5	3	3	5	4	3	-	4	4	4	2	5	3
6	5	5	3	2	1	4	-	4	4	4	5	3
7	3	3	3	4	3	4	4	-	2	4	1	3
8	3	3	3	4	3	4	4	2	-	4	3	1
9	5	5	3	4	3	2	4	4	4	-	3	5
10	4	4	4	3	4	5	5	1	3	3	-	2
11	4	4	4	5	4	3	3	3	1	5	2	-

≥ 24

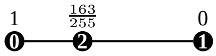
	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	2	4	3	6	4	7	7	5	6	6
1	1	-	3	3	4	5	5	6	6	6	7	7
2	2	3	-	6	5	8	4	5	5	5	8	8
3	4	3	6	-	1	6	2	7	7	5	6	6
4	3	4	5	1	-	5	1	6	6	6	7	5
5	6	5	8	6	5	-	4	7	9	3	10	6
6	4	5	4	2	1	4	-	7	7	7	8	4
7	7	6	5	7	6	7	7	-	6	10	3	9
8	7	6	5	7	6	9	7	6	-	10	9	3
9	5	6	5	5	6	3	7	10	10	-	7	7
10	6	7	8	6	7	10	8	3	9	7	-	6
11	6	7	8	6	5	6	4	9	3	7	6	-

≥ 42

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	3	7	7	8	9	10	10	10	10	10
1	1	-	4	6	8	7	10	9	9	11	11	11
2	3	4	-	10	8	11	6	7	7	7	11	11
3	7	6	10	-	2	9	4	11	11	9	9	11
4	7	8	8	2	-	7	2	9	9	9	11	9
5	8	7	11	9	7	-	7	10	12	4	14	8
6	9	10	6	4	2	7	-	11	11	11	13	7
7	10	9	7	11	9	10	11	-	8	14	4	12
8	10	9	7	11	9	12	11	8	-	14	12	4
9	10	11	7	9	9	4	11	14	14	-	10	12
10	10	11	11	9	11	14	13	4	12	10	-	8
11	10	11	11	11	9	8	7	12	4	12	8	-

$\geq 62; (\frac{23}{59}, \frac{36}{59})$

RHS = 64



TT-disjoint at 3, 4

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	0	2	5	5	5	5	6	8	8	8	9
1	0	-	2	5	5	5	5	6	8	8	8	9
2	2	2	-	7	7	7	5	4	6	8	8	7
3	5	5	7	-	8	6	6	5	3	7	9	10
4	5	5	7	8	-	8	2	3	5	5	5	8
5	5	5	7	6	8	-	6	7	7	9	3	6
6	5	5	5	6	2	6	-	1	5	7	7	6
7	6	6	4	5	3	7	1	-	6	6	6	7
8	8	8	6	3	5	7	5	6	-	10	10	7
9	8	8	8	7	5	9	7	6	10	-	6	3
10	8	8	8	9	5	3	7	6	10	6	-	9
11	9	9	7	10	8	6	6	7	7	3	9	-

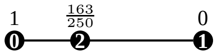
≥ 44

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	1	3	4	4	4	4	3	3	3	4
1	1	-	2	4	3	3	5	5	4	4	4	5
2	1	2	-	4	5	5	3	3	4	4	4	3
3	3	4	4	-	5	3	3	3	2	4	4	5
4	4	3	5	5	-	4	2	2	3	3	3	4
5	4	3	5	3	4	-	4	4	5	3	1	2
6	4	5	3	3	2	4	-	0	5	3	3	4
7	4	5	3	3	2	4	0	-	5	3	3	4
8	3	4	4	2	3	5	5	5	-	4	4	3
9	3	4	4	4	3	3	3	3	4	-	2	1
10	3	4	4	4	3	1	3	3	4	2	-	3
11	4	5	3	5	4	2	4	4	3	1	3	-

≥ 24

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	3	7	8	9	9	10	11	11	11	13
1	1	-	4	8	7	8	10	11	12	12	12	14
2	3	4	-	10	11	12	8	7	10	12	12	10
3	7	8	10	-	11	8	8	7	4	10	12	14
4	8	7	11	11	-	11	3	4	7	7	7	11
5	9	8	12	8	11	-	10	11	12	12	4	8
6	9	10	8	8	3	10	-	1	10	10	10	10
7	10	11	7	7	4	11	1	-	11	9	9	11
8	11	12	10	4	7	12	10	11	-	14	14	10
9	11	12	12	10	7	12	10	9	14	-	8	4
10	11	12	12	12	7	4	10	9	14	8	-	12
11	13	14	10	14	11	8	10	11	10	4	12	-

$\geq 64; (\frac{163}{255}, \frac{92}{255})$



TT-disjoint at 3, 4

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	0	2	5	5	5	5	6	8	8	8	9
1	0	-	2	5	5	5	5	6	8	8	8	9
2	2	2	-	7	7	5	7	4	6	8	8	7
3	5	5	7	-	8	6	6	5	3	7	9	10
4	5	5	7	8	-	2	8	3	5	5	5	8
5	5	5	5	6	2	-	6	1	5	7	7	6
6	5	5	7	6	8	6	-	7	7	9	3	6
7	6	6	4	5	3	1	7	-	6	6	6	7
8	8	8	6	3	5	5	7	6	-	10	10	7
9	8	8	8	7	5	7	9	6	10	-	6	3
10	8	8	8	9	5	7	3	6	10	6	-	9
11	9	9	7	10	8	6	6	7	7	3	9	-

≥ 44

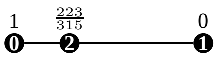
	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	1	3	4	3	4	4	3	3	3	4
1	1	-	2	4	3	4	3	5	4	4	4	5
2	1	2	-	4	5	4	5	3	4	4	4	3
3	3	4	4	-	5	4	3	3	2	4	4	5
4	4	3	5	5	-	1	4	2	3	3	3	4
5	3	4	4	4	1	-	3	1	4	2	2	3
6	4	3	5	3	4	3	-	4	5	3	1	2
7	4	5	3	3	2	1	4	-	5	3	3	4
8	3	4	4	2	3	4	5	5	-	4	4	3
9	3	4	4	4	3	2	3	3	4	-	2	1
10	3	4	4	4	3	2	1	3	4	2	-	3
11	4	5	3	5	4	3	2	4	3	1	3	-

≥ 24

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	3	7	8	8	9	10	11	11	11	13
1	1	-	4	8	7	9	8	11	12	12	12	14
2	3	4	-	10	11	9	12	7	10	12	12	10
3	7	8	10	-	11	9	8	7	4	10	12	14
4	8	7	11	11	-	2	11	4	7	7	7	11
5	8	9	9	9	2	-	9	2	9	9	9	9
6	9	8	12	8	11	9	-	11	12	12	4	8
7	10	11	7	7	4	2	11	-	11	9	9	11
8	11	12	10	4	7	9	12	11	-	14	14	10
9	11	12	12	10	7	9	12	9	14	-	8	4
10	11	12	12	12	7	9	4	9	14	8	-	12
11	13	14	10	14	11	9	8	11	10	4	12	-

$\geq 64; (\frac{163}{250}, \frac{87}{250})$

RHS = 80



TT-disjoint at 2, 4

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	3	6	8	6	7	8	10	10	8	10
1	1	-	4	5	7	7	8	9	9	9	9	11
2	3	4	-	9	11	7	6	11	7	7	7	11
3	6	5	9	-	8	2	3	10	10	10	8	10
4	8	7	11	8	-	6	7	8	10	12	4	14
5	6	7	7	2	6	-	1	8	10	10	10	12
6	7	8	6	3	7	1	-	7	11	11	11	13
7	8	9	11	10	8	8	7	-	12	4	10	8
8	10	9	7	10	10	10	11	12	-	8	14	4
9	10	9	7	10	12	10	11	4	8	-	14	12
10	8	9	7	8	4	10	11	10	14	14	-	10
11	10	11	11	10	14	12	13	8	4	12	10	-

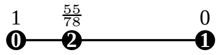
≥ 60

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	0	2	3	3	4	5	4	3	3	5	4
1	0	-	2	3	3	4	5	4	3	3	5	4
2	2	2	-	5	5	4	3	4	3	3	3	4
3	3	3	5	-	4	1	2	5	4	4	4	3
4	3	3	5	4	-	3	4	3	4	4	2	5
5	4	4	4	1	3	-	1	4	3	3	3	4
6	5	5	3	2	4	1	-	3	4	4	4	5
7	4	4	4	5	3	4	3	-	3	1	5	2
8	3	3	3	4	4	3	4	3	-	2	4	1
9	3	3	3	4	4	3	4	1	2	-	4	3
10	5	5	3	4	2	3	4	5	4	4	-	3
11	4	4	4	3	5	4	5	2	1	3	3	-

≥ 24

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	4	9	10	10	12	12	13	13	13	14
1	1	-	5	8	9	11	13	13	12	12	14	15
2	4	5	-	13	14	10	8	14	9	9	9	14
3	9	8	13	-	11	3	5	15	14	14	12	13
4	10	9	14	11	-	8	10	10	13	15	5	18
5	10	11	10	3	8	-	2	12	13	13	13	16
6	12	13	8	5	10	2	-	10	15	15	15	18
7	12	13	14	15	10	12	10	-	15	5	15	10
8	13	12	9	14	13	13	15	15	-	10	18	5
9	13	12	9	14	15	13	15	5	10	-	18	15
10	13	14	9	12	5	13	15	15	18	18	-	13
11	14	15	14	13	18	16	18	10	5	15	13	-

$\geq 80; (\frac{223}{315}, \frac{92}{315})$



TT-disjoint at 2, 5

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	3	6	5	8	7	8	10	10	8	10
1	1	-	4	5	6	7	8	9	9	9	9	11
2	3	4	-	9	8	11	6	11	7	7	7	11
3	6	5	9	-	1	8	3	10	10	10	8	10
4	5	6	8	1	-	7	2	9	9	9	9	11
5	8	7	11	8	7	-	7	8	10	12	4	14
6	7	8	6	3	2	7	-	7	11	11	11	13
7	8	9	11	10	9	8	7	-	12	4	10	8
8	10	9	7	10	9	10	11	12	-	8	14	4
9	10	9	7	10	9	12	11	4	8	-	14	12
10	8	9	7	8	9	4	11	10	14	14	-	10
11	10	11	11	10	11	14	13	8	4	12	10	-

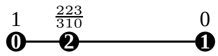
≥ 60

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	0	2	3	4	3	5	4	3	3	5	4
1	0	-	2	3	4	3	5	4	3	3	5	4
2	2	2	-	5	4	5	3	4	3	3	3	4
3	3	3	5	-	1	4	2	5	4	4	4	3
4	4	4	4	1	-	3	1	4	3	3	3	4
5	3	3	5	4	3	-	4	3	4	4	2	5
6	5	5	3	2	1	4	-	3	4	4	4	5
7	4	4	4	5	4	3	3	-	3	1	5	2
8	3	3	3	4	3	4	4	3	-	2	4	1
9	3	3	3	4	3	4	4	1	2	-	4	3
10	5	5	3	4	3	2	4	5	4	4	-	3
11	4	4	4	3	4	5	5	2	1	3	3	-

≥ 24

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	4	9	9	10	12	12	13	13	13	14
1	1	-	5	8	10	9	13	13	12	12	14	15
2	4	5	-	13	11	14	8	14	9	9	9	14
3	9	8	13	-	2	11	5	15	14	14	12	13
4	9	10	11	2	-	9	3	13	12	12	12	15
5	10	9	14	11	9	-	10	10	13	15	5	18
6	12	13	8	5	3	10	-	10	15	15	15	18
7	12	13	14	15	13	10	10	-	15	5	15	10
8	13	12	9	14	12	13	15	15	-	10	18	5
9	13	12	9	14	12	15	15	5	10	-	18	15
10	13	14	9	12	12	5	15	15	18	18	-	13
11	14	15	14	13	15	18	18	10	5	15	13	-

$\geq 80; (\frac{55}{78}, \frac{23}{78})$



TT-disjoint at 3, 5

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	2	6	6	7	7	8	10	10	10	12
1	1	-	3	7	7	6	8	7	11	11	11	13
2	2	3	-	8	6	9	5	10	8	10	10	10
3	6	7	8	-	8	11	7	8	4	10	12	14
4	6	7	6	8	-	3	1	8	8	10	10	10
5	7	6	9	11	3	-	4	11	7	7	7	11
6	7	8	5	7	1	4	-	9	9	9	9	11
7	8	7	10	8	8	11	9	-	10	12	4	8
8	10	11	8	4	8	7	9	10	-	14	14	10
9	10	11	10	10	10	7	9	12	14	-	8	4
10	10	11	10	12	10	7	9	4	14	8	-	12
11	12	13	10	14	10	11	11	8	10	4	12	-

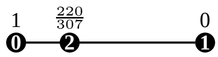
≥ 60

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	1	3	3	4	4	4	3	3	3	4
1	1	-	2	4	4	3	5	3	4	4	4	5
2	1	2	-	4	4	5	3	5	4	4	4	3
3	3	4	4	-	4	5	3	3	2	4	4	5
4	3	4	4	4	-	1	1	3	4	2	2	3
5	4	3	5	5	1	-	2	4	3	3	3	4
6	4	5	3	3	1	2	-	4	5	3	3	4
7	4	3	5	3	3	4	4	-	5	3	1	2
8	3	4	4	2	4	3	5	5	-	4	4	3
9	3	4	4	4	2	3	3	3	4	-	2	1
10	3	4	4	4	2	3	3	1	4	2	-	3
11	4	5	3	5	3	4	4	2	3	1	3	-

≥ 24

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	2	3	8	9	10	11	12	13	13	13	16
1	2	-	5	10	11	8	13	10	15	15	15	18
2	3	5	-	11	10	13	8	15	12	14	14	13
3	8	10	11	-	11	14	9	10	5	13	15	18
4	9	11	10	11	-	3	2	11	12	12	12	13
5	10	8	13	14	3	-	5	14	9	9	9	14
6	11	13	8	9	2	5	-	13	14	12	12	15
7	12	10	15	10	11	14	13	-	15	15	5	10
8	13	15	12	5	12	9	14	15	-	18	18	13
9	13	15	14	13	12	9	12	15	18	-	10	5
10	13	15	14	15	12	9	12	5	18	10	-	15
11	16	18	13	18	13	14	15	10	13	5	15	-

$\geq 80; (\frac{223}{310}, \frac{87}{310})$



TT-disjoint at 4, 6

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	2	5	7	6	8	9	9	9	9	11
1	1	-	3	6	8	5	9	8	10	10	10	10
2	2	3	-	7	7	8	6	11	11	11	7	13
3	5	6	7	-	8	1	3	8	10	10	8	10
4	7	8	7	8	-	7	11	4	10	12	8	14
5	6	5	8	1	7	-	4	9	9	9	9	11
6	8	9	6	3	11	4	-	7	7	7	11	11
7	9	8	11	8	4	9	7	-	14	14	10	10
8	9	10	11	10	10	9	7	14	-	8	12	4
9	9	10	11	10	12	9	7	14	8	-	4	12
10	9	10	7	8	8	9	11	10	12	4	-	8
11	11	10	13	10	14	11	11	10	4	12	8	-

≥ 60

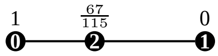
	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	1	3	3	4	4	3	3	3	4	4
1	1	-	2	4	4	3	5	4	4	4	5	3
2	1	2	-	4	4	5	3	4	4	4	3	5
3	3	4	4	-	4	1	1	4	2	2	3	3
4	3	4	4	4	-	3	5	2	4	4	3	5
5	4	3	5	1	3	-	2	5	3	3	4	4
6	4	5	3	1	5	2	-	3	3	3	4	4
7	3	4	4	4	2	5	3	-	4	4	5	3
8	3	4	4	2	4	3	3	4	-	2	3	1
9	3	4	4	2	4	3	3	4	2	-	1	3
10	4	5	3	3	3	4	4	5	3	1	-	2
11	4	3	5	3	5	4	4	3	1	3	2	-

≥ 24

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	2	3	8	9	10	11	12	12	12	13	15
1	2	-	5	10	11	8	13	12	14	14	15	13
2	3	5	-	11	10	13	8	15	15	15	10	18
3	8	10	11	-	11	2	3	12	12	12	11	13
4	9	11	10	11	-	9	14	5	13	15	10	18
5	10	8	13	2	9	-	5	14	12	12	13	15
6	11	13	8	3	14	5	-	9	9	9	14	14
7	12	12	15	12	5	14	9	-	18	18	15	13
8	12	14	15	12	13	12	9	18	-	10	15	5
9	12	14	15	12	15	12	9	18	10	-	5	15
10	13	15	10	11	10	13	14	15	15	5	-	10
11	15	13	18	13	18	15	14	13	5	15	10	-

$\geq 80; (\frac{220}{307}, \frac{87}{307})$

RHS = 92



TT-disjoint at 5, 7

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	3	6	7	7	7	9	7	8	10	9
1	1	-	4	7	6	8	8	10	8	7	11	10
2	3	4	-	9	10	8	10	6	10	11	7	10
3	6	7	9	-	1	7	9	3	7	8	8	7
4	7	6	10	1	-	6	10	4	8	9	9	8
5	7	8	8	7	6	-	4	10	10	11	7	8
6	7	8	10	9	10	4	-	6	10	7	11	10
7	9	10	6	3	4	10	6	-	6	9	9	6
8	7	8	10	7	8	10	10	6	-	9	3	6
9	8	7	11	8	9	11	7	9	9	-	6	3
10	10	11	7	8	9	7	11	9	3	6	-	9
11	9	10	10	7	8	8	10	6	6	3	9	-

≥ 54

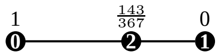
	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	1	3	4	5	6	5	6	7	5	6
1	1	-	2	4	3	6	5	6	7	6	6	7
2	1	2	-	4	5	4	7	4	7	8	4	7
3	3	4	4	-	1	6	5	2	7	6	6	7
4	4	3	5	1	-	5	6	3	6	7	7	6
5	5	6	4	6	5	-	3	8	9	10	6	7
6	6	5	7	5	6	3	-	5	10	7	7	10
7	5	6	4	2	3	8	5	-	5	8	8	5
8	6	7	7	7	6	9	10	5	-	9	3	6
9	7	6	8	6	7	10	7	8	9	-	6	3
10	5	6	4	6	7	6	7	8	3	6	-	9
11	6	7	7	7	6	7	10	5	6	3	9	-

≥ 42

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	2	4	9	11	11	13	13	13	15	15	15
1	2	-	6	11	9	13	13	15	15	13	17	17
2	4	6	-	13	15	11	17	9	17	19	11	17
3	9	11	13	-	2	12	14	4	14	14	14	14
4	11	9	15	2	-	10	16	6	14	16	16	14
5	11	13	11	12	10	-	6	16	18	20	12	14
6	13	13	17	14	16	6	-	10	20	14	18	20
7	13	15	9	4	6	16	10	-	10	16	16	10
8	13	15	17	14	14	18	20	10	-	18	6	12
9	15	13	19	14	16	20	14	16	18	-	12	6
10	15	17	11	14	16	12	18	16	6	12	-	18
11	15	17	17	14	14	14	20	10	12	6	18	-

$\geq 92; (\frac{67}{115}, \frac{48}{115})$

RHS = 94



TT-disjoint at 3, 5

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	4	8	8	9	10	12	12	12	14	16
1	1	-	5	7	9	10	11	11	13	13	15	17
2	4	5	-	12	8	11	6	16	8	10	12	14
3	8	7	12	-	4	15	6	16	10	10	8	10
4	8	9	8	4	-	11	2	12	10	12	12	14
5	9	10	11	15	11	-	9	11	13	5	9	17
6	10	11	6	6	2	9	-	14	12	14	10	12
7	12	11	16	16	12	11	14	-	12	16	16	6
8	12	13	8	10	10	13	12	12	-	8	4	14
9	12	13	10	10	12	5	14	16	8	-	12	14
10	14	15	12	8	12	9	10	16	4	12	-	10
11	16	17	14	10	14	17	12	6	14	14	10	-

≥ 76

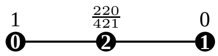
	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	6	11	14	14	16	16	20	22	22	22
1	1	-	7	10	15	15	17	15	21	23	23	23
2	6	7	-	17	12	18	10	22	14	18	18	22
3	11	10	17	-	5	23	7	23	17	15	15	15
4	14	15	12	5	-	18	2	18	16	20	18	18
5	14	15	18	23	18	-	16	16	22	8	14	24
6	16	17	10	7	2	16	-	20	18	22	16	20
7	16	15	22	23	18	16	20	-	16	22	24	8
8	20	21	14	17	16	22	18	16	-	14	8	24
9	22	23	18	15	20	8	22	22	14	-	22	22
10	22	23	18	15	18	14	16	24	8	22	-	16
11	22	23	22	15	18	24	20	8	24	22	16	-

≥ 116

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	5	9	11	11	13	14	16	17	18	19
1	1	-	6	8	12	12	14	13	17	18	19	20
2	5	6	-	14	10	14	8	19	11	14	15	18
3	9	8	14	-	4	18	6	19	13	12	11	12
4	11	12	10	4	-	14	2	15	13	16	15	16
5	11	12	14	18	14	-	12	13	17	6	11	20
6	13	14	8	6	2	12	-	17	15	18	13	16
7	14	13	19	19	15	13	17	-	14	19	20	7
8	16	17	11	13	13	17	15	14	-	11	6	19
9	17	18	14	12	16	6	18	19	11	-	17	18
10	18	19	15	11	15	11	13	20	6	17	-	13
11	19	20	18	12	16	20	16	7	19	18	13	-

$\geq 94; (\frac{143}{367}, \frac{224}{367})$

RHS = 110



TT-disjoint at 5, 7

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	2	5	6	7	9	8	9	9	11	9
1	1	-	3	6	5	8	8	9	10	10	10	10
2	2	3	-	7	8	7	11	6	11	11	13	7
3	5	6	7	-	1	8	8	3	10	10	10	8
4	6	5	8	1	-	7	9	4	9	9	11	9
5	7	8	7	8	7	-	4	11	12	10	14	8
6	9	8	11	8	9	4	-	7	14	14	10	10
7	8	9	6	3	4	11	7	-	7	7	11	11
8	9	10	11	10	9	12	14	7	-	8	12	4
9	9	10	11	10	9	10	14	7	8	-	4	12
10	11	10	13	10	11	14	10	11	12	4	-	8
11	9	10	7	8	9	8	10	11	4	12	8	-

≥ 60

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	3	6	7	7	7	9	7	9	8	10
1	1	-	4	7	6	8	8	10	8	10	7	11
2	3	4	-	9	10	8	10	6	10	10	11	7
3	6	7	9	-	1	7	9	3	7	7	8	8
4	7	6	10	1	-	6	10	4	8	8	9	9
5	7	8	8	7	6	-	4	10	10	8	11	7
6	7	8	10	9	10	4	-	6	10	10	7	11
7	9	10	6	3	4	10	6	-	6	6	9	9
8	7	8	10	7	8	10	10	6	-	6	9	3
9	9	10	10	7	8	8	10	6	6	-	3	9
10	8	7	11	8	9	11	7	9	9	3	-	6
11	10	11	7	8	9	7	11	9	3	9	6	-

≥ 54

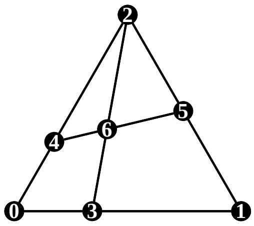
	0	1	2	3	4	5	6	7	8	9	10	11
0	-	2	5	11	13	13	16	16	16	18	19	19
1	2	-	7	13	11	15	16	18	18	20	17	21
2	5	7	-	16	18	14	21	11	21	21	24	14
3	11	13	16	-	2	14	17	5	17	17	18	16
4	13	11	18	2	-	12	19	7	17	17	20	18
5	13	15	14	14	12	-	7	19	21	17	24	14
6	16	16	21	17	19	7	-	12	24	24	17	21
7	16	18	11	5	7	19	12	-	12	12	19	19
8	16	18	21	17	17	21	24	12	-	14	21	7
9	18	20	21	17	17	17	24	12	14	-	7	21
10	19	17	24	18	20	24	17	19	21	7	-	14
11	19	21	14	16	18	14	21	19	7	21	14	-

$\geq 110; (\frac{220}{421}, \frac{201}{421})$

B.2 codim$(F) = 3$

The structure of the data sticks to the conventions described in Section A.2. Since the polytopes $\mathcal{P}_u$ allowed a visualization, we included illustrations of them.

RHS = 96



TT-disjoint at 3, 4

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	4	8	9	8	12	10	12	12	14	16
1	1	-	5	7	10	9	11	11	13	13	15	17
2	4	5	-	12	11	8	16	6	8	10	12	14
3	8	7	12	-	15	4	16	6	10	10	8	10
4	9	10	11	15	-	11	11	9	13	5	9	17
5	8	9	8	4	11	-	12	2	10	12	12	14
6	12	11	16	16	11	12	-	14	12	16	16	6
7	10	11	6	6	9	2	14	-	12	14	10	12
8	12	13	8	10	13	10	12	12	-	8	4	14
9	12	13	10	10	5	12	16	14	8	-	12	14
10	14	15	12	8	9	12	16	10	4	12	-	10
11	16	17	14	10	17	14	6	12	14	14	10	-

≥ 76

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	2	5	5	6	6	6	6	6	6	6
1	1	-	3	4	6	7	5	7	7	7	7	7
2	2	3	-	7	5	4	8	4	4	6	6	6
3	5	4	7	-	8	3	7	3	5	5	5	5
4	5	6	5	8	-	5	5	5	7	3	5	7
5	6	7	4	3	5	-	8	0	6	8	6	6
6	6	5	8	7	5	8	-	8	4	8	6	2
7	6	7	4	3	5	0	8	-	6	8	6	6
8	6	7	4	5	7	6	4	6	-	4	2	6
9	6	7	6	5	3	8	8	8	4	-	6	6
10	6	7	6	5	5	6	6	6	2	6	-	4
11	6	7	6	5	7	6	2	6	6	6	4	-

≥ 38

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	0	2	3	5	5	5	6	7	8	8	8
1	0	-	2	3	5	5	5	6	7	8	8	8
2	2	2	-	5	7	5	7	4	5	6	6	8
3	3	3	5	-	8	2	8	3	6	5	5	5
4	5	5	7	8	-	6	6	7	8	3	5	9
5	5	5	5	2	6	-	6	1	6	7	7	7
6	5	5	7	8	6	6	-	7	6	7	9	3
7	6	6	4	3	7	1	7	-	7	8	6	8
8	7	7	5	6	8	6	6	7	-	5	3	9
9	8	8	6	5	3	7	7	8	5	-	8	8
10	8	8	6	5	5	7	9	6	3	8	-	6
11	8	8	8	5	9	7	3	8	9	8	6	-

≥ 42

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	3	6	7	7	9	8	9	9	10	11
1	1	-	4	5	8	8	8	9	10	10	11	12
2	3	4	-	9	8	6	12	5	6	8	9	10
3	6	5	9	-	11	3	11	4	7	7	6	7
4	7	8	8	11	-	8	8	7	10	4	7	12
5	7	8	6	3	8	-	10	1	8	10	9	10
6	9	8	12	11	8	10	-	11	8	12	11	4
7	8	9	5	4	7	1	11	-	9	11	8	9
8	9	10	6	7	10	8	8	9	-	6	3	10
9	9	10	8	7	4	10	12	11	6	-	9	10
10	10	11	9	6	7	9	11	8	3	9	-	7
11	11	12	10	7	12	10	4	9	10	10	7	-

$\geq 56; (\frac{143}{218}, \frac{75}{218}, 0)$

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	6	11	13	13	17	16	19	20	22	24
1	1	-	7	10	14	14	16	17	20	21	23	25
2	6	7	-	17	17	13	23	10	13	16	18	22
3	11	10	17	-	22	6	24	9	16	15	13	15
4	13	14	17	22	-	16	16	15	20	7	13	25
5	13	14	13	6	16	-	18	3	16	19	19	21
6	17	16	23	24	16	18	-	21	18	23	25	9
7	16	17	10	9	15	3	21	-	19	22	16	20
8	19	20	13	16	20	16	18	19	-	13	7	23
9	20	21	16	15	7	19	23	22	13	-	20	22
10	22	23	18	13	13	19	25	16	7	20	-	16
11	24	25	22	15	25	21	9	20	23	22	16	-

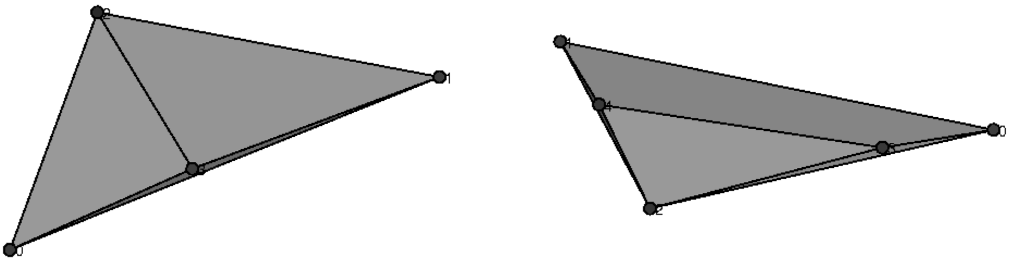
$\geq 116; (\frac{11}{17}, 0, \frac{6}{17})$

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	4	8	9	11	11	12	13	14	14	14
1	1	-	5	7	10	12	10	13	14	15	15	15
2	4	5	-	12	11	9	15	8	9	12	12	14
3	8	7	12	-	15	5	15	6	11	10	10	10
4	9	10	11	15	-	10	10	11	14	5	9	15
5	11	12	9	5	10	-	14	1	12	15	13	13
6	11	10	15	15	10	14	-	15	10	15	15	5
7	12	13	8	6	11	1	15	-	13	16	12	14
8	13	14	9	11	14	12	10	13	-	9	5	15
9	14	15	12	10	5	15	15	16	9	-	14	14
10	14	15	12	10	9	13	15	12	5	14	-	10
11	14	15	14	10	15	13	5	14	15	14	10	-

$\geq 78; (0, \frac{25}{51}, \frac{26}{51})$

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	5	9	11	12	14	14	16	17	18	19
1	1	-	6	8	12	13	13	15	17	18	19	20
2	5	6	-	14	14	11	19	9	11	14	15	18
3	9	8	14	-	18	5	19	7	13	12	11	12
4	11	12	14	18	-	13	13	13	17	6	11	20
5	12	13	11	5	13	-	16	2	14	17	16	17
6	14	13	19	19	13	16	-	18	14	19	20	7
7	14	15	9	7	13	2	18	-	16	19	14	17
8	16	17	11	13	17	14	14	16	-	11	6	19
9	17	18	14	12	6	17	19	19	11	-	17	18
10	18	19	15	11	11	16	20	14	6	17	-	13
11	19	20	18	12	20	17	7	17	19	18	13	-

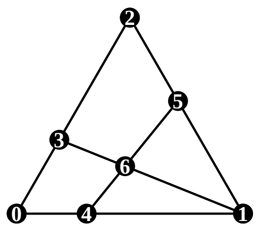
$\geq 96; (\frac{13}{34}, \frac{75}{374}, \frac{78}{187})$



(a) $\mathcal{P}_3$ (b) $\mathcal{P}_4$

Figure B.2.1: The two polytopes $\mathcal{P}_3$ and $\mathcal{P}_4$ defining regular subdivisions of the standard simplex for RHS = 96

RHS = 108



TT-disjoint at 3, 4

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	5	9	11	10	12	14	14	15	17	19
1	1	-	6	8	12	11	13	13	15	16	18	20
2	5	6	-	14	14	9	7	19	9	12	14	16
3	9	8	14	-	18	5	7	19	13	12	10	12
4	11	12	14	18	-	13	11	13	15	6	10	20
5	10	11	9	5	13	-	2	14	12	15	15	17
6	12	13	7	7	11	2	-	16	14	17	13	15
7	14	13	19	19	13	14	16	-	14	19	19	7
8	14	15	9	13	15	12	14	14	-	9	5	17
9	15	16	12	12	6	15	17	19	9	-	14	16
10	17	18	14	10	10	15	13	19	5	14	-	12
11	19	20	16	12	20	17	15	7	17	16	12	-

≥ 90

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	0	2	3	5	5	5	5	7	8	8	8
1	0	-	2	3	5	5	5	5	7	8	8	8
2	2	2	-	5	7	3	3	7	5	6	6	6
3	3	3	5	-	8	2	2	8	6	5	5	5
4	5	5	7	8	-	6	6	6	8	3	5	9
5	5	5	3	2	6	-	0	6	6	7	7	7
6	5	5	3	2	6	0	-	6	6	7	7	7
7	5	5	7	8	6	6	6	-	6	7	9	3
8	7	7	5	6	8	6	6	6	-	5	3	9
9	8	8	6	5	3	7	7	7	5	-	8	8
10	8	8	6	5	5	7	7	9	3	8	-	6
11	8	8	6	5	9	7	7	3	9	8	6	-

≥ 40

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	3	6	7	8	8	8	8	9	9	9
1	1	-	4	5	8	9	9	7	9	10	10	10
2	3	4	-	9	8	5	5	11	5	8	8	8
3	6	5	9	-	11	4	4	10	8	7	7	7
4	7	8	8	11	-	7	7	7	9	4	6	10
5	8	9	5	4	7	-	0	10	8	11	9	9
6	8	9	5	4	7	0	-	10	8	11	9	9
7	8	7	11	10	7	10	10	-	6	11	9	3
8	8	9	5	8	9	8	8	6	-	5	3	9
9	9	10	8	7	4	11	11	11	5	-	8	8
10	9	10	8	7	6	9	9	9	3	8	-	6
11	9	10	8	7	10	9	9	3	9	8	6	-

≥ 52

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	4	7	9	9	10	11	11	12	13	14
1	1	-	5	6	10	10	11	10	12	13	14	15
2	4	5	-	11	11	7	6	15	7	10	11	12
3	7	6	11	-	14	4	5	14	10	9	8	9
4	9	10	11	14	-	10	9	10	12	5	8	15
5	9	10	7	4	10	-	1	12	10	13	12	13
6	10	11	6	5	9	1	-	13	11	14	11	12
7	11	10	15	14	10	12	13	-	10	15	14	5
8	11	12	7	10	12	10	11	10	-	7	4	13
9	12	13	10	9	5	13	14	15	7	-	11	12
10	13	14	11	8	8	12	11	14	4	11	-	9
11	14	15	12	9	15	13	12	5	13	12	9	-

$\geq 70; (\frac{43}{69}, 0, \frac{26}{69})$

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	7	12	15	15	17	19	21	23	25	27
1	1	-	8	11	16	16	18	18	22	24	26	28
2	7	8	-	19	20	12	10	26	14	18	20	22
3	12	11	19	-	25	7	9	27	19	17	15	17
4	15	16	20	25	-	18	16	18	22	8	14	28
5	15	16	12	7	18	-	2	20	18	22	22	24
6	17	18	10	9	16	2	-	22	20	24	20	22
7	19	18	26	27	18	20	22	-	20	26	28	10
8	21	22	14	19	22	18	20	20	-	14	8	26
9	23	24	18	17	8	22	24	26	14	-	22	24
10	25	26	20	15	14	22	20	28	8	22	-	18
11	27	28	22	17	28	24	22	10	26	24	18	-

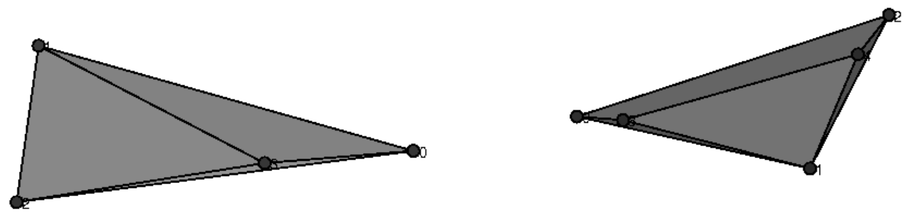
$\geq 128; (\frac{172}{249}, \frac{77}{249}, 0)$

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	5	9	11	13	13	13	15	17	17	17
1	1	-	6	8	12	14	14	12	16	18	18	18
2	5	6	-	14	14	8	8	18	10	14	14	14
3	9	8	14	-	18	6	6	18	14	12	12	12
4	11	12	14	18	-	12	12	12	16	6	10	18
5	13	14	8	6	12	-	0	16	14	18	16	16
6	13	14	8	6	12	0	-	16	14	18	16	16
7	13	12	18	18	12	16	16	-	12	18	18	6
8	15	16	10	14	16	14	14	12	-	10	6	18
9	17	18	14	12	6	18	18	18	10	-	16	16
10	17	18	14	12	10	16	16	18	6	16	-	12
11	17	18	14	12	18	16	16	6	18	16	12	-

$\geq 90; (0, \frac{77}{181}, \frac{104}{181})$

	0	1	2	3	4	5	6	7	8	9	10	11
0	-	1	6	10	13	14	15	16	18	20	21	22
1	1	-	7	9	14	15	16	15	19	21	22	23
2	6	7	-	16	17	10	9	22	12	16	17	18
3	10	9	16	-	21	6	7	22	16	14	13	14
4	13	14	17	21	-	15	14	15	19	7	12	23
5	14	15	10	6	15	-	1	18	16	20	19	20
6	15	16	9	7	14	1	-	19	17	21	18	19
7	16	15	22	22	15	18	19	-	16	22	23	8
8	18	19	12	16	19	16	17	16	-	12	7	22
9	20	21	16	14	7	20	21	22	12	-	19	20
10	21	22	17	13	12	19	18	23	7	19	-	15
11	22	23	18	14	23	20	19	8	22	20	15	-

$\geq 108; (\frac{2}{5}, \frac{77}{215}, \frac{52}{215})$



(a) $\mathcal{P}_3$ (b) $\mathcal{P}_4$

Figure B.2.2: The two polytopes $\mathcal{P}_3$ and $\mathcal{P}_4$ defining regular subdivisions of the standard simplex for RHS = 108

Appendix C

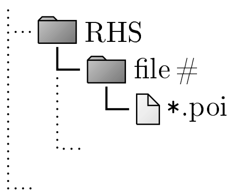
How the data was generated

Several scripts were implemented in order to obtain the data in Appendices A and B. They can be found on the attached CD in the directory `Utils/scripts`. When planning to compute tilting complexes for GTSP(n) with $n > 12$, these scripts may prove to be useful. Hence we give a short step-by-step description of their usage within the process of data generation.

Step 1) Extract the non-NR facets G, whose intersection with STSP(n) has co-dimension $c > 1$, from the database for GTSP(n):

```
./extractFacets <n> <c>
```

This retrieves the information on the facets from the database and copies the corresponding .poi files to the following folder structure created by the script



Step 2) Compute the NR-facets containing $G \cap \text{STSP}(n)$ for all .poi files:

```
./findFacets4RHS <n> <min> <max>
```

This applies the program `findFacets` to all non-NR facets G with a corresponding right hand side within the closed interval [`min`,`max`] and in each case creates a .ieq file that stores the NR-facets containing $G \cap \text{STSP}(n)$. This way it is possible to run several instances simultaneously for pairwise disjoint right hand side ranges, thus allowing to distribute the computation to different servers.

Step 3) Compute the tilting complexes for all $G \cap \text{STSP}(n)$:

```
./tiltingComplex4RHS <n> <min> <max>
```

Aside from the fact that this script applies the program `tiltingComplex` to the .ieq files from the previous step, the usage is identical to `findFacets4RHS`. The additional LaTeX output generated by `tiltingComplex` is appended to a specified .tex file, in which the data for different right hand sides is gathered in separate subsections.

List of Algorithms

List of Figures

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List of Symbols

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