Separation for the Max-Cut Problem using Graph Contraction

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University of Heidelberg

Joint work with:
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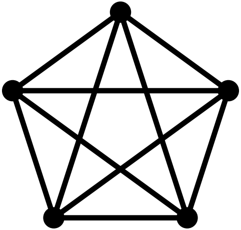
Gerhard Reinelt (University of Heidelberg)
Giovanni Rinaldi (IASI, Rome)

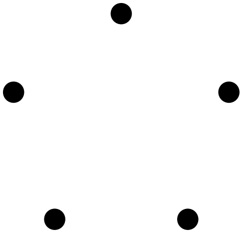
HGS Annual Colloquium
Heidelberg, November 19, 2010

Max-Cut Problem



Given: $G = (V, E, \mathbf{w})$

Max-Cut Problem



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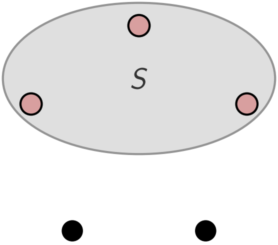

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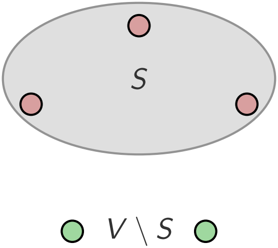

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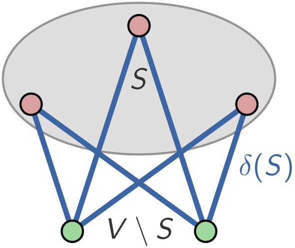

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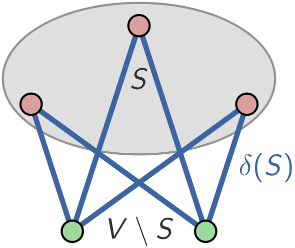
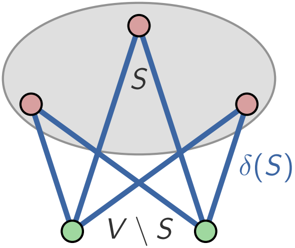


Given: $G = (V, E, \mathbf{w})$

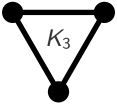
Find: $\max_{S \subseteq V} \mathbf{w}\big(\delta(S)\big)$

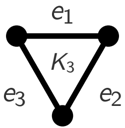
Max-Cut Problem



Given:	$G = (V, E, \mathbf{w})$
Find:	$\max_{S \subseteq V} \mathbf{w}\big(\delta(S)\big)$
Appl.:	• Statistical Physics • VLSI Chip Design

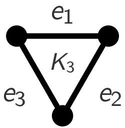
Cut Polytope





$\mathbf{e} \in \{0, 1\}^3$

Cut Polytope



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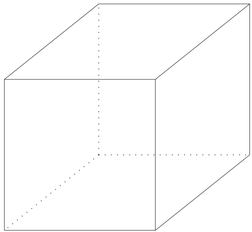

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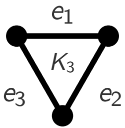

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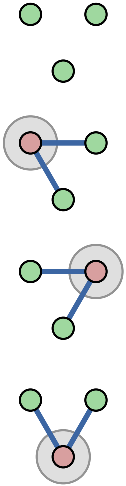

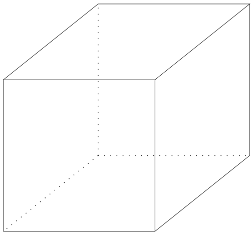

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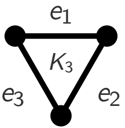

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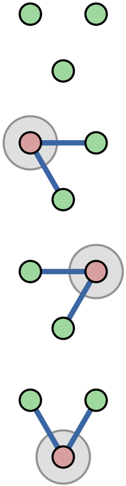
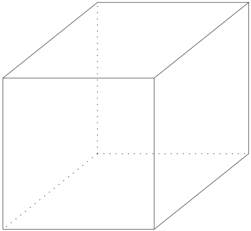


$(0, 0, 0)$

$(1, 0, 1)$

$(1, 1, 0)$

$(0, 1, 1)$



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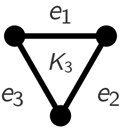

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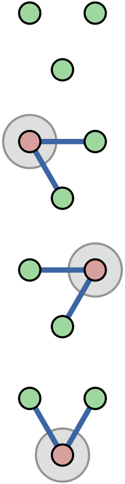

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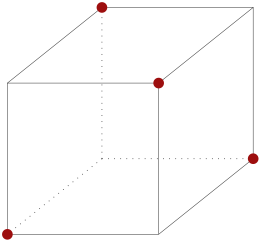
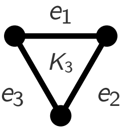
$(1, 1, 0)$

$(0, 1, 1)$





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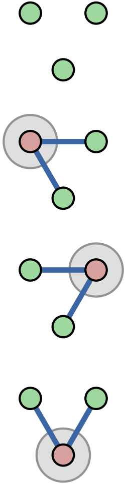

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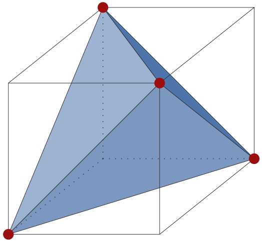
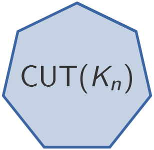
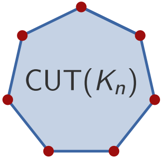


$\mathrm{CUT}(K_3)$


CUT(K_n)

Separation for Max-Cut on Complete Graphs



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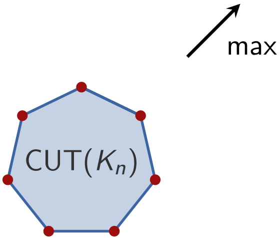

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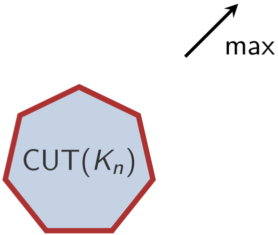

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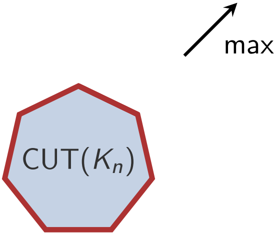
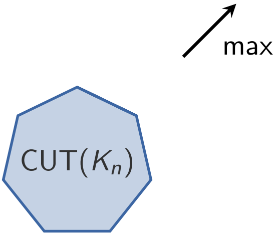


E. g. CUT(K_9) has 256 vertices
but over $1.2 \cdot 10^{13}$ facets!

Separation for Max-Cut on Complete Graphs



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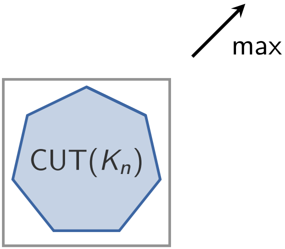

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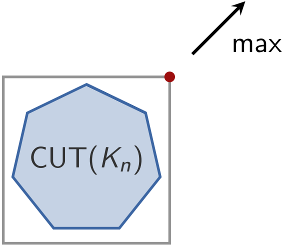

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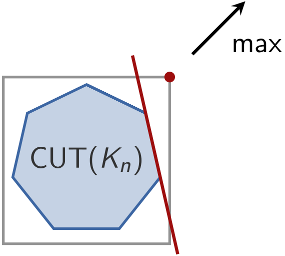

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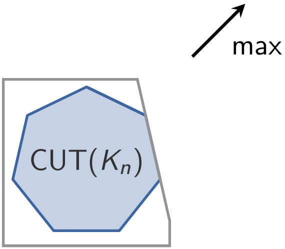

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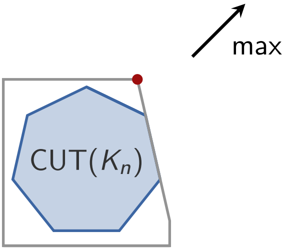
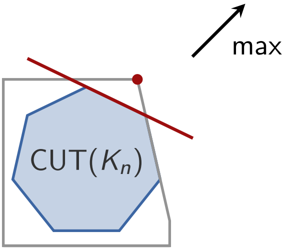
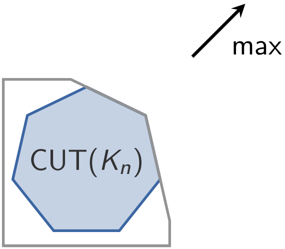



max
CUT(K_n)

Separation for Max-Cut on Complete Graphs



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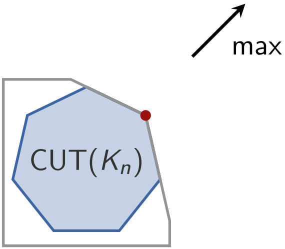

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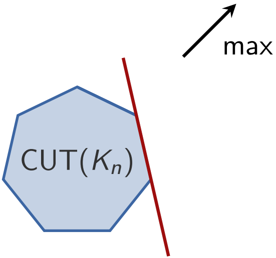

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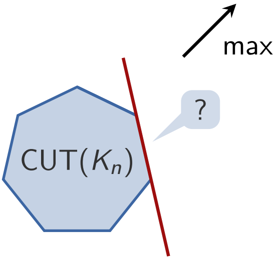

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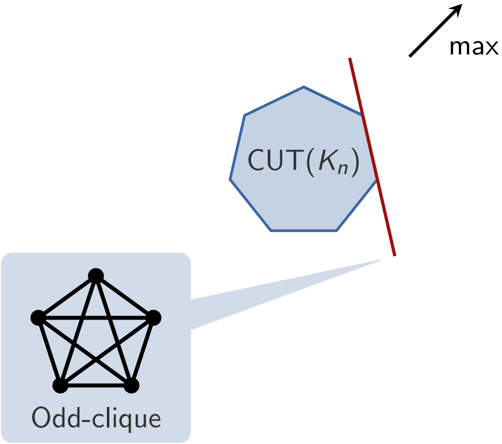

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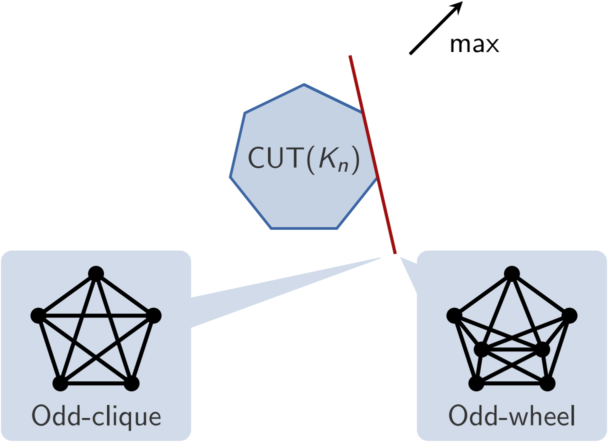
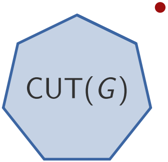


Separation for Max-Cut on Sparse Graphs



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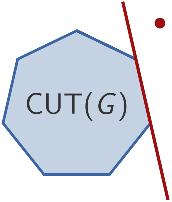

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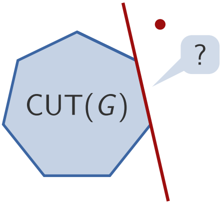

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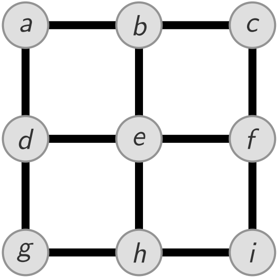

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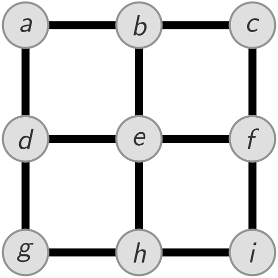
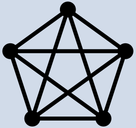
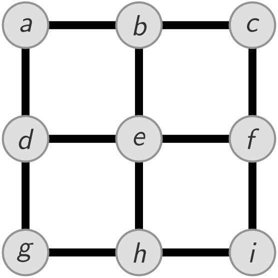
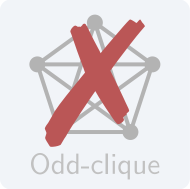




Odd-clique

Separation for Max-Cut on Sparse Graphs





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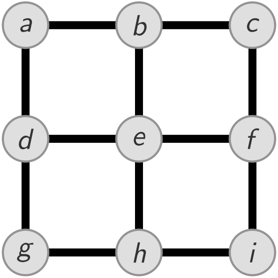

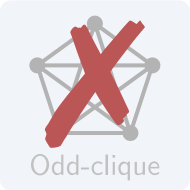

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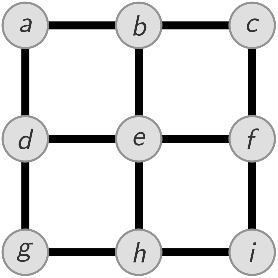

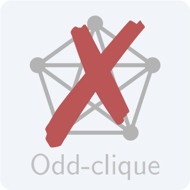

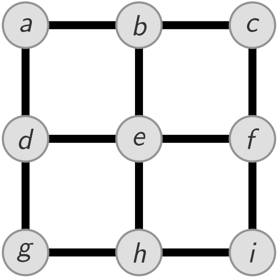
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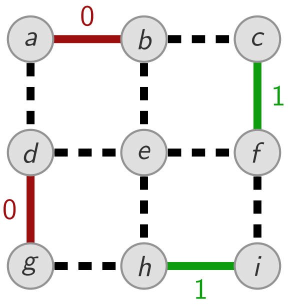
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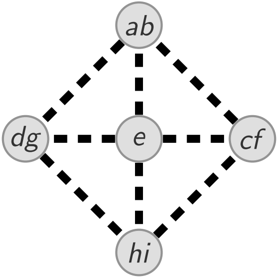
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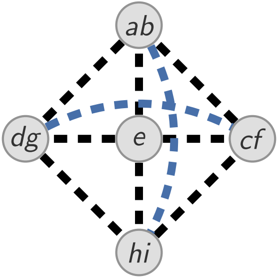

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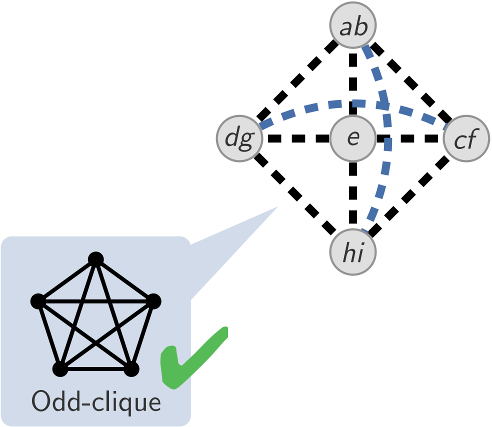
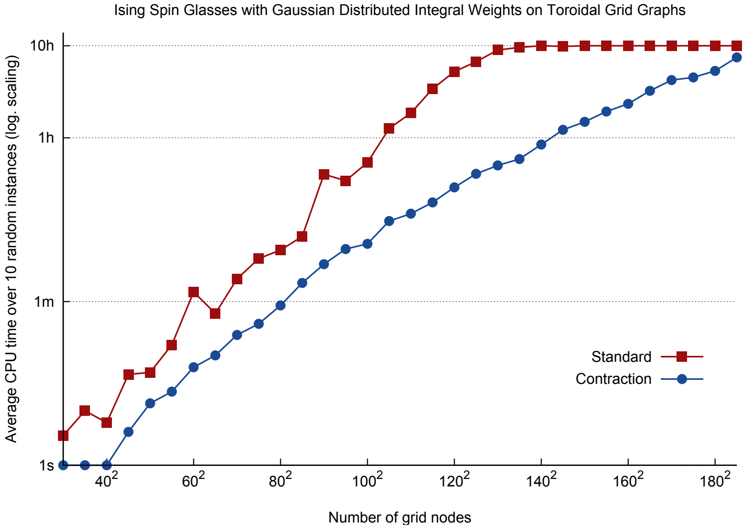


Computational Results for Spin Glasses



[Intel Xeon 2.8 GHz, 8GB shared RAM. **CPU time per instance limited to 10h.**]